INVESTIGATION INTO THE SIGNIFICANCE OF STRENGTH CHARACTERISTICS IN INELASTIC TORSIONAL SEISMIC RESPONSE

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SUMMARY

Depending on the element stiffness to strength relationship, elastically identical structural systems can have different strength distributions. The seismic response of elastically identical structures but with different strength characteristics were compared using time history dynamic analysis. For these torsionally susceptible single storey structures the maximum inelastic displacement and ductility demands were obtained for the lateral load resisting elements. A significant difference in the response from one strength distribution to another was observed for all eccentricities and range of system properties. Systems that were balanced in strength with respect to the centre of mass, but unbalanced in stiffness, responded in both translation as well as rotation even in the inelastic range. Elastically balanced systems, those with no stiffness eccentricity but unbalanced in strength distribution, were also found to have a significant torsional response, resulting in increased ductility demands on the stiffer and stronger element. For all of the prescribed design ductilities considered, strength eccentricity was found to influence the system response. The stiffness as well as the strength distribution were shown to have a significant impact on the overall system torsional response and both should be considered in the analysis of torsionally susceptible inelastic structures.

INTRODUCTION

Torsional behaviour of buildings when subject to lateral seismic ground motion continues to be observed as one of the contributing factors in the damage, and at times collapse, of structures during earthquakes. To mitigate the risk of additional demands on the lateral resisting system due to torsion, additional provisions are specified in modern building codes. In areas of high seismicity, structures are generally designed for ductile response. However, the torsional design provisions, as the lateral design provisions, are mainly based on elastic analyses. Previous studies in torsional inelastic seismic response have focused on verifying the adequacy of current code provisions through numerical modelling [Chandler et. al. 1997; Humar & Kumar 1998; Tso & Zhu 1992; Wong & Tso 1994]. In most of these studies, the strength of the lateral resisting elements was based on design calculations, such as the minimum requirements of equivalent static or response spectrum methods. Thus, the strength of the elements became a function of a design procedure and was based on the elastic properties of the system. However, as was pointed out by Paulay [1997, 1998 a,b,c], the calculated design strength of a structure is related to its stiffness, although it is not directly proportional to it. The relationship between an element's stiffness and its strength is dependant upon the type of lateral resisting system, member dimensions and element material properties and not on calculations from prescribed design procedures. As a result, different stiffness to strength relationships in the elements are possible and it is these properties that control the strength distribution in the lateral resisting system.

Traditionally, research into the torsional response of structures used the elastic stiffness distribution as means of introducing irregularity and hence induce torsional response. Other studies [Paulay 1997, 1998 a,b,c] have focused on the strength distribution alone as the controlling parameter in the torsional response of ductile structures because the maximum demands occur when the structure is in the inelastic part of the response. In an

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effort to understand the inelastic torsional behaviour, the objective of this paper is to determine the influence of the strength distribution on the inelastic response by using different stiffness-to-strength relationships for torsionally susceptible structural systems. A number of simple relationships between an element stiffness and strength are outlined. The seismic response of a single storey model of varying system parameters to encompass a wide range of structural systems is analysed using time history dynamic analysis. The results presented here are part of a collaborative research project between The University of British Columbia and The University of Auckland. The findings of the numerical investigation will be used as basis for preparing an experimental model for shake table testing.

STIFFNESS TO STRENGTH RELATIONSHIP

To illustrate one possible variance of strength to stiffness relationships, consider an elasto-plastic rectangular element "*i*" of elastic modulus E, yield strength f_y, section width b_i, depth h_i and element length L_i. The lateral bending stiffness and strength of each element can be defined proportional to these properties for fix-fix or fix-pin boundary conditions as $k_i \propto E b_i h_i^3 / L_i^3$ and $V_i \propto f_y b_i h_i^2 / L_i$. Assuming consistent material properties for all elements, the stiffness and strength are related based on the geometric properties only. In a structural system, two elements in any one principal direction are sufficient to define the centre of rigidity CR and the centre of strength CV, leading to the lateral load resisting element arrangement of elements 1 and 2 shown in the simple one storey structure in Figure 1.

