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ESTIMATION OF DEEP UNDERGROUND VELOCITY STRUCTURES BY INVERSION OF SPECTRAL RATIO OF HORIZONTAL TO VERTICAL COMPONENT IN P-WAVE PART OF EARTHQUAKE GROUND MOTION

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SUMMARY

A new method for estimating deep underground velocity structures to the uppermost layer in the earth's crust is proposed. The method uses records of small or medium-size earthquakes observed at ground surface and inverts the spectral ratio of horizontal to vertical component in the P-wave part. The methodology is based upon the fact that the spectral ratio in P-wave part strongly depends on the transfer function of sedimentary layers to a P-wave incidence. A genetic algorithm (GA) is applied for the inversion, however three kinds of constraints and an elite preservation are incorporated into a GA in order to make powerful the search ability. A horizontally layered half space is assumed as a structural model in the inversion and the S-wave velocity, the P-wave velocity and the thickness of each layer and the P-wave incident angle are chosen as the parameters to be optimized. The applicability of the inversion method is investigated using synthetic and observed data. Consequently, It is demonstrated that the proposed method is useful as a convenient method for estimating both the S-wave velocity and the P-wave velocity structures of sedimentary layer ling over the uppermost layer in the earth's crust.

INTRODUCTION

The clarification of deep underground velocity structures, in particular S-wave velocity structures to the uppermost layer of the earth’s crust is one of the most important subjects in earthquake engineering because the effects of deep underground velocity structures on strong ground motions are considerably large. The reflection method and the refraction method are generally used to investigate deep underground structures. However, those methods are unsuitable for the investigation of S-wave velocity structures, therefore the inversion method of surface-wave dispersion data obtained from microtremor array observations has become of interest recently as one of useful tools to extract S-wave velocity structures [e.g., Horike,1985; Okada et al. ,1990; Sato et al.,1998].

On the other hand, earthquake records have not been positively used to investigate subsurface structures up to now. However, characteristics of subsurface structures are always contained in earthquake records regardless of the size of earthquakes. So, we tried to estimate velocity structures of deep sedimentary layers using records of small or medium-size earthquakes observed at ground surface. A lot of records have been accumulated in sites where earthquake observations have already been done. It is an aim of this study to put those earthquake records to practical use. The spectral ratio of horizontal to vertical component in P-wave part of earthquake ground motions is inverted for the purpose. The methodology is based upon the fact that the spectral ratio strongly depends on the transfer function of deep underground structures to a P-wave incidence [e.g., Kobayashi et al,1990; Abe et al,1991]. A genetic algorithm (GA) is applied for the inversion. GA’s have been already used in several other fields [e.g., Goldberg,1989] and in seismology and earthquake engineering[e.g., Stoffa and Sen,1991;Yamanaka and Ishida,1996]. It is pointed out from those applications that GA’s are effective as an inversion algorithm and have high robustness. In our inversion, the S-wave velocity, the P-wave velocity and the thickness of each layer and the incident angle of P-wave are chosen as the parameters to be optimized because those parameters affect the spectral ratio.

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Accordingly, it is a goal of this study to investigate whether both of the S- and P-wave velocity structures can be estimated by our method or not.

In this paper, first, we explain the procedure of the genetic inversion method. Next, the inversions of synthetic data are carried out using realistic structural models in order to confirm the validity of the method. Finally, the applicability of the method to observed data is investigated. The data used are observed at a site where PS-logging was carried out to the uppermost layer in the earth's crust, so the accuracy of the inverted model is confirmed directly comparing with the PS-logging model.

2. PROCEDURE OF THE INVERSION

2.1 HANDLING OF PARAMETERS

A horizontally layered half-space is assumed as a structural model of sedimentary layers in the inversion. A theoretical spectral ratio of horizontal to vertical component compared with observed one is calculated assuming a P-wave incidence because the spectral ratio in P-wave part of earthquake ground motion is the target of the inversion. In the inversion, the S-wave velocity, the P-wave velocity and the thickness of each layer and the incident angle of P-wave are chosen as the parameters to be optimized as mentioned above. However the P-wave velocity in the basement is fixed at 5.5km/s because P-wave velocities of about 5.5km/s have been reported a lot to the basement. This has an effect to avoid a trade-off between velocities and thickness. The remained parameters, namely the density and the Q value of each layer are handled as follows:

The density of each layer is set linking to an S-wave velocity by the following equation (1).

$$\rho=0.67Vs^{1/2}+1.40 \qquad (1)$$

in which ρ is the density in g/cm^3 and *Vs* is S-wave velocity in km/s. Equation (1) was obtained from the regression analysis of the existing deep PS-logging data at six sites in Japan [Omote et al,1984; Kionoshita, 1986; Shimizu et al.,1986]. The spectral ratio is not affected very much even if densities change somewhat. Therefore equation (1) can be used without trouble.

