INFERENCE OF LOAD-DEFORMATION CHARACTERISTICS FROM PROPOSED MACRO-MODEL FOR CANTILEVER PRECAST PC BEAMS ASSEMBLED BY PRESTRESSING TO COLUMNS WITH ROUND PC TENDON

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SUMMARY

The authors propose a method by which to estimate the load-deformation characteristics in the ultimate region of prestressed concrete (PC) beams assembled prestressing with round PC tendons. The conclusions are as described below. 1) Under the assumed appropriate deformation conditions, draw out of the PC tendons is computed on the basis of the bond characteristics of PC tendons, and a macro model which enables rotational deformations to be obtained is proposed. 2) the analysis results thus obtained about load-deformation characteristics of beams are in good agreement with the experimental results, and 3) in the aforesaid load-deformation characteristics, strains at extreme fibers of concrete and depth of neutral axis can be estimated with sufficient accuracy.

INTRODUCTION

In order to introduce performance design to concrete member assemblies, it is necessary to evaluate the deformation performance and strength of the members corresponding to the damage degree of the concrete as material. In the recent years, many buildings have been designed and constructed using precast (PCa) concrete column and beam members assembled by prestressing force. However, no methods have yet been established to predict the load-deformation behaviors of such precast concrete frames assembled by prestressing, especially such behaviors in the large deformation domain (ultimate domain) at time of a severe earthquake. This report is to deal with a fundamental study for the purpose of predicting the load-deformation relationship of prestress assembly type PC beams in the ultimate domain relating it with the compression edge strain intensity which represents the damage degree of the concrete on the compression side. Specifically, it was intended to build a macro-model of prestressed assembly type cantilever PC beam based on the basic data obtained from experiments wherein a monotonous positive-negative alternating load is applied on specimens consisting of a cantilever PCa PC beam assembled to a column stub and thereby show that the result of the analysis on the load-deformation relationship of the PC cantilever beam calculated from said model sufficiently corresponds to the result of the experiment.

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GENERAL DESCRIPTION AND ASSUMPTIONS OF THE MODEL

This model is based on the assumption that, as shown in Fig. 1, the PC steel member draw-out deformation on the tension side dependent on the bond characteristics causes rotational deformation while conforming to the deformation itself centering around the neutral axis like the strain conformance conditions in the case of the bending moment-curvature analysis in section as applied on the compression shrinkage in the compression concrete domain (compression plasticity domain) α-times the neutral axis depth Xn. For the construction of this model, therefore, the experimental data concerning the properties of the deformation conformance coefficient α as well as the bond characteristics of the PC steel member are indispensable.

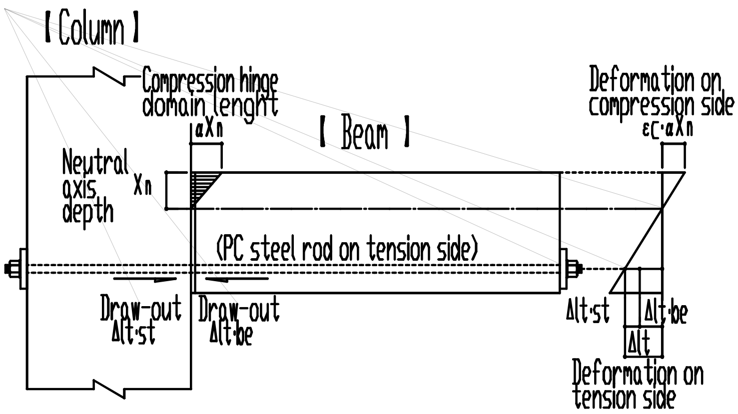


Figure 1: Outline of Macro-Model

Deformation conformance coefficient(α)

Fig. 2 shows the result of the experiment on coefficient α. It reveals the relationship of the PC steel member position displacement by elongation (εc (dp-Xn)) sought from the deformation conformance condition where the compression edge concrete strain intensity is assumed as εc and the compression plasticity domain length as Xn with the tension PC steel member elongation (Δlt) obtained from the experiment. The PC steel member elongation as obtained from experiment was calculated from the measured values of the foil strain gauges adhered to the PC steel member and was the total of the draw-out from the column stub (Δlt•st) and the draw-out from the whole length of the beam (Δlt•be). According to said figure, a general proportional relation is seen between Δlt and εc(dp-Xn) regardless of the magnitude of the compression edge concrete strain intensity (εc). The deformation conformance coefficient αis obtained as its proportion gradient.

