THE SIGNIFICANCE OF THE DIRECTION OF GROUND MOTION ON THE STRUCTURAL RESPONSE

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SUMMARY

This paper aims to develop an improved understanding of the critical response of structures to multicomponent seismic motion characterized by three uncorrelated components that are defined along its principal axes: two horizontal and the vertical component. The critical response is defined as the largest value of response for all possible incident angles of the horizontal components with respect to the structural axes. An explicit formula has been derived to calculate the critical response. The ratio r_{cr}/r_{srss} between the critical and the SRSS response --corresponding to the principal horizontal components of ground acceleration applied along the structure axes—is bounded by 1 and $\sqrt{(2/1+\gamma^2)}$, where γ is the spectrum intensity ratio for the two principal horizontal components. This implies that the critical response never exceeds $\sqrt{2}$ times the result of the SRSS analysis, and this ratio is about 1.13 for typical values of γ, say 0.75. The upperbound of r_{cr}/r_{srss} can be reached by axial forces in columns of symmetric-plan buildings or can be approximated by lateral displacements in elements of unsymmetrical buildings.

INTRODUCTION

Translational ground motion is decomposed usually into three orthogonal components: two in the horizontal plane and one in the vertical direction. When defined along its principal axes, the ground motion components are uncorrelated. These principal axes are oriented such that the major axis is horizontal and directed toward the epicenter of the earthquake and the minor axis is vertical (Penzien and Watabe, 1975). The components of the ground motion along any other orthogonal system of axes are obviously correlated. Because the location of the epicenter is not known, it is necessary to determine the structural response as a function of the incident angle (the angle between the principal axes of ground motion and the reference axes of the structure) and design for the largest or critical response. To determine this response, the CQC3 rule has been developed (Menun and Der Kiureghian, 1998). Because the CQC3 equation provides a formula for determining the critical angles, it is not necessary to determine the response for various values of the incident angle. The CQC3 equation, evaluated numerically for these critical angles, provides the critical response. This paper aims to: (1) develop an explicit formula for the critical response; (2) present an upper bound for the critical response; and (3) identify the ground motion and system parameters that influence the critical response and the variation of the response with the incident angle.

1. CRITICAL RESPONSE OF STRUCTURES

The excitation is defined in terms of spectra associated with the principal (uncorrelated) directions of the translational components of ground motion, which are oriented along the two horizontal axes 1 and 2 and the vertical axis z, as shown in Fig. 1. The reference axes of the structure are x, y and z. The angle θ denotes the orientation of axis 1 relative to axis x. The spectra are denoted as $A(T_n)$ for the major principal axis 1, $\gamma A(T_n)$ for the intermediate principal axis 2, where $\gamma \leq 1$, and $A_z(T_n)$ for the minor principal axis z; T_n is the vibration period

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of a SDOF system. Accounting for the correlation among ground motion components, the mean peak total response $r(\theta)$ is given by the CQC3 rule (Smeby and Der Kiureghian, 1985; López and Torres, 1997; Menun and Der Kiureghian, 1998):

$$r(\theta) = \left\{ \left[r_x^2 + (\gamma r_y)^2 \right] \cos^2\theta + \left[(\gamma r_x)^2 + r_y^2 \right] \sin^2\theta + 2(1-\gamma^2) r_{xy} \sin\theta \cos\theta + r_z^2 \right\}^{1/2} \tag{1}$$

where r_x and r_y are the mean peak values of response quantity r due to a single component of ground motion defined by the spectrum $A(T_n)$ applied first along the x-direction and then along the y-direction, respectively; and r_z is the mean peak value of r due the vertical component of ground motion defined by the spectrum $A_z(T_n)$. The peak response, r_k (k = x, y, z), to these individual components of ground motion is given by the response spectrum method using the CQC combination rule (Wilson, Der Kiureghian and Bayo, 1981). The term r_{xy} in Eq. 1 is a cross-term of the modal responses that contribute to r_x and r_y:

