USE OF FRAGILITY MODELS AND DAMAGE RISK MATRICES IN STRUCTURAL DAMAGEABILITY EVALUATION

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SUMMARY

The analysis of the damage states of a structure subjected to earthquake motions is considered. The damage levels are quantified in terms of damage indices and seismic damage risk matrices, and the losses associated with this kind of damages are evaluated using damage factor matrices. These are matrices with stochastic entries which are obtained by means of adapted fragility models.

INTRODUCTION

There exists a large spectrum of sources to damages in structures, besides the “natural” degradation processes due to ageing, corrosion, fatigue and others. Large earthquake motions can induce major damages to any type of structures (buildings, lifelines, roads – highways, bridges, industrial facilities including nuclear power plants, and others). An essential problem consists in the evaluation of the “remaining” reliability of a given structure or of a given class of (similar) structures situated in a more-or-less homogeneous seismic area. The probabilistic approach is unavoidable, and a wide range of probabilistic / stochastic models is available for the evaluation of the reliability and seismic risk of structural systems (either under design or existing). This is just why an appropriate selection of the models must be assumed, depending on the type of structure as well as on the nature of the most likely factors able to induce structural damages. The damage risk due to earthquakes has to be evaluated in terms of specific models, and the so-called (seismic) *fragility models* have gained a wide use, in the last two decades, in the safety and risk analysis and prediction of structures and equipment in nuclear facilities [1,2]. However, such models have proved, as well, their utility in the stochastic seismic damage estimation of rather general types of structures like RC frames [3,4]. The concept of *damage state* (due to earthquake events) can be approached by use of fragility curves which express the conditional probabilities of reaching or exceeding various damage states at given levels of ground motions.

Relationships between earthquake ground motion severity and structural damage, along with seismic site hazard analysis, are frequently used for assessing possible regional losses due to earthquakes. *Ground motion* - versus - *damage* relationships characterize the level of damage reached by a specific class of structures as a function of a relevant ground motion parameter (like the PGA or PGV). Various types of matrices can be used for handling available evidence on structural damages due to earthquakes of evaluated intensities. As a preliminary stage, both the damage levels and the seismic intensities must be discretized within the intervals of possible values. This question is addressed in Section 1, in connection with the fragility models adapted for the evaluation of structural damageability : the “classical” PGA capacity met in earlier papers like [1] is replaced by d = the damage state parameter, which covers an interval $[0,D]$. We go deeper into the quantification of structural damageability in Section 2, where damage risk functionals are also considered. A variant of Park & Ang damage index is adapted for the damage risk quantification. Damage level risk matrices are presented in Section 3, together with damage probability matrices. The latter ones are derived from the former in terms of a damage threshold, and they can be used as matrix factors yielding a damage risk vector. The paper closes with a couple of remarks on the applicability of these models under time dependent approaches.

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DAMAGE STATES AND SEISMIC FRAGILITY MODELS

The prediction of the lifetime of buildings in a seismic area needs some mathematical models to be built, able to describe (as close as possible) the evolution of the structural damage. Certainly, any such model should take into account as many sources of uncertainty / randomness as possible, but without becoming practically unemployable. The randomness in some relevant structural parameters is taken into account in [5], with certain assumed distributions for them. But it is hard to select the "best" (or - at least - the most appropiate) set of structural parameters for describing the damage states induced by earthquakes in structural components and systems.