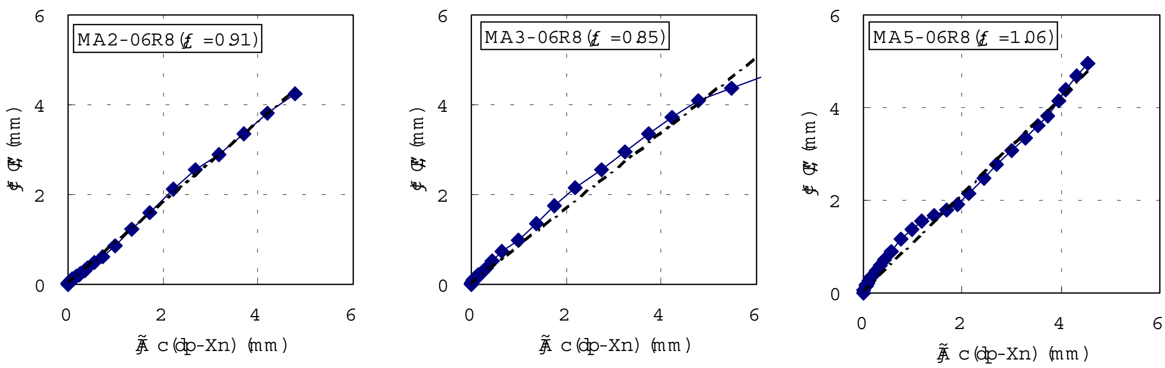


Figure 2: PC Steel Member Position Elongation Sought from Deformation Conformance and PC Steel Member Elongation Obtained from Experiment

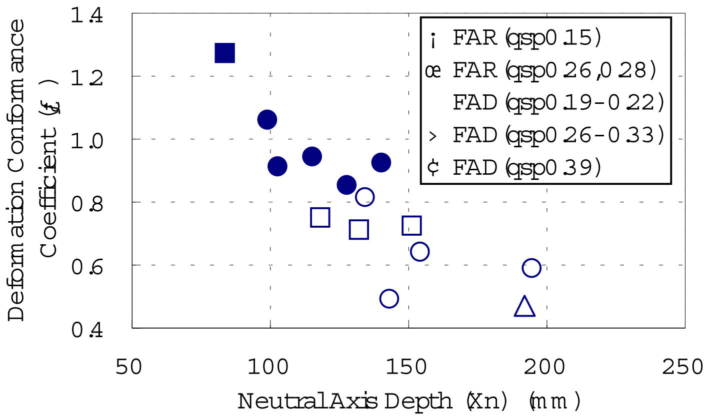


Figure 3: Relation between Neutral Depth (Xn) and Deformation Conformance Coefficient (α)

Fig. 3 shows the relation between the neutral axis depth Xn and the coefficient α of each specimen. The value of α differs specimen to specimen. The value of α in the round steel rod prestressing type (R-series) was larger than that in the deformed steel rod prestressing type (D-series). Further, the value of α was seen to generally decrease linearly as the neutral axis depth Xn increases. In the analysis of the relation between load (P) and deformation (δ) by this model, as will be described hereinafter, the value of α obtained from experiment with each specimen will be adopted.

Bonding characteristics

Fig. 4 shows, by solid line, an example of the relation between PC steel tendon bond stress (τ) and slip (S) as measured at 100 □ 200mm inside the stub from the critical section surface. The figure shows that the τ - S relations of the PC steel tendons in different specimens are quite alike in shape, and even a small slip (S) has a peak in the bonding stress (τ), and thereafter τ tends to decrease to converge to a certain value. The purpose of the present study was to pursue the load-deformation relation in the ultimate domain of the PC beam, therefore in the case of the τ - S model of the tension side PC steel tendon in the present study, a rigid- plasticity type τ - S relation having a convergence value of τ in a large slip range, as shown by broken line in the figure, was adopted, and its bonding stress was assumed to be approximately 6 kg/cm^2.