$$r_{xy} = \sum_i \sum_j \rho_{ij} r_{xi} r_{yj} \tag{2}$$

where -ρ_{ij} is the modal correlation coefficient for modes i and j. If the principal components of ground acceleration are applied along the structural axes, the total response is given using Eq. 1 with θ = 0° when the major principal component is oriented in the x-direction, and Eq. 1 with θ = 90° when the major principal component is oriented in the y-direction. Therefore, $r(\theta{=}0°)=\{r_x^2+(\gamma r_y)^2+ r_z^2\}^{½}$ and $r(\theta{=}90°)=\{(\gamma r_x)^2+r_y^2+ r_z^2\}^{½}$. These equations represent the SRSS combination of uncorrelated responses to the individual uncorrelated components of ground motion. Here, the larger of these two response values will be defined as the SRSS response, r_{srss}:

$$r_{srss} = \max\,[r(\theta = 0^\circ)\,;\, r(\theta = 90^\circ)] \tag{3}$$

Observing the structure of Eq. 2, it is apparent that r_{xy} measures the correlation between responses r_x and r_y to ground motions that are perfectly correlated. The correlation coefficient α for responses r_x and r_y is defined as:

$$\alpha = \frac{r_{xy}}{r_x r_y} \tag{4}$$

which is defined for $r_x \neq 0$ and $r_y \neq 0$. It can be shown that α is bounded between –1 and +1 for any structure and spectral shape (López, Chopra and Hernández, 1999). The limiting values of α, 0 and ±1, denote that responses r_x and r_y (to perfectly correlated ground motions) are uncorrelated and perfectly correlated, respectively.

Differentiating Eq.1 with respect to θ and setting the derivative equal to zero gives the critical angles:

$$\theta_{cr} = \frac{1}{2}\tan^{-1}\left[\frac{2r_{xy}}{r_x^2 - r_y^2}\right] \tag{5}$$

Equation 5 leads to two values of θ between 0° and 180°, separated by 90°, which give the maximum (r_{max}) and minimum (r_{min}) response values. Note that the critical values of θ are independent of the spectrum intensity ratio γ. To determine the critical response r_{cr}, usually the two numerical values of θ_{cr} are substituted for θ in Eq. 1. We can, however, derive (López et. al., 1999) an explicit equation for r_{cr} by recognizing that they represent the combined response to three components of ground motion acting in directions 1, 2 and z, with $\theta = \theta_{cr}$, as shown in Fig. 1:

$$r_{cr} = r_{\max} = \left\{ \left(1+\gamma^2\right)\left(\frac{r_x^2 + r_y^2}{2}\right) + (1-\gamma^2)\sqrt{\left(\frac{r_x^2 - r_y^2}{2}\right)^2 + r_{xy}^2} + r_z^2 \right\}^{1/2} \tag{6}$$

The explicit formula given by Eq. 6 is convenient for design purposes because it avoids computation of the two critical angles, as required in previous works (Smeby and Der Kiureghian, 1985; López and Torres, 1997), and provides a rational basis to determine the critical response from r_x, r_y, r_z and r_{xy}. Eq. 6 is not computationally demanding, requiring calculation of terms that are readily available if the conventional CQC modal combination rule is implemented in the dynamic analysis software to calculate r_x and r_y. The ratio of the critical response due to horizontal ground motion, Eq. 6 with r_z = 0, and the response from SRSS analysis (Eq.3), is given by:

$$\frac{r_{cr}}{r_{srss}} = \left\{ \frac{(1+\gamma)^2 (\frac{1+\beta^2}{2}) + (1-\gamma^2)\sqrt{(\frac{1-\beta^2}{2})^2 + (\alpha\beta)^2}}{1+(\gamma\beta)^2 \quad or \quad \gamma^2 + \beta^2} \right\}^{1/2} \tag{7}$$