To fix the ideas, let us first take into account a single damage state parameter as the story drift ratio δ for a multistory building. It is assumed that the possible values of δ cover an interval $[0, D]$. This interval has to be partitioned into a finite number of subintervals :

$$[0, D] = \bigcup_{k=1}^{\ell-1} [d_{k-1}, d_k) \cup [d_{\ell-1}, d_\ell]. \tag{2.1}$$

Each interval corresponds to a category of structural damage (from, e.g., no damage to severe damage). Obviously, the random damage indicator Δ can take values in one of the subintervals of (2.1) with a specific probability. If the impact of the external actions would not be taken into account, such a probability would be given by

$$P\left(\Delta \in [d_{k-1}, d_k)\right) = F_\Delta(d_k) - F_\Delta(d_{k-1}) \text{ for } 1 \le k \le \ell, \tag{2.2}$$

where $\boldsymbol{F}_\Delta$ is the *cdf* (cumulative distribution function) of Δ. But such an oversimplifying assumption cannot be accepted. The damage state of the system essentially depends on the actions from the environment, including the impact of earthquake motions. A parameter describing the intensity of the seismic excitation at the site of the system has to be considered, for instance the PGA A like in the most fragility models. The possible values of A also cover an interval $[0, \alpha)$ which has to be partitioned :

$$[0, \alpha) = \bigcup_{j=1}^{m} [a_{j-1}, a_j) \quad \text{with} \quad a_0 = 0. \tag{2.3}$$

Conditional probabilities have now to be introduced for the damage indicator $\Delta \in [d_{i-1}, d_i)$ provided $\boldsymbol{A} \in [\boldsymbol{a}_{j-1}, \boldsymbol{a}_j)$:

$$P\left(\Delta \in [d_{i\text{-}1}, d_i) \mid A \in [a_{j\text{-}1}, a_j)\right) = q_{ij}, \tag{2.4}$$

for $1 \le i \le \ell$ and $1 \le j \le m$. These conditional probabilities are the entries of a stochastic matrix

$$Q = [q_{ij}]_{\ell, m} \tag{2.5}$$

that can be called the *damage level risk matrix* (DLRM). Obviously, the entries of Q have to satisfy the condition

$$\sum_{i=1}^{\ell} q_{ij} = 1 \tag{2.6}$$

since - for any earthquake input intensity - the structural component / system has to be in one and only one damage state / damage level, falling in one (and only one) of the intervals :

$$[d_{i-1}, d_i) \subseteq [0, D) \quad \text{with} \quad P(\Delta \in [0, D) = 1). \tag{2.6a}$$

This model is connected with the concept of fragility [1,7]. In most references, lognormal distributions are assumed for the PGA capacity = the ground acceleration A corresponding to failure, where : $A = \tilde{A}\,\varepsilon_R \varepsilon_U$, $\tilde{A}$ = the median of A and $\varepsilon_R, \varepsilon_U$ are two random variables with unit medians that represent the inherent randomness about the median and the uncertainty in the median value, respectively. The logarithmic standard deviations of $\varepsilon_R,\ \varepsilon_U$ are $\beta_R,\ \beta_U$. The three parameters $\tilde{A}, \beta_R, \beta_U$ are enough for expressing the failure probability at a level at most equal to a of the ground acceleration :

$$\boldsymbol{P_f(a)} = \Phi\left[\frac{\ln(\boldsymbol{a}/\tilde{\boldsymbol{A}})}{\beta_{\boldsymbol{R}}}\right] \quad \text{with} \quad f_{\tilde{A}}(c) = \frac{1}{\sqrt{2\pi}\, c\beta_U} \exp\left[-\frac{1}{2}\left(\frac{\ln(c/\tilde{A})}{\beta_U}\right)^2\right]. \tag{2.7}$$

This model makes possible to evaluate the fragility (that is, the failure frequency conditional on a seismic intensity a) of the structure by

$$F(a, p_0) = \Phi\left[\frac{1}{\beta_R} \ln\left(\frac{a}{\tilde{A}\exp(-\beta_U\, \Phi^{-1}(p_0))}\right)\right], \tag{2.8}$$

where Φ is the standard normal *cdf* and $\boldsymbol{p_0}$ is the probability that determines a corresponding fragility curve ($p_0 =$ 0.5 determines the median curve). This formulation is adapted from [2].