By varying the element dimensions alone, different locations of CV can exist for the same location of CR. For example, with the above mentioned proportionalities in stiffness and strength, CV coincides with CR for any ratio b_1 / b_2, assuming $h_1 / h_2 = L_2 / L_1 = 1.0$. This relationship is referred to as stiffness-to-strength distribution A. On the other hand, CV will not coincide with CR for any ratio L_2 / L_1 where $h_1 / h_2 = b_1 / b_2 = 1.0$ except for $L_2 / L_1 = 1.0$ and is referred to as strength distribution B. Also, CV will not coincide with CR for any ratio of h_1 / h_2 where $b_1 / b_2 = L_2 / L_1 = 1.0$ except for $h_1 / h_2 = 1.0$ and is referred to as strength distribution C. Hence, each location of CR has a unique location of CV depending on the stiffness-to-strength relationship used. Given the location of CM, the stiffness eccentricity e_R and strength eccentricity e_V can be calculated. Figure 2 illustrates the resulting relationship between e_R and e_V for the above examples of stiffness-to-strength distributions for two locations of CM. In these examples, the ratio of only one of the dimensions was varied from 0 to 1.0, while the other ratios remained unity. A positive value of stiffness or strength eccentricity implies that CR or CV respectively is located in the positive x direction from CM, i.e. between CM and element 2. In reality buildings are likely to have more complex relationships.

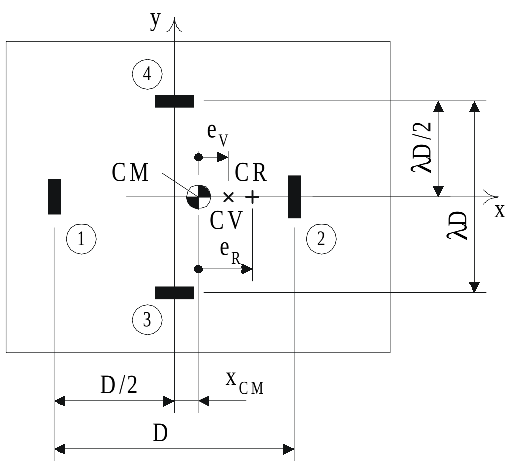

Figure 1: Model layout in plan view

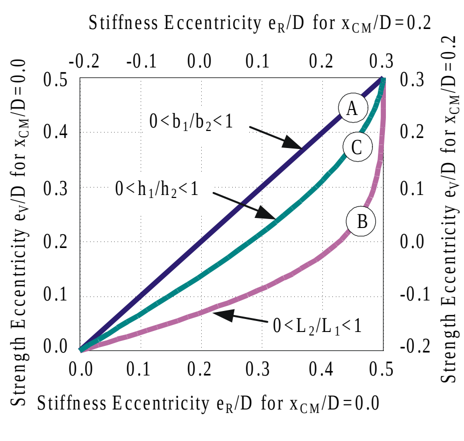

Figure 2: Stiffness-to-strength relationships

STRUCTURAL MODEL

The structural model used for the analysis consisted of a rigid rectangular diaphragm connected to the lateral load resisting elements. Since the system response is not sensitive to the number of lateral elements along the direction of ground motion [Goel & Chopra 1990], two lateral load resisting elements were used in each of the two principal directions. They were symmetrically placed about the reference coordinate axis at a distance D and λ D as shown in Figure 1, where λ represented the ratio of the spacing between the elements in the two principal directions. Three element arrangements were considered, λ = 0, 0.5 and 1.0. The elements were idealised to have a bilinear force deformation relationship with 3% post yield stiffness. The out-of-plane stiffness and strength of each element was assumed to be negligible. Hence, elements 1 and 2 provided lateral stiffness and strength in the y-direction and elements 3 and 4 in the x-direction only.

The total system mass M was assumed to be concentrated at CM. In general, dead and live load distributions often result in asymmetric CM locations with respect to the lateral resisting elements. Hence, the position of CM was defined with respect to the coordinate axis by the distance x_{CM}. Due to the complexity and the number of parameters already considered, the mass was placed symmetrically between the elements in the y-direction. Two positions of CM were investigated in this study, $x_{CM} / D = 0$ and 0.2.