wherein β is the response ratio defined as $\beta = r_y / r_x$. The denominator in Eq. 7 has two alternatives expressions: the first is valid if $r(\theta = 0°) \geq r(\theta = 90°)$, implying that $r_x \geq r_y$ or $\beta \leq 1$; the second applies if $r(\theta = 0°) \leq r(\theta = 90°)$, implying that $r_x \leq r_y$ or $\beta \geq 1$. Note that the ratio r_{cr}/r_{srss} depends on dimensionless parameters α, β, and γ. It can be shown that r_{cr}/r_{srss} is identical for β values that are reciprocal of each other. Figure 2 plots Eq. 7 as a function of correlation coefficient α for several values of β and four values of γ. For γ= 1, r_{cr}/r_{srss} =1, independent of α and β, implying that the SRSS analysis is correct only if both horizontal components of ground motion have the same intensity. For fixed values of γ< 1 and β, the response ratio r_{cr}/r_{srss} is largest at $\alpha = \pm 1$, i.e., when responses r_x and r_y are perfectly correlated. Among all values of β, r_{cr}/r_{srss} is largest for β = 1, i.e., $r_x = r_y$. For α = 0, r_{cr}/r_{srss} = 1, independent of β and γ, implying that the SRSS analysis is correct if responses r_x and r_y are uncorrelated. For a fixed value of γ, r_{cr}/r_{srss} is largest when $\alpha = \pm 1$ and β = 1, simultaneously; the latter condition implies that $\theta_{c\ r}$ = 45° or 135°. The critical response is bounded as follows (López, Chopra and Hernández, 1999):

$$r_{srss} \leq r_{cr} \leq r_{srss}\sqrt{\frac{2}{1+\gamma^2}} \tag{8}$$

The upper bound value, $(r_{cr}/r_{srss})_{\max}$, has values √2, 1.26, 1.13, and 1.08 for γ= 0, 0.5, 0.75, and 0.85, respectively, implying that the critical response value does not exceed √2 times the result of the SRSS analysis, as also noted by Torres, 1996. For typical values of the spectrum intensity ratio γ, say 0.75, this ratio is 1.13. Eq. 16 and the resulting conclusions are valid for any structure and any spectral shape.

3. ONE-STORY SYMMETRICAL-PLAN BUILDINGS

Consider the idealized one-story system shown in Fig. 3. The height of the columns is 0.4 times the bay length, *L*. The first mode involves uncoupled motion in the x-direction at period T_x, and the second mode describes uncoupled motion in the y-direction at period T_y. The damping ratio is assumed to be 5 percent in both modes. The ground motion consists of two horizontal components. The major principal component is defined by the design spectrum shown in Fig. 3. The selected response quantities are the base shear V_b in the x-direction and the axial forces in columns a, b, c, and d, respectively (Fig. 3). It can be shown (López, Chopra and Hernández, 1999) that the correlation coefficient α (Eq. 4) for the axial force in columns a and c is given by $+\rho_{12}$, and for columns b and d is given by $-\rho_{12}$, where ρ_{12} is the well-known modal correlation coefficient, for modes 1 and 2, found in textbooks (Chopra, 1995). The ratio β is equal to zero for V_b and equal to $A(T_y)/A(T_x)$ for the axial forces in columns a, b, c, d.

The variation of V_b, normalized relative to the structural weight w, with θ, is presented in Fig. 4a for systems with $T_x = 0.5$ sec. and several values of γ. Because $r_y = 0$ and $r_{xy} = 0$ (Eq. 2), these results are not dependent on T_y. The value of r_x is 1.355 w. Note in Fig. 4a, if $\gamma = 1$, V_b is independent of θ. For a fixed θ, the response increases as γ increases, indicating increasing intensity of the weaker component of ground motion. As we shall see later, these two observations apply to all response quantities, but the following observations are restricted to V_b: for any value of $\gamma < 1$, V_b is largest if $\theta = 0°$and smallest if $\theta = 90°$. The variation of the axial force N_a in column a, normalized relative to w, with θ, is presented in Fig. 4b for a system with identical periods $T_x = T_y = 0.5$ sec. For this response quantity, r_x, r_y, and r_{xy} are all non-zero and influenced by both T_x and T_y. For this system, $r_x = r_y = 0.1wA(T_x)/g$, $\alpha = +1$ and $\beta = 1$. Observe in Fig. 4b that the axial force for any angle θ may be larger or smaller than the axial force for $\theta = 0°$ or $\theta = 90°$. The maximum and the minimum values of N_a occur for incident angles when $\theta = 45°$ and $\theta = 135°$, respectively. As γ increases, the response value increases for all values of θ, consistent with intuition.