Judging that the effect, upon the rotational deformation, of the shrinkage of the PC steel member on the compression side is smaller than that of the draw-out on the tension side so that only the effect upon the synthetized compressive force may be considered and in order to simplify the derivation of the calculation formula, the steel member force to contribute to the synthetized compressive force was considered using the strain conformance coefficient (F-value). The F-value was assumed to be 0.1 to 0.15.

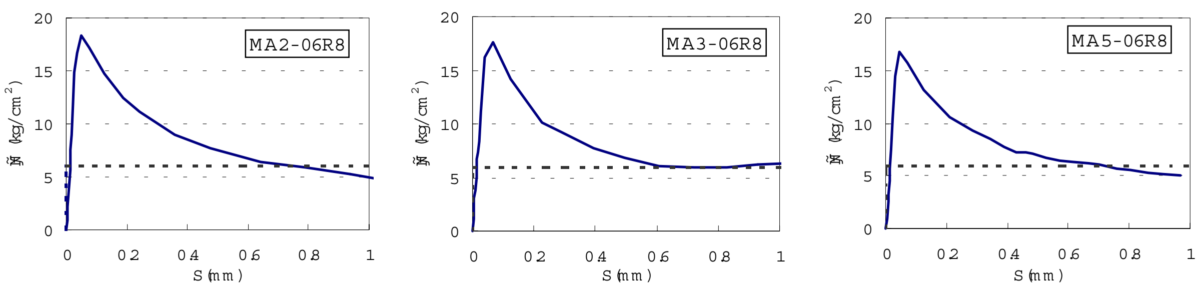


Figure 4: Relation between Bonding Stress (τ) and Slip (S) of PC steel Tendon

Force shared by the cutoff compression steel bars

In the case the prestressed type PCaPC beam, compression steel bars are cut off near the critical section surface and do not penetrate the prestressed interface, but the compressive force shared by these compression steel bars was measured in the present experiment. Fig. 5 shows some examples of the distribution of strain on compression steel bars from the prestressed interface. The figure shows that the compression steel bars, though cut off, share the compressive force, and in the case of a member angle in the large deformation domain, the yield strain is exceeded.

Therefore, in the case of our model which is intended for the ultimate domain, it was assumed that the ordinary steel bars on the compression side share the compressive force equivalent to the yield strength at the prestressed interface.

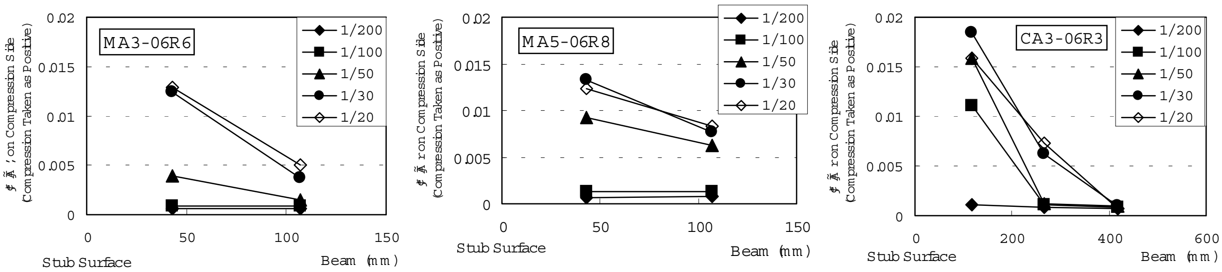


Figure 5: Compression Steel Bar Strain Distribution

DERIVATION OF THE ROTATIONAL DEFORMATION INFERENCE FORMULA

The load-deformation inference formula of the prestressed round steel tendon assembly type PC beam by the proposed macro-model can be sought as follows from the forces equilibrium equation (Equation (1)) and the deformation conformance conditions (Equation (3)). The stress block coefficients k1k3(x) and k2(x) of the stress (σ)- strain (ε) relationship of concrete will be shown in Supplement 1.