The two stiffness-to-strength relationships identified as A and B in Figure 2 were considered. For strength distribution A, the location of CR coincided with CV in all cases. For distribution B and $x_{CM} / D = 0$, CV was significantly closer to CM than CR. A number of relationships between e_R and e_V of note exist for $x_{CM} / D = 0.2$ and strength distribution *B*. An elastically balanced system, i.e. where $e_R / D = 0$, can have an irregularity in the inelastic range, namely a significant strength eccentricity. On the other hand, systems with no strength irregularity, i.e. where $e_V / D = 0$, can have significant stiffness eccentricity. Eccentricity values between these two latter cases result in stiffness and strength eccentricities of opposite sign. Hence, although CR is located between CM and element 2, CV is between CM and element 1. Lastly for $x_{CM} / D = 0.2$ and strength distribution B, negative stiffness eccentricities are associated with larger values of negative strength eccentricity, i.e., $|e_R| < |e_V|$.

Element Stiffness

The natural period of vibration of systems where CM coincided with CR resulted in modes of vibration that are purely translational or purely rotational. These periods are referred to as nominal periods of vibration T_{nX}, T_{nY} and T_θ for the x direction, y direction and rotational elastic modes of vibration respectively. In this study, the nominal translational period of vibration was assumed identical in both principal directions, i.e. $T_n = T_{nX} = T_{nY}$. A wide spectrum of nominal lateral periods was considered, T_n = 0.2, 0.4, 0.8, 1.4 and 2.0 seconds. The system elastic lateral stiffness *K* was then computed using the predetermined nominal lateral period of vibration and mass of the system. The total stiffness was redistributed between elements 1 and 2 to achieve the desired stiffness eccentricity e_R in the y-direction while being evenly distributed between elements 3 and 4 in the x-direction. As a result, the system was asymmetric in only one direction.

A recent study by Humar & Kumar [1998] emphasised that the ratio of translational to rotational period of vibration calculated as $\Omega = T_n / T_\theta = \sqrt{K_\theta / (K \cdot r^2)}$ is the most important parameter governing the torsional response, where K_θ is the torsional stiffness of the lateral elements about CR and r is the radius of gyration about CM of the system. Numerically convenient, varying the radius of gyration can represent structures of different plan geometry or mass distribution. The rotational period was therefore varied by increasing and decreasing r to achieve three values of $\Omega = 1/\sqrt{2}$, 1.0 and $\sqrt{2}$, corresponding to systems ranging from nominally torsionally dominant to nominally translationally dominant.

Ground motion and Element Strength

Ground motions obtained during the 1971 San Fernando earthquake as recorded at 234 Figueroa Street in Los Angeles, California, were used for the non-linear time history dynamic analysis. Two horizontal components, N37E and S53E, were applied in the analyses for the y-direction and x-direction ground acceleration input respectively. This set of acceleration values was chosen based on similarities of its elastic response spectrum to the Newmark and Hall design spectrum [Newmark & Hall 1982], which is often used as basis in modern building codes. The peak ground accelerations recorded 41 km from the epicentre on stiff soil is nearly identical in magnitude for the two components, approximately 0.2g. Furthermore, this earthquake record has been used as one of a number of records in previous studies on torsional response of asymmetric structures [Humar & Kumar 1998, Tso & Zhu 1994, Wong & Tso 1992].

For the purpose of investigating inelastically responding structural systems of varying degrees of inelasticity, the design ductility μ_d was varied as $\mu_d = 2$, 4 and 6. Considering that only one earthquake record was used and in an effort to reduce the uncertainty in the correlation between any single design spectrum and the earthquake ground input, the ductility response spectra of the earthquake itself were used for the design of the system considered. These ductility spectra, as plotted in Figure 3, were obtained from an inelastic single degree of freedom SDF response with 5% damping and 3% strain hardening. Based on the design ductility and nominal period of vibration, the spectral acceleration S_a and consequently the base shear capacity V_b were determined for each principal direction as $V_b = S_a \cdot M$. The base shear was then numerically distributed to the lateral load resisting elements based on the element stiffness to reflect the two different stiffness-to-strength relationships A and B shown in Figure 2. A range of stiffness eccentricities $-0.2 \le e_R / D \le 0.2$ and their corresponding strength eccentricities were considered. This resulted in a comparison of structural systems that are elastically identical but with different strength distributions, i.e. different locations of CV.