For V_b, the ratio r_{cr}/r_{srss} is always equal to one and $\theta_{cr} = 0°$ because V_b is not affected by the y-component of ground motion; $r_y = \beta = 0$. For the axial force in column a, the ratio r_{cr}/r_{srss} is presented in Fig. 5 for a system with fixed $T_y = 0.5$ sec and T_x over a range of values. The SRSS analysis gives the correct critical response if the vibration periods, T_x and T_y, are well separated, because the responses r_x and r_y are then essentially uncorrelated ($\alpha \cong 1$). The ratio r_{cr}/r_{srss} is largest for systems with $T_x = T_y$, as this condition implies that responses r_x and r_y are perfectly correlated ($\alpha = 1$); the largest value of r_{cr}/r_{srss} is equal to the upperbound in Eq. 8. Thus the discrepancy between r_{srss} and r_{cr} may be significant for systems with closely spaced periods of vibration and smaller values of the spectrum intensity ratio.

4. ONE-STORY UNSYMMETRICAL-PLAN BUILDINGS

Consider an idealized one-story unsymmetrical-plan building with a rigid slab supported by any number of lateral resisting elements oriented along directions x and y (Fig. 5). The system has three degrees of freedom: translations x and y of the center of mass (CM) and rotation of the slab about the z axis. The system has mass and stiffness properties symmetrical about the y-axis but unsymmetrical about the x-axis, with $e/r = 0.3$, where e is the distance from the center of rigidity CR to the CM; r is the radius of gyration of the floor about the z axis. Damping ratio is 5%. T_x, T_y and T_θ are the periods of the corresponding symmetrical-plan system with $e = 0$, but the mass and x-, y-, and θ-stiffnesses are identical to the coupled system: $T_y = 0.66$ sec., T_x and T_θ are varied, but $T_x/T_\theta = 1$. The principal components of ground motion are defined by $A(T_n)$ and $\gamma A(T_n)$, respectively, applied at incident angle θ (Fig. 6), where $A(T_n)$ is the design spectrum of Fig. 5. Ground motion in the x-direction excites the two natural vibration modes (periods T_1 and T_2) that contain coupled x-lateral and torsional motion. Ground motion in the y-direction excites only the mode (period T_3) that describes uncoupled motion in the y-lateral direction. T_1, T_2 and T_3 are plotted against T_x/T_y in Fig. 7a.