1) Assuming that the incremental tensile force of the PC steel tendon upon loading is expressed by T, the following equation can be obtained from the "forces equilibrium conditions":

$k1k3(x)\bullet b\bullet Fc\bullet Xn+F(1-dpc/Xn)\ \varepsilon c\bullet Ep\bullet Apc-Tp0c+Arc\sigma ry=T+Tp0$ (1)

where, Tp0, Tp0c : PC steel tendon initial introduction force on tension side and compression side respectively

F : strain conformance coefficient of compression PC steel tendon

From Equation (1), the neutral axis Xn will be :

$Xn = \{-K2+T+Tp0+\sqrt{(K2-T+Tp0)^2-4K1\bullet K3}\}/(2K1)$ (2)

where, $K1 = k1k3(x)b\bullet Fc$, $K2 = F\bullet\varepsilon c\bullet Ep\bullet Apc-Tp0c+Arc\sigma ry$, $K3 = -F\bullet dpc\bullet\varepsilon c\bullet Ep\bullet Apc$

2) From the "deformation conformance conditions" on the compression side and the tension side :

$\Delta lc/\Delta lt = Xn/(dp-Xn)$ (3)

Assuming the compression hinge domain length as $\alpha\bullet Xn$, then the shrinkage of the compression hinge Δlc will be:

$\Delta lc = \varepsilon c\bullet\alpha\bullet Xn$ (4)

From (3) and (4), the elongation on the tension side Δlt with reference to the neutral axis Xn will be expressed by:

$\Delta lt = \varepsilon c\bullet\alpha\ (dp-Xn)$ (5)

3) From Equation (2) of 1) and Equation (5) of 2) :

$\Delta lt = \varepsilon c\bullet\alpha\ [dp-\{K2+T+Tp0+\sqrt{(K2-T+Tp0)^2-4K1\bullet K3}\}/(2K1)]$ (6)

4) On the other hand, the PC steel tendon draw-out from the column stub $\Delta lt\bullet st$ and the PC steel tendon draw-out from the entire length of the beam $\Delta lt\bullet be$ will be expressed as tensile steel tendon force factor of T+Tp0, which will be divided into the following two cases depending on whether non-yielding or yielding:

In case of (A) $T+Tp0 \le Tpy$:

$\Delta lt\bullet st = (2T-\tau\bullet\phi\bullet Lst)Lst/(2Ep\bullet Ap)$ (7)

where, Lst : length of the PC steel tendon in the stub

$\Delta lc\bullet be = (2T-\tau\bullet\phi\bullet Lbe)Lbe/(2Ep\bullet Ap)$ (8)

where, Lbe : length of the PC steel tendon in the cantilever beam

In case of (B) $T+Tp0 > Tpy$:

$\Delta lt\bullet st = (\varepsilon py - \varepsilon p0)Lst+(T+Tp0-Tpy)Lst/(E'p\bullet Ap)$ (9)

$\Delta lt\bullet be = (\varepsilon py - \varepsilon p0)Lbe+(T+Tp0-Tpy)Lbe/(E'p\bullet Ap)$ (10)

5) From Equation (6) of 3) and Equation (7), (8), (9) and (10) of 4), the incremental tensile force of the PC steel tendon T can be sought as follows:

(A) In case of T+Tp0 <= Tpy :

$T = \{-K8+\sqrt{\ } (K8^2-K7 \bullet K9)\}/K7$ (11)

K4=Ep•Ap•α•εc/K1, K5=-Ep•Ap•α•εc/K1-2(Lst+Lbe),
$K6=2Ep \bullet Ap \bullet \alpha \bullet \varepsilon c\{dp+(K2-Tp0)/(2K1)\}+\tau \bullet \phi\ (Lst^2+Lbe^2)$, $K7=K4^2-K5^2$,
$K8=K4^2(Tp0-K2)-K5 \bullet K6$, $K9=K4^2\{(Tp0-K2)^2-4K1 \bullet K3\}-K6^2$

(B) In case of T+Tp0 > Tpy :

$T = \{-K15+\sqrt{\ } (K15^2-K14 \bullet K16)\}/K14$ (12)

K11=E' p•Ap•α•εc/(2K1), K12=-E' p•Ap•α•εc/(2K1)-(Lst+Lbe),
K13=E' p•Ap•α•εc{dp+(K2-Tp0)/(2K1)}- {E' p•Ap(εpy - εp0)+Tp0-Tpy}(Lst+Lbe),
$K14=K11^2-K12^2$, $K15=K11^2(Tp0-K2)-K12 \bullet K13$, $K16=K11^2\{(Tp0-K2)^2-4K1 \bullet K3\}-K13^2$

6) Neutral axis Xn will be sought from Equations (11) and (12) of 5) and Equation (2) of 1), and then the compression hinge shrinkage Δlc and tension side elongation Δlt can be calculated by substituting said Xn in Equations (4) and (5) of 2).