We are going to see how a fragility model could be employed for building an DLRM as the one given (2.5) with the definition (2.4) of its entries $\boldsymbol{q_{ij}}$ An exact application of expression (2.4) would be (according to the definition of the conditional probabilities)

$$\boldsymbol{q_{ij}} = \frac{\boldsymbol{P}[\{\Delta \in [\boldsymbol{d_{i-1}}, \boldsymbol{d_i})\} \cap \{\boldsymbol{A} \in [\boldsymbol{a_{j-1}}, \boldsymbol{a_j})\}]}{\boldsymbol{P}[\{\boldsymbol{A} \in [\boldsymbol{a_{j-1}}, \boldsymbol{a_j})\}]}\ . \tag{2.9}$$

Alternatively, a representative (central) value may be taken in each interval of the partition (2.3) : $\hat{a}_j \in [a_{j-1}, a_j)$; then, the probabilities (2.4) of the stochastic matrix Q can be evaluated by

$$\boldsymbol{q_{ij}} = \boldsymbol{P}[\Delta < \boldsymbol{d_i} \,|\, \tilde{\boldsymbol{a}}_{\boldsymbol{j}}] - \boldsymbol{P}[\Delta < \boldsymbol{d_{i-1}} \,|\, \tilde{\boldsymbol{a}}_{\boldsymbol{j}}] = \boldsymbol{P}[\Delta \geq \boldsymbol{d_{i-1}} \,|\, \tilde{\boldsymbol{a}}_{\boldsymbol{j}}] - \boldsymbol{P}[\Delta \geq \boldsymbol{d_i} \,|\, \tilde{\boldsymbol{a}}_{\boldsymbol{j}}]. \tag{2.10}$$

Let us assume that a double lognormal format is acceptable for the damage index Δ:

$$\boldsymbol{F_\Delta(a, C)} = \Phi\left[\frac{\ln(\boldsymbol{a}/\boldsymbol{C})}{\beta_{\boldsymbol{R}}}\right] \quad \text{with}$$

$$f_C(c) = \frac{1}{\sqrt{2\pi}\, c\beta_U} \exp\left[-\frac{1}{2}\left(\frac{\ln(c/\tilde{\Delta})}{\beta_U}\right)^2\right]. \tag{2.11}$$

In (2.11) C is the median capacity of the damage index Δ, while $\tilde{\Delta}$ is the median of C . Then we could express the entries of the DLRM Q as

$$\Phi\left[\frac{1}{\beta_R}\ln\left(\frac{\hat{a}_j}{\tilde{\Delta}\exp[-\beta_U\, \Phi^{-1}(p_{i-1})]}\right)\right] - \Phi\left[\frac{1}{\beta_R}\ln\left(\frac{\hat{a}_j}{\tilde{\Delta}\exp[-\beta_U\, \Phi^{-1}(p_i)]}\right)\right], \tag{2.12}$$

where p_i is the probability that determines the damage-fragility curve corresponding to the damage level $d_i \in (0, D)$ $(1 \le i \le \ell)$ (see Eq.(2.8)).

The fragility model can be discretized in terms of DPDs (discrete probability distributions), as shown in [1]. Two families of curves are defined as

$$[\gamma] \equiv \{<q_j, \gamma_j(a)>\}, \ [\Delta] \equiv \{<p_i, \Delta_i(a)>\}, \tag{2.13}$$

where Δ is fragility curve of the DI depending on the seismic acceleration, while γ_j is a seismicity curve (also known as a hazard curve). A rather widely accepted analytical expression for γ involves Fréchet's type II "extreme value" distribution :

$$\gamma(a) = P[A > a] = 1 - \exp\left[-(\sigma/a)^{\alpha}\right], \tag{2.14}$$

where $\sigma =$ the size parameter and $\kappa =$ the shape parameter. Such an expression can be found, e.g., in [6]. Families of hazard curves and fragility curves are presented in Fig.1.