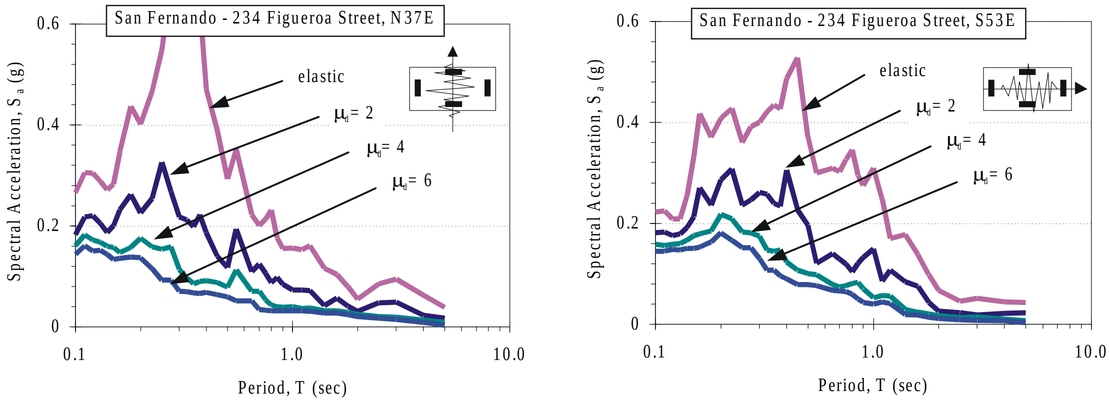

Figure 3: Earthquake response spectra in y-direction (N37E) and x-direction (S53E)

Since the objective of this study was to identify the influence of strength characteristics and not conformance to design code procedures, no additional or accidental eccentricity was incorporated in establishing the design strengths of the lateral load resisting elements.

SEISMIC RESPONSE

Time history non-linear dynamic analysis was used to calculate the response to the bi-directional earthquake ground acceleration. The maximum displacement $\Delta_{\max}$ of each element and the maximum rotation θ of the diaphragm were recorded. The ductility demand μ was calculated for each element by comparing the maximum displacement to the yield displacement Δ_y as $\mu = \Delta_{\max} / \Delta_y$. Although only one earthquake record was used, a number of distinctive behaviours and relevant trends were observed [Dusicka 2000]. Examples of the responses are shown for illustration and trends observed in the response of systems of other system parameters reported.

To quantify the torsional response, the element displacement as well as ductility demands were normalised as a percentage of the response of a SDF system, which had the same total base shear capacity V_b and total stiffness *K*. This was achieved by comparing the element demand to the SDF displacement Δ_{SDF} or ductility demand μ_{SDF} as appropriate. In effect, the response of a SDF system represents the response of a one storey structure where CM coincides with CR as well as CV and therefore responds only in translation. The difference between the observed demands and SDF demands represent an increase or decrease as compared to the response in simple translation.

Response of Different Stiffness to Strength Distributions

Examples of displacement and ductility demands are shown in Figure 4 for the two locations of CM. For strength distribution B, $x_{CM}/D=0$, the displacement demand on the element which recorded an increase in demand was less than on strength distribution A for all values of the eccentricity. This result is further emphasised when looking at the ductility demands. This difference from displacement to ductility response is due to the different yield displacements of the elements of same elastic properties, as a consequence of different stiffness-to-strength relationships. For both locations of CM the plots for the displacement and ductility demands are identical for strength distribution A because the element strength is directly proportional to its stiffness. However, for strength distribution B, the ductility response is different from the displacement response as it has a different stffness-to-strength relationship. In the case of $x_{CM}/D=0$ and positive values of e_R, element 2 is the stiffer as well as the stronger element and has a smaller yield displacement than element 1, i.e. $\Delta_{y2}<\Delta_{y1}$. Despite a decrease in displacement demand in element 2 as compared to the SDF system, an increase in ductility demand was calculated. Conversely, a large increase in displacement demand on element 1 as compared to the SDF system resulted in a smaller increase in ductility demand. Interestingly, if the response was forced to have no rotation at all and element 1 reached ductility μ_1, then element 2 would reach ductility $\mu_2=\mu_1\cdot\Delta_{y1}/\Delta_{y2}$, i.e. larger ductility demand than element 1 despite a simply translational response.