7) Rotation angle θ and the deformation δ can be sought by θ= (dp - Xn)/Δlt or θ= Xn/Δlc and δ=θ•a (a: shear span length), and the bending moment M and the load P can be sought by M = (T+Tp0)•(dp-k2•Xn) and P=M/a.

Supplement 1 : Calculation formula for bending/compression stress block coefficients

In case 0<=X<1 :

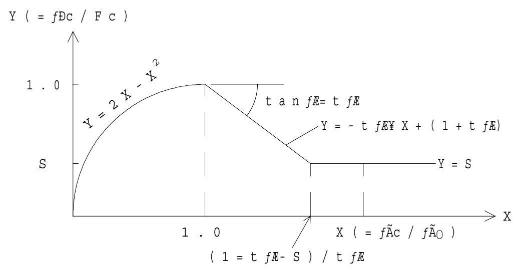
$k1k3(X)=1+\{(1-X)^3-1\}/(3X)$
$k2(X)=1-[1/2+(1-X)^3/(3X) + \{(1-X)^4-1\}/(1\ 2X^2)]/k1k3(X)$

In case 1<=X<(1+tθ-s)/tθ :

k1k3(X)= -(tθ/2)X+(1+tθ)-(1/3+tθ/2)/X
$k2(X)=1-[-(t\theta/3)X+(1+t\theta)/2-(1+2t\theta)/(12X^2)]/k1k3(X)$

In case X>(1+tθ-s)/tθ :

$k1k3(X)=s+\{(1+t\theta-s)^2/(2t\theta)-(t\theta/2+1/3)\}/X$
$k2(X)=1-[s/2+(1+t\theta-s)^3/(6 \bullet t\theta^2 \bullet X^2)-(1+2t\theta)/(12X^2)]/k1k3(X)$



Supplemental figure: σ-ε Relation of Concrete

COMPARISON BETWEEN ANALYTICAL RESULT AND EXPERIMENTAL RESULT, AND DISCUSSIONS

Fig. 6 shows some example comparisons between the experimental result and the analytical result of the relationship between load (P) and deformation (δ). In this figure, the fine line represents the experimental result and the bold line the analytical result. The symbols □,□,□ and ○ show the positions where the member angle is 1/200, 1/100, 1/50 and 1/30 respectively in the P-δcurve or the P-δ envelope in the experimental result, and the symbols ■,□,▲ and ● show those points on the analytical P-δ line where the compression edge strain intensities (εc) are respectively the same as on the afore-mentioned positions. It could be understood from Fig. 6 that the analytical result generally follows the characteristics of the experimental result that generally the rigidity begins to drop drastically, compared with the initial rigidity, after the member angle of 1/200, under the effect of the PC steel tendon draw-out so that the load gradually increases toward the maximum strength as the deformation increases. Further, the figure well predicts the strength and deformation at the time when the load generally ceases to increase due to yielding of the PC steel member. Furthermore, the void symbols experimental and those of the black ones analytical where the value of
εc respectively coincides with each other are positioned correspondingly, or in other words, they show generally the same deformations, therefore it is considered that the experimental result coincides with the analytical result as for the distribution of strain on sections such as at the position of the neutral axis in the member section.

Fig. 7 shows some comparisons between the experimental value and the analytical value in the relationship between the draw-out (elongation on tension side: Δlt) and the deformation (δ) of the tensile PC steel member. In this figure, the symbols ○ and □ respectively show the draw-out of the PC steel member out of the column stub Δlt•st and the draw-out of the PC steel member out of the whole length of the beam Δlt•be as obtained from the experiments, and the symbols ● and ▲ respectively show them as obtained from the analyses. Further, the fine line without symbol marks shows the experimental result of the total draw-out Δlt and the bold line without symbol marks shows the analytical result of the total draw-out Δlt. This figure suggests that the PC steel draw-out Δlt•st out of the column stub at a certain deformation as obtained by analysis and expressed by the symbol ● is slightly larger than that as obtained by experiment and expressed by the symbol ○, but both of them generally allow good predictions. Likewise, as to the PC steel member draw-out from the whole beam length Δlt•be, the analytical value expressed by the symbol ▲ is slightly larger than the experimental value expressed by the symbol □, but their prediction accuracies are generally acceptable.