We will study the edge displacement of the system in the y-direction ($d_x/r = 1.225$ for a square plan). For this response quantity, the correlation coefficient α, the response ratio β, the critical angle θ_{cr}, and the critical response r_{cr} are computed and plotted in Figure 7 as a function of the uncoupled period ratio T_x/T_y, for a single ground motion component (γ=0). The response has been normalized relative to the displacement of the corresponding symmetric system , which is given by $A(T_y) \div (T_y/2\pi)^2 = 14.67$ cm. Also plotted are the response values r ($\theta = 0°$) and r ($\theta = 90°$), for the two special cases of ground motion applied in the x-direction and in the y-direction, respectively. The latter response is independent of T_x/T_y because the response to ground motion in the y-direction, the axis of symmetry, is independent of T_x and T_y is fixed. For $\theta = 0°$, the displacement increases as T_x becomes larger. For the cases when $T_x/T_y << 1$ or $>>1$, the periods T_1 and T_2 are well separated from T_3 (Fig. 7a), α approaches zero (Fig. 7b), θ_{cr} approaches 90° when $T_x/T_y << 1$ and 0° when $T_x/T_y >> 1$ (Figure 7c), and r_{cr} approaches r ($\theta = 90°$) for $T_x/T_y << 1$ and r ($\theta = 0°$) when $T_x/T_y >> 1$ (Fig.7d). On the contrary, observe that when one of the natural vibration periods, T_1 or T_2, is equal to vibration period T_3 in Fig. 7a, α is close to -1 or +1, respectively, β tends to 1, θ_{cr} is close to 135° or 45°, respectively, and r_{cr} has two peaks that exceed the two responses r ($\theta = 0°$) and r ($\theta = 90°$). As pointed out previously in Fig. 2, the simultaneity of β approaching 1 and α approaching –1 or +1 leads to an increase in the critical response with respect to the responses r ($\theta = 0°$) and r ($\theta = 90°$), as confirmed in Fig. 7d.

The ratio of the critical value of the response to its SRSS value, r_c/r_{srss}, is computed from Eq. 7 and plotted against the period ratio T_x/T_y, in Fig. 8, for several values of γ. The largest value of r_c/r_{srss} is 1.24 when $\gamma = 0$ and $T_x/T_y = 1.15$. When $\gamma = 0.5$ and $\gamma = 0.75$, the largest value of r_c/r_{srss} is reduced to 1.15 and 1.08, respectively. These values are below the upperbound values given by Eq. 8.

5. CONCLUSIONS

1. An explicit formula has been derived to calculate the critical structural response to two principal components of horizontal ground motion acting along any incident angle and the vertical component of ground motion; the critical response is defined as the largest value of response for all possible incident angles. This formula is convenient for design purposes, especially code applications, because it avoids computation of the critical incident angles.

2. The ratio between the critical value of response and the SRSS response—corresponding to the principal components of ground acceleration applied along the structural axes—depends on three dimensionless parameters: the spectrum intensity ratio γ between the two principal components of horizontal ground motion; the correlation coefficient α of responses r_x and r_y; and $\beta = r_y/r_x$. The correlation coefficient α depends on the structural properties, but is always bounded between -1 and 1

3.The ratio r_{cr}/r_{srss} is bounded by 1 and $[2/(1+\gamma^2)]^{1/2}$, implying that the critical response is not grater than 1.13 times the SRSS response for typical values of the spectrum intensity ratio, say 0.75; and never exceeds $\sqrt{2}$ times the SRSS response. For a fixed value of γ, the ratio r_{cr}/r_{srss} is largest if $\beta = 1$ and $\alpha = \pm 1$. The parametric variations presented for one-story buildings indicate that this condition can be satisfied by axial forces in columns of symmetrical buildings or can be approximated by lateral displacements in resisting elements of unsymmetrical buildings.

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REFERENCES

Chopra, A. K. (1995), *Dynamics of Structures*, New York: Prentice Hall.

López, O. A. and Torres R. (1997), "The critical angle of seismic incidence and the maximum structural response," *J. Earthquake Engrg. Struc. Dyn.*, **26**, 881-894.

López O. A., Chopra A. K. and Hernández J. J. (1999), " Critical response of structures to multicomponent earthquake excitation", [to be published].

Menun, C.-H. and Der Kiureghian, A.D. (1998), "A replacement for the 30%, 40%, and the SRSS rules for multicomponent seismic analysis," *Earthquake Spectra*, **14**(1), 153-163.

Penzien, J. and Watabe, M. (1975), "Characteristics of 3-dimensional earthquake ground motion," *J. Earthquake Engrg. Struc. Dyn.*, **3**, 365-374.

Smeby, W. and Der Kiureghian, A.D. (1985), "Modal combination rules for multicomponent earthquake excitation," *J. Earthquake Engrg. Struc. Dyn.*, **13**, 1-12.