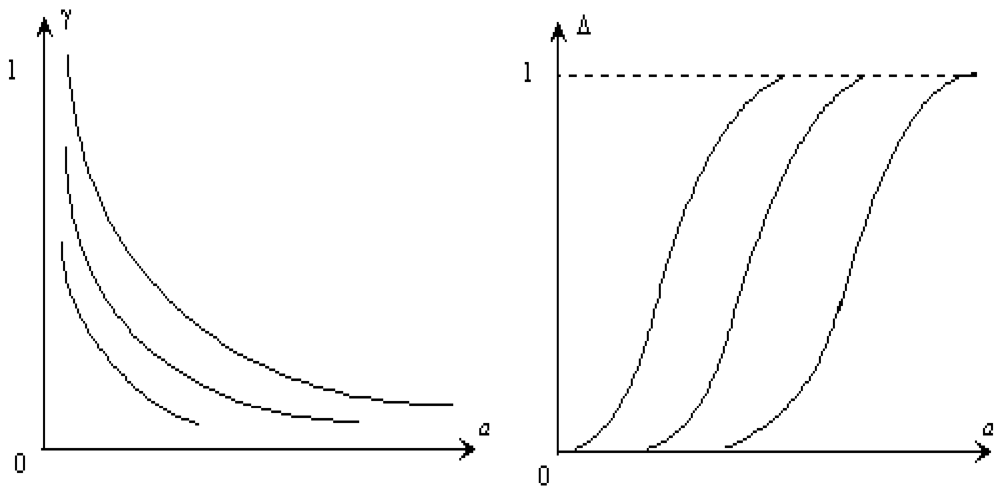

Fig. 1. Seismicity curves $\gamma_j(a)$; **Fragility curves** $\Delta_j(a)$

The two families of curves can be composed resulting in a "two-dimensional" DPD

$$[\varphi] = \{<p_{ij}, \varphi_{ij}>\} \text{ with } p_{ij} = p_{ij} q_{ij} \text{ and } \varphi_{ij} = -\int_0^{\infty} \Delta_i \frac{d\gamma_j}{da} da \tag{2.15}$$

Let us recall that the fragility curve Δ_i is determined by the probability p_i while a hazard curve γ_j will be entirely determined by specific values on its size and shape parameters (σ_j , κ_j) The derivative under the integral in (2.15) - following from the analytical assumption (2.14) - is the (specific) *pdf* of Fréchet's distribution :

$$f_A^{(j)}(a) = \frac{\kappa_j}{\sigma_j}\left(\frac{\sigma_j}{a}\right)^{\kappa_{j+1}} \exp\left[\left(\frac{\sigma_j}{a}\right)^{\kappa_j}\right]. \tag{2.16}$$

As regards the probabilities $q_k = P[a_{k-1} \le A \le a_k]$, the can be derived from Eq.. (2.14) by

$$q_k = P[A \ge a_{k-1}] - P[A > a_k] = \exp[-(a_j/a_k)^{\kappa_j}] - \exp[-(a_j/a_{k-1})^{\kappa_j}]. \tag{2.17}$$

This probability that the earthquake intensity A falls in the interval $[\boldsymbol{a}_{k-1}, \boldsymbol{a}_k]$ in represented in Fig.2.