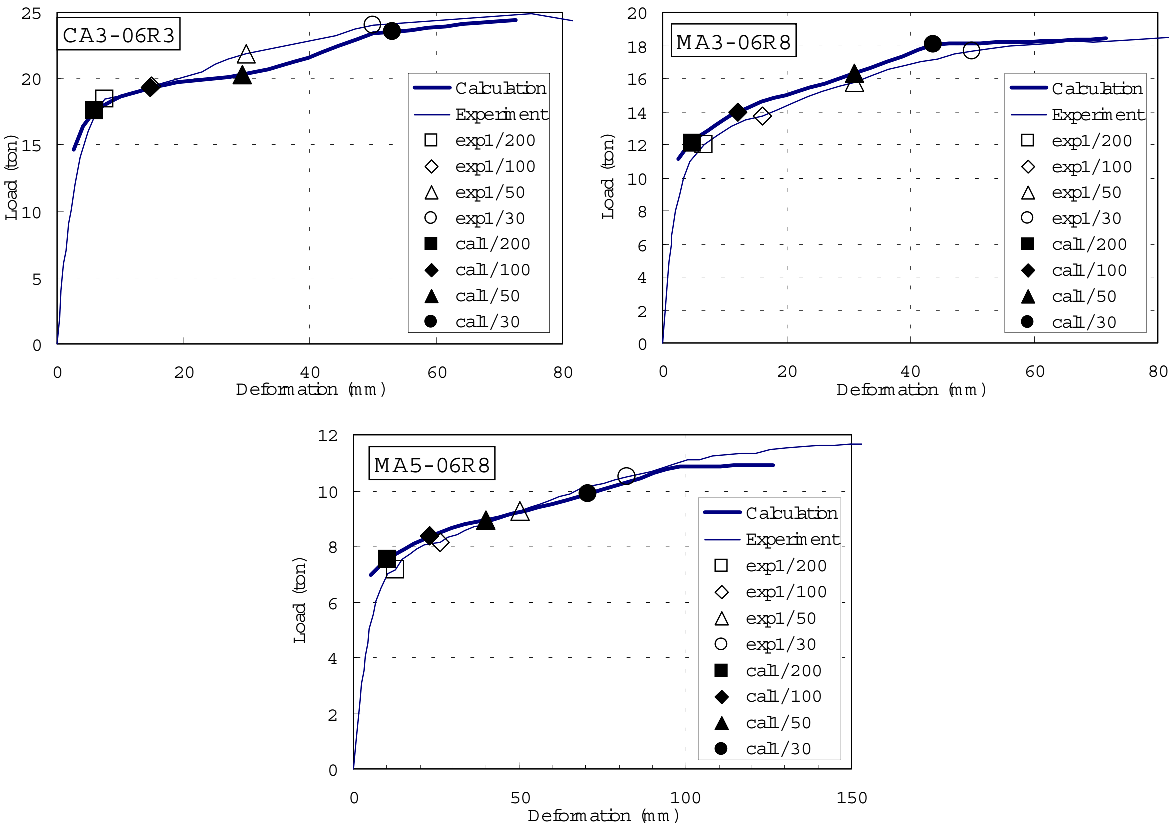


Figure. 6 Relation between Load(P) and Deformation(σ)

In conclusion, when looking at the total draw-out Δlt which is the total of both, the bold line without symbols showing the value analytically calculated as the total of the symbols ● and ▲ is, on some specimens, evaluated larger than the total draw-out at the deformation point of the fine line without symbol marks showing the value experimentally obtained as the total of the symbols ○ and □. However, those analytical result and experimental result generally coincide with each other. Further, from the same figure, it could be considered that in the case of the present specimens, the percentage of the draw-out from the whole length of the beam Δlt•be occupying in the PC steel total draw-out is larger than the percentage of the draw-out from the column stub Δlt•st occupying in the PC steel total draw-out. Supposedly, it is because that in these experiments the distance from the joint plate at the free end of the cantilever beam to the critical section is longer than the distance from the joint plate on the outside of the stub to the critical section. Therefore, in the case of the present model where the rotational angle is inferred from the draw-out deformation value of the PC steel on the tension side depending on the interface characteristics, the position on the member where that steel begins to slip will be the important point.