Torres R. (1996), "Influencia de la direccionalidad del movimiento sismico sobre la respuesta dinamica de estructuras," Trabajo de Grado para Magister Scientarium, IMME, Universidad Central de Venezuela.

Wilson, E.L., Der Kiureghian A., and Bayo, E. P. (1981), "A replacement of the SRSS method in seismic analysis," *J. Earthquake Engrg. Struc. Dyn.*, **9**, 187-194.

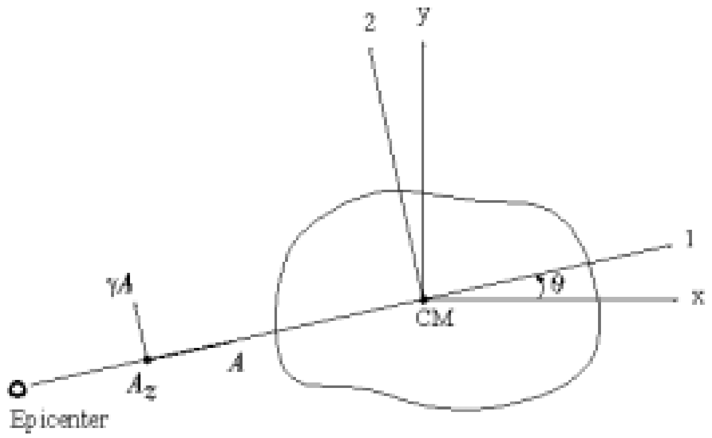

Figure 1. Principal axes of ground motion and structural axes.

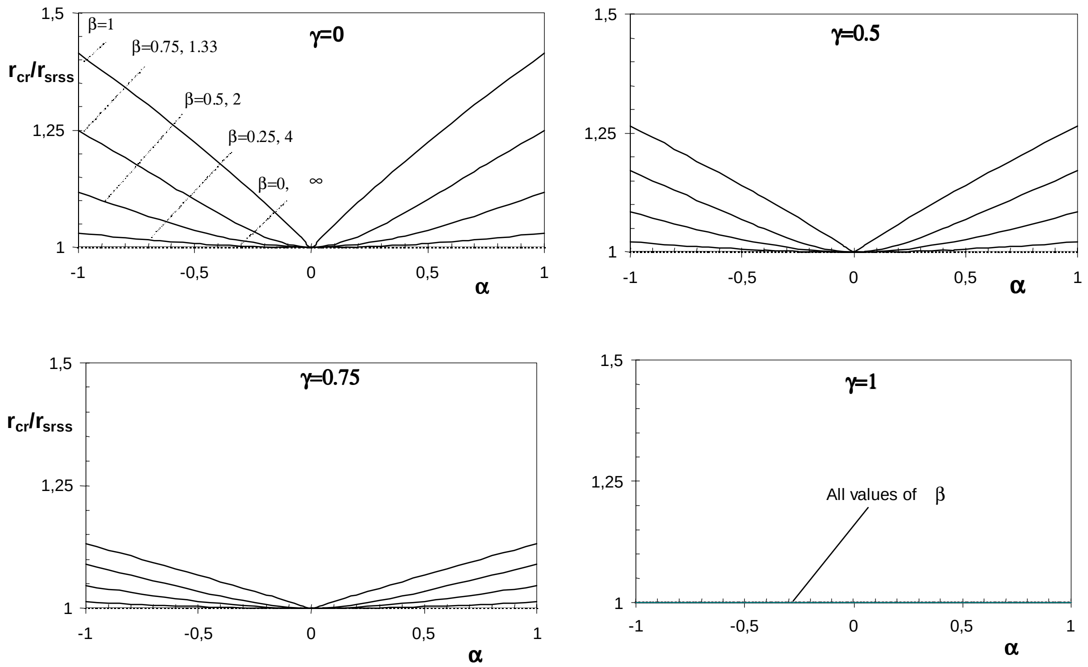

Figure 2. Ratio of critical and SRSS response as a function of γ for selected values of α and β.