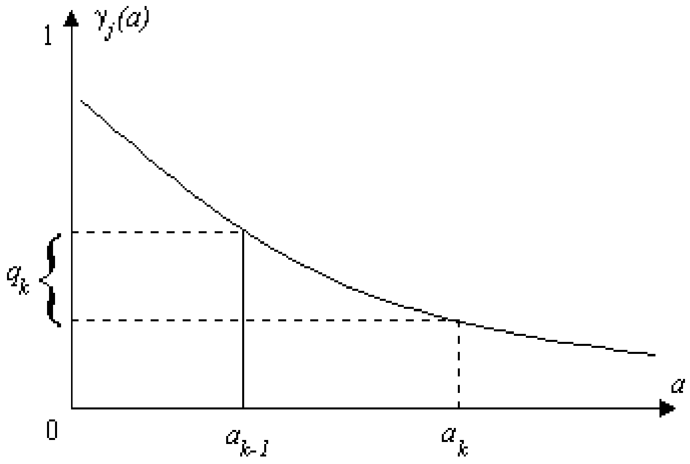

Fig.2 Probabilities of earthquake level intensities

At the same time , the hazard curve γ_j is assumed – under the DPD approach – with its specific probability q_j (not the same with q_k). The DPDs for the hazard curves are considered as independent from the ones for the fragility curves Δ_i. This independence justifies the expression of $p_{Ij} = p_i \; q_j$ given in Eq. (2.15) as the probability of the fragility φ_{Ij}.

SEISMIC DAMAGEABILITY QUANTIFICATION AND DAMAGE RISK FUNCTIONALS

The level of losses due to possible damage states usually quantified in terms of a *damage factor* that can be defined (see, e.g., the Report ATC-13 [8]) as the ratio between the cost of repair and the total cost of the system / structure. Seven classes for such damage states (with respect to the losses) are considered in [8]. These damage states are clearly dependent on the earthquake intensity, and they should be probabilistically described. A (pseudo-)lognormal distribution can be used for describing the seismic fragility of a structure with respect to the earthquake intensity MMI; we keep the notations Δ, δ (as in Section 2) for the damage factor, but we denote by $m \in [0, M]$ the values of the MMI. Thus, the fragility curve for the loss factor can be expressed as

$$P\left[MMI \geq m \,|\, a\right] = \int_0^m \frac{1}{\sqrt{2\pi}\;\mu\;\sigma_{ln\,A}}\; exp\left[-\frac{1}{2}\left(\frac{ln\,\mu\; - \;E\left[ln\,\mu\right]}{\sigma_{ln\,A}}\right)^2\right] d\,\mu\;. \qquad (3.1)$$

Certainly, such a model would be not complete without some distributional assumptions on the (damage) lossfactor conditional on the parameter MMI = m. A Beta distribution can be accepted for this dependence. Therefore, the conditional *pdf* of Δ would be

$$f_\Delta\left(\delta \,|\, m\right) = \frac{1}{B\left(r,s\right)}\,\delta^{\,r-1}\left(1-\delta\right)^{s-1}\;, \qquad (3.2)$$

where B is Euler's Beta function. The density in (3.3) does not explicitly depend on m, but this dependence is implicit and the estimations can be found for *low, mean* and *high* damage factors. Let us also mention that the simple expression for the Beta *pdf* is obtained from the general one for $q \rightarrow r$, $r \rightarrow s$, a=0 and b=0 ; in other words, the damage loss factor is not measured in percents but it is normalized to $[0,1]$. Alternatively, we suggest that the two parameters that characterize the *pdf* of (3.2) could be statistically estimated using the expected value and the variance of the Beta distribution :

$$E\left[\Delta\right] = \frac{r}{r+s}\ ,\ Var\left[\Delta\right] = \frac{r\,s}{(r+s)(r+s+1)}\ . \tag{3.3}$$

Then, a *damage factor risk matrix* (DFRM) could be obtained, following the procedure presented in Section 2 with the necessary modifications. The damage factors are defined as percentages of replacement value / repair cost with respect to the total value. Seven damage categories can be considered, and (for instance) five earthquake intensity levels (resulting in 7-by-5 DFRMs). Then the probability of (earthquake induced) damage of a structure to exceed the damage state d_i at a specified level of the ground motion intensity can be expressed as

$$P_{i\,|\,k} = P\left(D \ge d_i \middle| Y = y_k\right) = P\left(I_D \ge \ell_i \middle| Y = y_k\right) = \sum_{j=i}^{m} P_{j\,|\,k}\ . \tag{3.4}$$