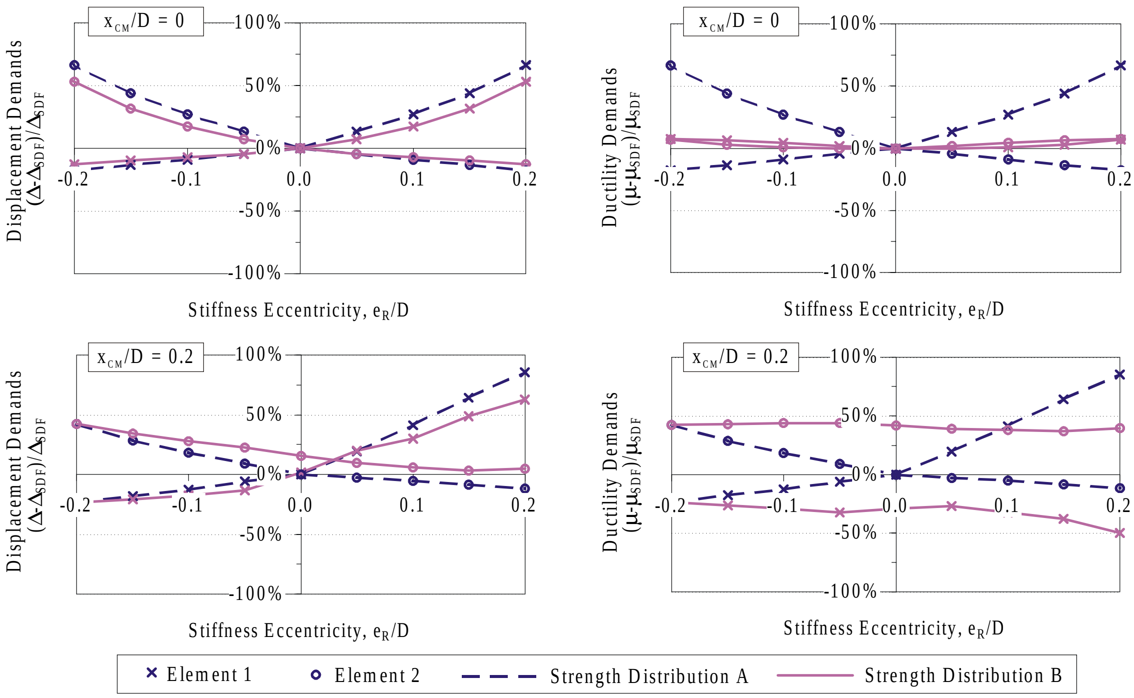

Figure 4: Normalised displacement and ductility demands for $T_n=0.8$ sec, $\Omega=1/\sqrt{2}$, $\lambda=0.5$ and $\mu_d=6$

From the general trends obtained from additional analyses, which are not included here due to space limitations, an increase in stiffness or strength eccentricity resulted in an increase in displacement and ductility demand on one or both of the elements. For the case of $x_{CM}/D=0$, the demands of elements 1 and 2 are mirror images about the y-axis due to symmetry in both stiffness and strength distributions. For these systems, the ductility demands of strength distribution B were usually closer to the SDF response than systems of strength distribution A. For the case of $x_{CM}/D=0.2$, symmetry in response about the y-axis was not observed. The approximately constant nature of ductility demand on strength distribution B of systems where $x_{CM}/D=0.2$, was observed for other values of T_n, Ω, λ and μ_d, although not always to the same degree of consistency. This indicates that an optimal value of stiffness and strength eccentricity may exist, whereby the ductility demand on the elements is less affected by the stiffness irregularity in the system.