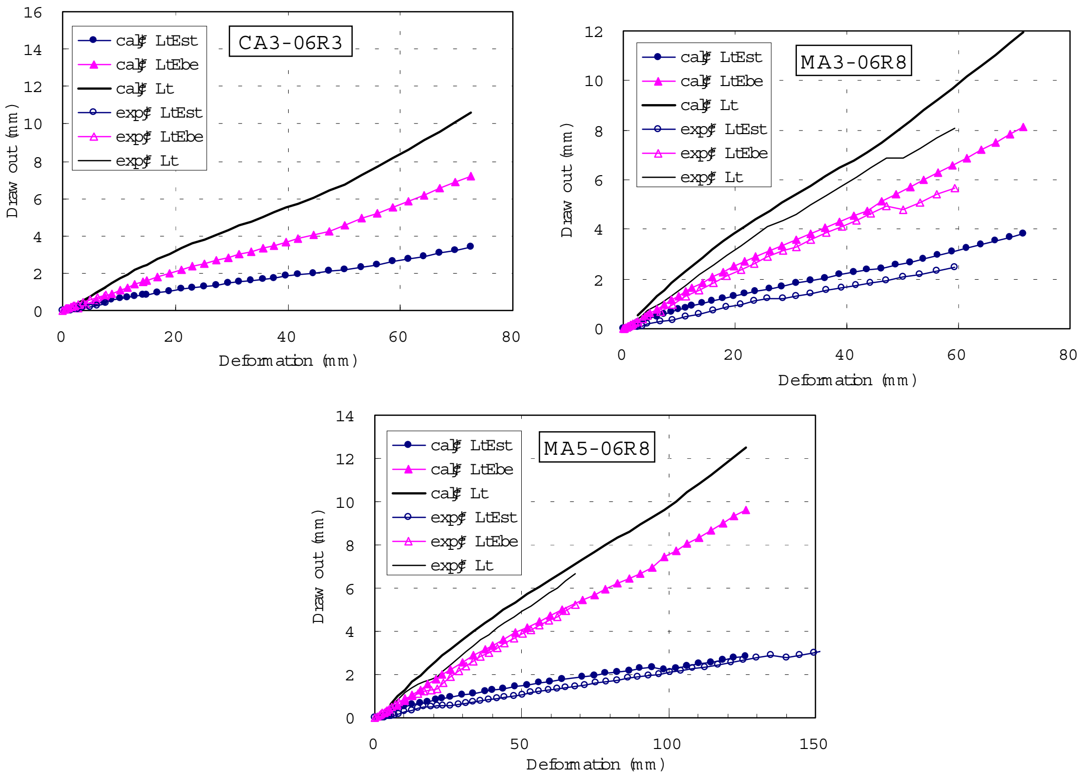


Figure. 7 Relation between PC Steel-Draw-Out and Deformation

CONCLUSION

The conclusion of this report follows. 1) Concerning the round PC steel tendon prestressed type cantilever beam, a macro-model was proposed and thereby the rotational deformation was sought by calculating the steel tendon draw-out giving certain joint characteristics for the tension PC steel tendon under assumed deformation conformance conditions on the compression side and the tension side. 2) The result of the analysis of the load-deformation relationship of the prestressed assembly type cantilever PC beam calculated by means of said macro-model sufficiently conforms to the result of the experiment. 3) Section surface strain distribution, concrete compression edge intensity εc, neutral axis depth Xn, etc. at any selected point in the load-deformation relation can be estimated with sufficient accuracy so that the damage degree of concrete as material can be correlated to the load-deformation relationship in the ultimate domain which represents the assembly performance. 4) It is thought that in the future this model should be applied to commoner. PCa PC column•beam framing and that said model should be developed into a model capable of allowing gripping of the rotational performance in the hinge domain in case PC steel with a high assembly performance is used.