The probability in Eq.(3.4) cannot be directly evaluated. In many studies, a damage index is used to quantify the damage level experienced by a structure. This random damage index I_D is a function I of structural (strength) parameters and demand (load) variables. Thus I_D=I(R,S). In the model of [4], the demand / load vector and the capacity vector are random scalar functions that respectively depend on the seismic variables S and on resistance variables R that is $L = L(S)$ & $C = C(R)$. The damage index I_D is a composed function through these ("synthetic") variables :

$$I_D = I(C, L) = I(C(R), L(S))\ . \tag{3.5}$$

Each conditional probability as in Eq.(4) results in a fragility curve for damage state In many references, the set of damage state is "linguistically" stated. For instance, it could be (as in [4]) :D={none, minor, moderate, severe, collapse}. Such a qualitative description of the damage states is customary in the fuzzy approaches, and it gives a suggestion for employing fuzzy models in the reliability evaluation of degrading structures. In terms of the damage state set, this description would result in D=[d_0, d_1, d_2, d_3, d_4]. But even when the approach is not fuzzy but a quantitative evaluation of the damage levels is not available, the necessity of using damage indices becomes evident.

The choice of proper (local and global) damage intensity indices is important, and a large diversity of such damage measures have been proposed in more or less recent references. Naturally, the choice essentially depends on the nature of the structure under analysis as well as on the type of loads the component / structure has to support. One of the widely used index is Park & Ang's index ; it is used in [3,4] for the stochastic seismic estimation and Bayesian updating of fragilities in RC frames. It is a problem to evaluate to what extent the use of P & A index is proper for structures in NPP's. Let $D_\ell \left(\ell = \overline{1, L}\right)$ denote the *local* damage intensity (index) at location l or of component l among the L possible locations / components under observation. Then a *global* damage intensity index can be naturally defined as the weighted average of D_l's:

$$D_T = \sum_{\ell=1}^{L} \omega_\ell\, D_\ell \quad \text{with} \quad \omega_\ell \in [0,1]\ \&\ \sum_{\ell=1}^{L} \omega_\ell = 1\,. \tag{3.6}$$

In [4], such weights are associated to the $P\,\&\,A$ index

$$D = \frac{\theta_m}{\theta_u} + \frac{\beta}{M_y\,\theta_u} \int dE \tag{3.7}$$

by $\omega_\ell = E_\ell \,/\, \sum_{\ell=1}^{L} E_\ell$ where E_ℓ = the energy dissipated at location ℓ.

The damage indices (including the $P\,\&\,A$ index) are often defined as taking values in the interval [0,1]. For the five damage states model (earlier mentioned) this interval could be partitioned as $[0, 1] = [0, 0.2) \cup [0.2, 0.4) \cup$

$\cup\ [0.4, 0.6) \cup [0.6, 0.8) \cup [0.8, 1]$. If a weighted average as in Eq.(3.6) is applied on local damage indices $D_l \in [0, 1]$ it clearly follows from this equation that $D_l \in [0, 1]$, too. There are other studies where weights are applied to several classes of structural typology in order to assess their relative importance, but these weights W_i are not subjected to the second condition following Eq.(3.6), that is $W_I \in [0 , 1]$ but it is no more required for their sum to be =1. Anyway, if applied on damage indices, such weights keep them in [0 , 1] : $D_I \in [0 , 1] \Rightarrow W_I D_I \in [0 , 1]$ Problems like the short-term and long-term prediction of the damage states of a structure are considered in [6]. The DSP model there presented also takes into account the behavior in time, including Markov chains. Such an analysis gets beyond the methods we have considered in this contribution, but we suggest that the DLRM's and DFRM's could be adapted for such time-dependent analyses.