For a direct comparison between the two stiffness-to-strength distributions, the difference between the ultimate demands for strength distribution A and B on each of the elements was calculated. Figure 5 illustrates this demand difference of the system considered in Figure 4. In addition this figure also includes responses of elastically identical systems but of different design ductility considerations, i.e. varying μ_d. It is evident that a significant difference exists between elastically identical systems but of different strength distributions or different design base shear capacities. For the relatively small irregularities, the difference in response between the two strength distributions can easily reach 50% of the demand on the SDF system.

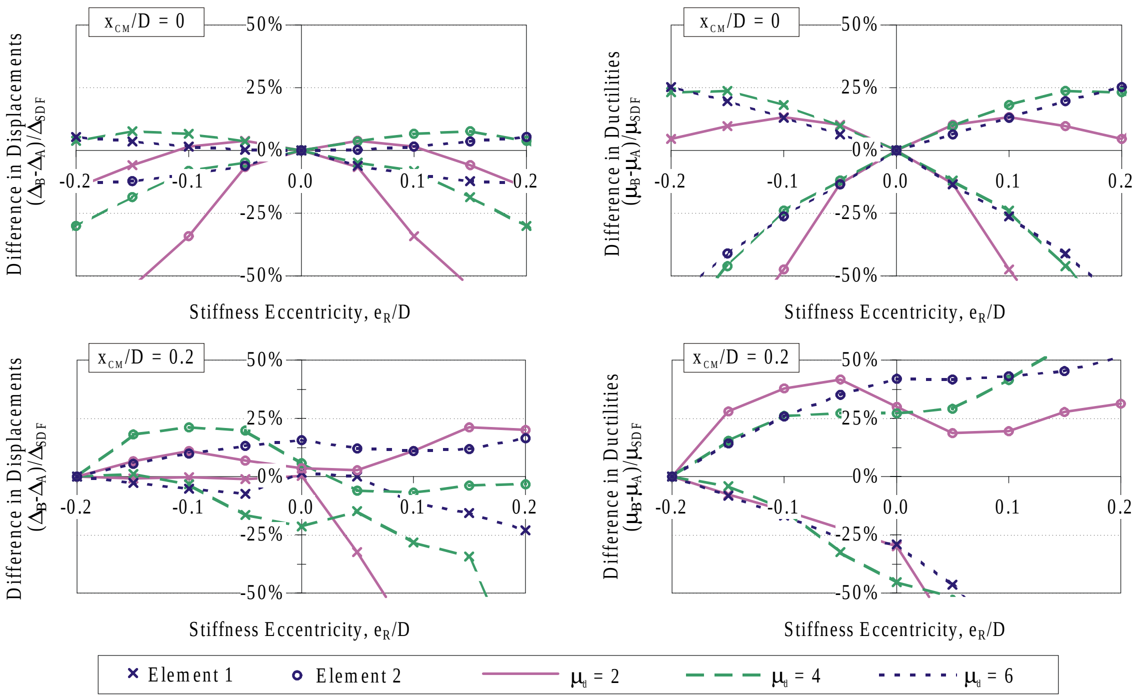

Figure 5: Normalised demand difference between distribution A and B for $T_n = 0.8$ sec, $\Omega = 1/\sqrt{2}$ and $\lambda = 0.5$

For all of the values shown in Figure 5 as well as all the other systems considered, both element displacement as well as ductility difference was higher for the stiffer and stronger of the two elements in any one system. That is element 1 for the negative values of e_R and $x_{CM}/D = 0$ and element 2 for all of the other remaining systems. Furthermore, when considering ductility demand alone, the stiffer and stronger element was observed to generally have positive difference in demand. This indicates a higher ductility demand on the strong element for the stiffness-to-strength distribution B than A. The opposite was observed for the more flexible and weaker element.