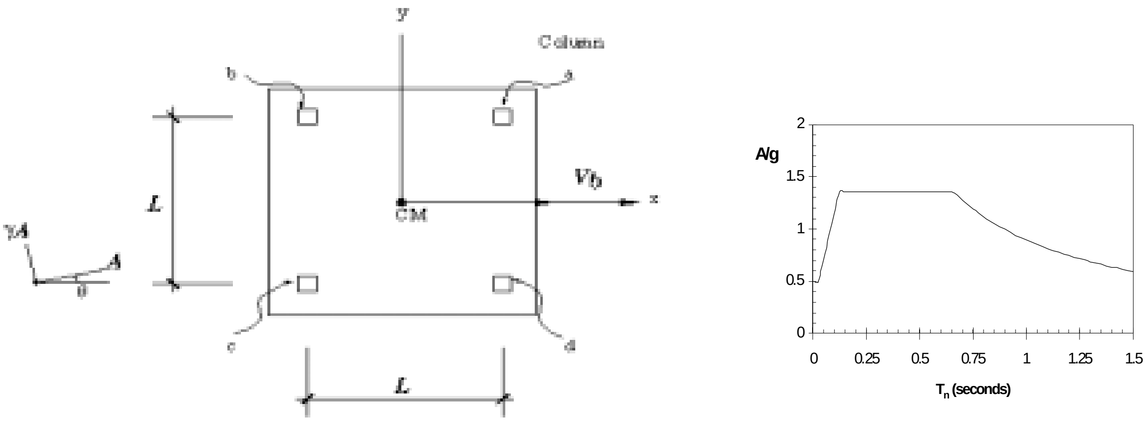

Figure 3. One-story square building with a rigid slab and design spectrum.

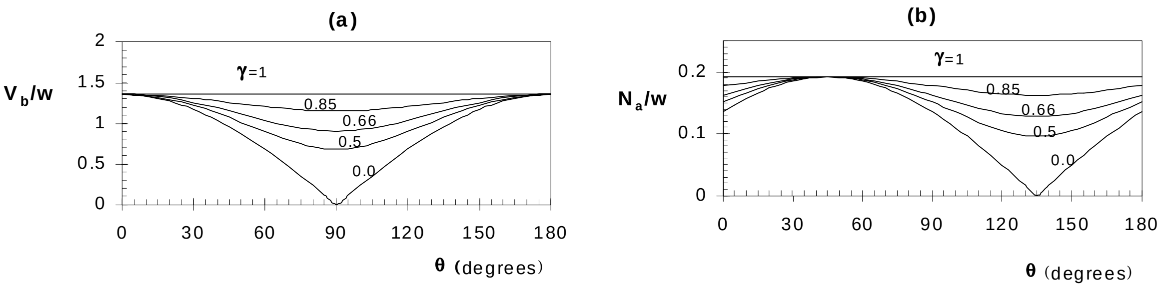

Figure 4. Variation of response with incident angle for several values of γ; (a) base shear in x-direction, (b) axial force in column a.

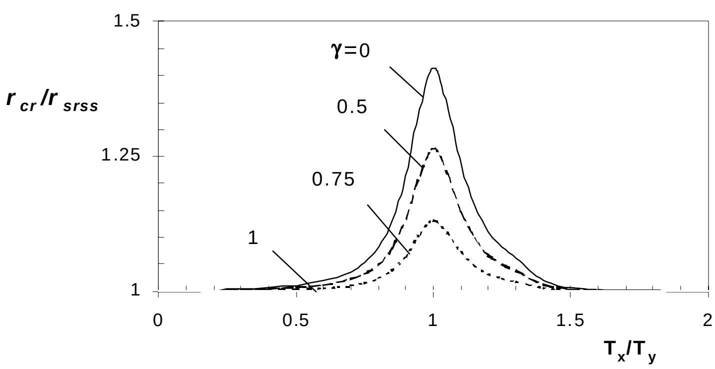

Figure 5. Variation of r_{cr}/r_{srss} with T_x/T_y and γ for the axial force in column a.