LEVEL OF RISK AND SEISMIC DAMAGE PROBABILITY MATRICES

Another stochastic measure over the damage state set is the probability that the structural member / structure enters a damage state d_j after an earthquake occurrence provided its pre-earthquake damage state was d_I

$$P_{i\,j} = P\left(D_{\text{post}} = d_j \mid D_{\text{pre}} = d_i\right). \tag{4.1}$$

Here, d_I , d_j are called damage levels, but they may be clearly assimilated as damage states. In fact, the probabilities in Eq.(4.1) are conditional probabilities, but with a different meaning than t0he ones in Eq.(3.4). These probabilities can be considered as the entries in a stochastic transition matrix denoted (as in [6]) by CDTP - *c*onditional *d*amage *t*ransition *p*robability matrix. Hence

$$\text{CDTP} = \lfloor P_{i\,j} \rfloor_{m+1,\, m+1} \tag{4.2}$$

with $P_{i,j}$ of Eq.(4.1). It is natural to extend the probabilities in (4.1) by an additioonal conditioning on the earthquake intensity, resulting in twice-conditional probabilities, that is probabilities of damage state transitions conditional on a certain earthquake intensity level :

$$P_{i\,j\,k} = P\left(D_{\text{post}} = d_j \mid D_{pre} = d_i \;\&\; Y = y_k\right). \tag{4.3}$$

In Eqs. (4.1) & (4.3) it is natural to assume that $d_j \geq d_i$ This implies that $P_{I\,j} = P\,(D_{post} = d_j \mid D_{pre} = d_i \;\&\; Y = y_k)$ if $j < i$. Consequently, the CDTP matrix is upper-triangular. As regards the conditional probabilities in Eq.(4.3), they would lead to a 3-dimensional matrix

$$\text{CEDTP} = \lfloor P_{i\,j\,k} \rfloor_{m+1, m+1, n} \tag{4.4}$$

i.e., a *c*onditional *e*arthquake (-induced) *d*amage *t*ransition *p*robability matrix. These models could be turned into time-dependent ones if the probabilities in Eqs. (4.1) & (4.3) are considered as being time-dependent :

$$P_{i\,j} = P_{i\,j}(t) \quad \& \quad P_{i\,jk} = P_{i\,jk}(t)\,. \tag{4.5}$$

Certainly, this dependence of time should be specifically modeled for the transition probabilities. Thus, the conditioning event in Eq.(4.1) would have a larger probability for increasing t due to the aging process of the structure or structural member. As regards the earthquake occurrence events, they clearly depend on the earthquake hazard model accepted. The probabilities in Eqs.(4.5) should be respectively defined as

$$P_{i\,j}(t\,,\, t+\Delta t) = P\left(D(t+\Delta t) = d_j \mid D(t) = d_i\right), \tag{4.6}$$

and similarly for $P_{i\,j\,k}(t, t+\Delta t)$. These latter probabilities should be meant as follows : $P_{i\,j\,k}(t, t+\Delta t) =$ the probability that the component / structure enters the damage state d_j at moment $t + \Delta t$ provided it has been in state d_i at $t \in (0 , T)$ a nd an earthquake of intensity (class) Y_k ($1 \leq k \leq n$) has occurred at some moment $\tau \in (t , t + \Delta t)$. The equations (4.1) , (4.3) & (4.6) could be reformulated by means of a damage intensity index, but not this is the essential point.

A (3-dimensional) CEDTP matrix as in Eq.(4.4) can be turned into a (2-dimensional) *s*eismic *r*isk *t*ransition *p*robability matrix - abbreviated SRTP - if it is postmultiplied by a (column) vector of earthquake occurrence probabilities. Let us recall that a vector of earthquake intensities $\mathbf{Y} = [\, y_1 \ldots y_k \ldots y_n]^T$ was earlier considered in Section 2, and the probabilities associated with it can be written as $\boldsymbol{P}\,(\mathbf{Y}) = [\, P\,(y_1) \ldots P\,(y_k) \ldots P\,(y_n)\,]^T$,

$$\text{SRTP} = [Q_{ij}]_{m+1,\,m+1} = \text{CEDTP} \bullet P(Y) = \left[\sum_{k=1}^{n} P_{ijk} \; P(y_k) \right]_{m+1,\,m+1} . \tag{4.7}$$