The source of the torsional response originates from the irregularity between elements 1 and 2. Due to the resulting rotations of the rigid diaphragm the demand on elements 3 and 4 are also affected. The degree of increase is under investigation, but the increase in the demand becomes more significant with $\lambda = 1.0$, most likely due to geometry considerations.

Influence of Design Ductility

For some systems in Figure 5, the difference in response is greater for lower values of design ductility. Although not a consistent trend, it does illustrate that a significant difference due to inelastic response was also observed for systems of high strength and therefore limited ductility response. The observed demands for these low design ductility capacity systems was observed to have more scatter in the response as a function of the increasing stiffness eccentricity than higher ductility values. The interaction of elastic and inelastic contributions to the overall torsional response can be one of the causes. Nonetheless, one can expect the difference in the response from one strength distribution to another to diminish as the systems approach purely elastic behaviour. This transition is expected to occur at much smaller design ductility values than considered here, i.e. $\mu_d << 2$. Systems with design ductility much greater than 6 are expected to have low contribution to the torsional response from stiffness eccentricity as the response would be mostly inelastic. However, the higher values of design ductilities used in this study, i.e. $\mu_d = 6$, which reflect some of the maximum values used in current building codes, were also found to have a significant contribution due to elastic stiffness eccentricity.

Stiffness and Strength Eccentricity Contributions

While responding elastically, systems with no stiffness eccentricity $e_R = 0$, respond in pure translation. In the inelastic response however, rotation of the diaphragm does occur in these systems when the elements yield at different displacements, i.e. when $e_R = 0$ but $e_V \neq 0$. This was observed for the case where $x_{CM} / D = 0.2$ and element stiffness-to-strength relationship B. The resulting strength eccentricity $e_V / D = -0.13$ introduced irregularity in the inelastic range and therefore resulted in a combination of translational and rotational response once the elements yielded. It is the difference in the location of the force resultant with respect to CM in the inelastic range, as stressed by Paulay [1997, 1998 a,b,c], which causes the rotation of the diaphragm. The resulting combination of translation and rotation in the response induced additional demands on the elements and reduced the demand on others.

Figure 6 illustrates the displacement and ductility demands on elements 1 and 2 caused by the strength eccentricity alone for a range of nominal natural periods of vibration and ratios of nominal translational to rotational periods. The stiffer and stronger element, i.e. element 2 for $x_{CM} / D = 0.2$, was found to generally have a larger displacement demand while the less stiff and weaker element had a smaller demand as compared to the response of a SDF system. The increase in displacement is highlighted by the associated ductility demand increase, which in the example shown can be more than 40% of the SDF response.

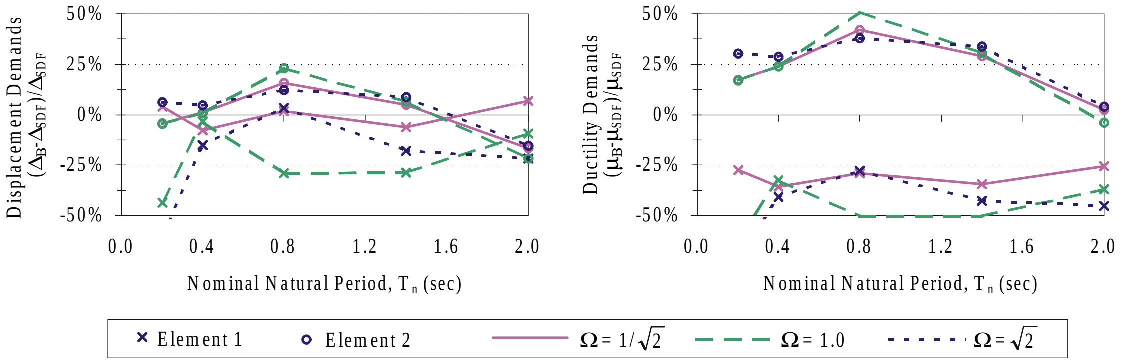

Figure 6: Demands on elastically balanced systems $e_R / D = 0$, $x_{CM} / D = 0.2$, $\lambda = 0.5$, $\mu = 6$

A structural system where $x_{CM} / D = 0.2$, $e_R / D = 0.2$ and element stiffness to strength relationship B has a very small strength eccentricity, $e_V / D = -0.02$. Due to the insignificant strength eccentricity, no additional rotations were introduced while both elements have yielded. Nonetheless, this structural system had a rotational component in the response from the start of the ground motion and due to rotational momentum, the motion continued into the inelastic range. The maximum demands occurred in the inelastic part of the response, but it is clear that the strength eccentricity alone is not the sole contributor to the torsional response in ductile structures.