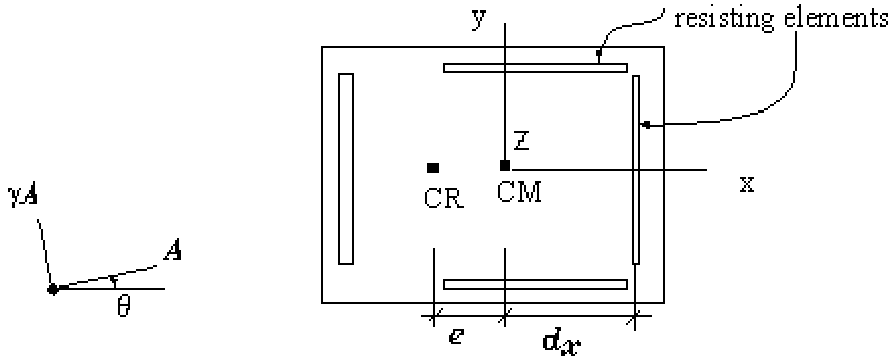

Figure 6. One-story unsymmetrical building

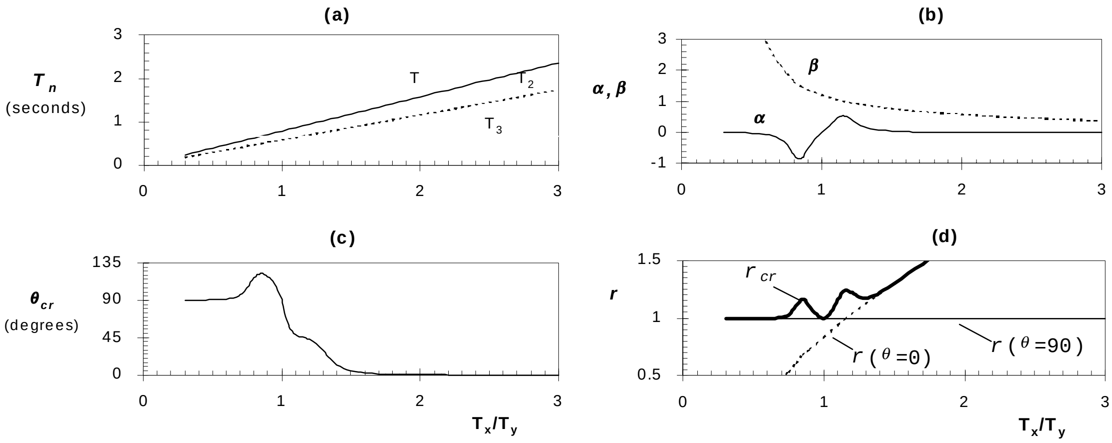

Figure 7. (a) Vibration periods, (b) parameters α and β, (c) critical angle, and (d) normalized response for a one story asymmetrical building.

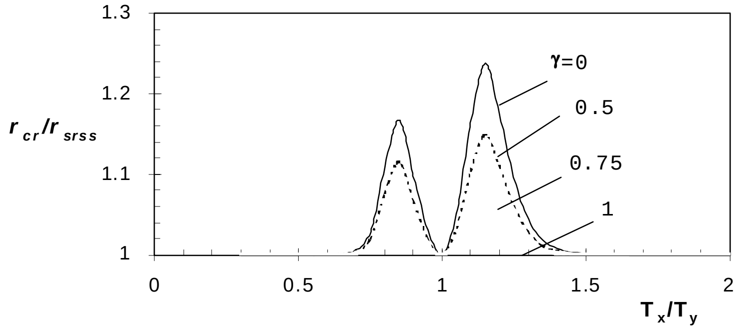

Figure 8. Variation of r_{cr}/r_{srss} with T_x/T_y and γ for the edge displacement in the unsymmetrical building.