As earlier mentioned, the time-dependent risk analysis makes necessary to the entries in such a SRTP matrix to depend on time. The seismic vulnerability indices of [8] seem to be more appropriate for such an approach. A (seismic) vulnerability function is defined by

$$F_D(d, T) = \int_0^T \int_0^d \int_0^{y_{\max}} f(\delta | y) \; f_Y(y, \tau) \, dy \; d\delta \; d\tau \tag{4.8}$$

where $f(d \mid y)$ is the conditional damage density function over the Y earthquake intensity and $f_y(y, t)$ is the time-dependent intensity density function. $F_D\ (d\ ,\ T\ )$ in Eq.(4.8) is the damage cumulative distribution function over the lifetime / operational cycle $[\ 0\ ,\ T\ ]$ of the component or structure. It becomes a time-dependent damage *cdf* if T is replaced by $t \in [\ 0\ ,\ T\ ]$ in Eq.(4.8). The model given by Eq.(4.8) is applicable when the variables there involved can be considered as being random, independent and continuous on their definition domains. Certainly, specific distribu-tional assumptions must be assumed for these variables.

CONCLUDING REMARKS

The damage / loss states are discretized into a finite number of classes that induce a partition over the interval of possible δ-values. The range of the earthquake intensity parameter is also partitioned, and the classical fragility model is reformulated so that DLRM (damage level risk matrices) and DFRM (damage factor risk matrices) can be defined and used. The discrete probability distribution (DPD) approach is also used, making possible an alternative approach for estimating the probabilities of the stochastic DLR and DFR matrices.

REFERENCES

[1] Kaplan, S., Perla, H.F., Bley, D.C.: A methodology for seismic risk analysis of NPPs. *Risk Analysis* **3**(3), 169-180, 1983

[2] Yamaguchi, A.: Seismic fragility analysis of the heat transport problem of LMFBR considering partial correlation of multiple failure modes. *Trans. SMiRT 11* (Tokyo-Kyoto), Vol.M, 91-96, 1991

[3] Singhal, A.: Stochastic seismic damage estimation in reinforced concrete frames, *Proc. 11 WCEE*, Paper No. 1863, pp.1-8, Acapulco MX, June 1996

[4] Singhal, A., Kiremidjian, A.S.: Bayesian updating of fragilities with application to RC frames. *Journal of Structural Engineering*, Vol.124, pp.922-929, 1998

[5] Huo, J.-R., Hwang, H.H.W.: Fragitlity in Memphis buildings. *Proc. 11wcee* (Acapulco, June 1996), Paper 1056, Abstracts p.1119 & preprint, 1996

[6] Hamamoto, T.: Stochastic long-term damage prediction of multi-story buildings. *Proc. 11 WCEE*, P.989, pp.1-8, Acapulco MX, June 1996

[7] Kunnath, S.K., Valles-Mattox, R.E., Reinhorn, A.M.: *Proc. 11 WCEE* Acapulco, June 1996), paper No.1300, 1996

[8] Applied Technology Council: *Earthquake Damage Evaluation Data for California*, ATC-13, Redwood City, California: ATC, 1985

[9] Vulpe A., Carausu, A.: Evalualuation of seismic damagebility of structures using fragility models and damage risk matrices, *Trans. SMIRT 14* (Lyon), Vol.M, 175-182, 1997

[10] Carausu, A., Vulpe, A.: Fragility estimation for seismically isalated models structures by HCLPF values and by bi-linear regression, *Trans. SMIRT 12* (Frankfurt), Vol.A, 237-242, 1993