CONCLUSIONS

Two variations of element stiffness-to-strength relationships were analysed in single storey systems using time history dynamic analysis for one set of bi-directional earthquake ground motions. A wide range of torsionally susceptible systems were considered and a significant difference in response for the two strength distributions was observed. In the two cases considered, stiffness distribution B was found to have a larger response difference as compared to A for the stiffer and stronger element. Systems that are perfectly balanced elastically can still have a significant torsional response due to strength irregularities. Yet, strength eccentricity alone was not the sole contributor to the torsional response because systems that have no strength eccentricity but do have stiffness eccentricity also showed significant torsional response. It was found that both the stiffness as well as the strength eccentricities contribute to the torsional response of ductile structures. These conclusions are not limited to systems designed to specific level of design ductility for the range considered. Therefore, in order to investigate the torsional response of ductile structural systems, both stiffness and strength irregularities should be considered and consequently previous studies on inelastic torsional response may have to be revisited.

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REFERENCES

Chandler, A.M., Correnza, J.C. and Hutchinson G.L (1997). Inelastic Response of Code-designed Eccentric Structures Subject to Bi-directional Loading, Structural Engineering and Mechanics, Vol. 5, No. 1, 51-58.

Dusicka, P. (2000) Numerical and Experimental Investigation of Inelastic Torsional Response of Single Storey Structural Systems, Masters Thesis, The University of British Columbia (in preparation).

Goel, R. K. and Chopra, A. K. (1990). Inelastic Seismic Response of One-Storey, Asymmetric-plan Systems: Effects of Stiffness and Strength Distribution, Earthquake Engineering and Structural Dynamics, Vol. 19, 949-970.

Goel, R. K. and Chopra, A. K. (1991). Inelastic Seismic Response of One-Storey, Asymmetric-Plan Systems: Effects of System Parameters and Yielding, Earthquake Engineering and Structural Dynamics, Vol. 20, 201-222.

Humar, J.L. and Kumar P. (1998). Torsional Motion of Buildings During Earthquakes: I. Elastic Response, II Inelastic Response, Canadian Journal of Civil Engineering, Vol. 25, 898-934.

Newmark, N. M. and Hall, W. J. (1982), Earthquake Spectra and Design, Earthquake Engineering Research Institute.

Paulay, T. (1997). A behaviour-based Design Approach to Earthquake-induced Torsion in Ductile Buildings, Seismic Design Methodologies for the Next Generation of Codes, Fajfar & Krawinkler, Balkema, Rotterdam.

Paulay, T. (1998a). A Simple Seismic Design Strategy Based on Displacement and Ductility Compatibility, Asia-Pacific Workshop on Seismic Design & Retrofit of Structures, Chinese Taipei, 207-221.

Paulay, T. (1998b). Mechanism in Ductile Building Systems as Affected by Torsion, New Zealand National Society for Earthquake Engineering Conference, Wairakei, Taupo, March 1998, 111-118.

Paulay, T. (1998c). Torsional Mechamisms in Ductile Building Systems, Earthquake Engineering and Structural Dynamics, Vol. 27, 1101-1121.

Tso, W. K. and Zhu, T. J. (1992). Design of Torsionally Unbalanced Structural Systems Based on Code Provisions I: Ductility Demand, Earthquake Engineering and Structural Dynamics, Vol. 21, 609-627.

Wong, C. M. and Tso, W. K. (1994). Inelastic Seismic Response of Torsionally Unbalanced Systems Designed Using Elastic Dynamic Analysis, Earthquake Engineering and Structural Dynamics, Vol. 23, 777-798.