The Qs and *Qp* values of each layer are fixed according to equations (2) and (3) referring to previous studies.

$$Qs=50f \qquad (2)$$

$$Qp=Qs/2 \qquad (3)$$

where *f* is frequency. Equation (2) was obtained for the whole sedimentary layer of Quaternary and Tertiary from deep borehole data in the Kanto area by Kinoshita (1986). Equation (3) was obtained from shallow borehole data in the Sendai area by Abe et al. (1993) and from deep borehole data at Tomioka, Fukushima Pref. by Tohdo et al. (1995). Though regional characteristics of *Q* values should be considered, equations (2) and (3) might be used to first approximation.

2.2 INVERSION ALGORITHM

We first used a simple GA as the basic algorithm of the inversion referring to Stoffa and Sen(1991), and Yamanaka and Ishida(1996). However it was found that our purpose cannot be achieved with high accuracy by a simple GA. because there are many parameters to be optimized in our problem. So, we modified a simple GA to make powerful the search ability. A simple GA is essentially composed of three operators, namely the selection, the crossover and the mutation. In our algorithm, three kinds of constraints and an elite preservation are incorporated into a simple GA as shown in Figure 1. Those three constraints are as follows: The first constraint uses PS-P time, that is the difference in time between a PS-converted wave generated at the basement (the uppermost of the earth's crust) and a direct P-wave. The PS-P time depends only on velocity structure of sedimentary layers lying over the basement, therefore it is very effective as a constraint condition. The constraint of PS-P time can be incorporated easily into the inversion because Kobayashi et al. (1998) indicated that the PS-P time can be detected easily from the same data used in this inversion by means of the receiver function proposed by Langston(1979). In this constraint, the permitted limits of PS-P time are set within ±10% of the detected value. The second constraint uses the velocity formation of deep sedimentary layers. In this constraint, we assume that velocities become faster as layers become deeper. The assumption might be approximately appropriate in deep sedimentary layers. The third constraint uses the depth of basement. In this constraint, the permitted limits of the basement depth are set within the average±standard deviation calculated from the basement depths of all individuals in each generation. There seem to be a problem in this constraint because the true depth of the basement is not a priori information. However, the possibility that the true depth deviate from the permitted limits is very little because the standard deviation is very large when a generation is young, on the other hand, the average approaches the true depth when a generation evolves. The effectiveness of this constraint will be shown later in the inversion of synthetic and observed data.

The whole procedure and the elite preservation are as follows: In the GA, the search areas for each unknown parameter are defined and each parameter in the search areas is coded with an *n*-bit binary string. The binary strings of all parameters are concatenated in the genetic operation. After the above preparation, an initial population with *N* individuals which satisfy the two constraints of PS-P time and velocity formation is generated in the search areas using random numbers. In the calculation of fitness, fitness values of each individual are calculated based upon a predetermined fitness function. In our algorithm, the fitness function is defined by the reciprocal of a misfit function that is the squares-sum of the residuals between observed and calculated spectral ratios. In the elite preservation, a comparison of the best individual in the current generation with that in the previous generation is made and the latter is added in the current generation only if the latter has higher fitness than the former. In the selection, a new population is reproduced according to the fitness of each individual. The roulette rule is used well as a selection method, where each current individual is selected probabilistically in proportion to its fitness. Therefore, individuals with higher fitness have higher probability of contributing offspring in the next generation. In the crossover, all individuals reproduced in the selection are mated at random and some lower partial strings for the mated individuals are exchanged each other only if a generated random number is smaller than a given crossover probability. The crossover position in the string is selected randomly. In the mutation, randomly selected bits in the concatenated strings are changed from 0 to 1 or vise versa. After the mutation, three constraints by the PS-P time, the velocity formation and the basement depth are executed in order. Consequently, the individuals that satisfy all the three constraints are selected. Though the number of surviving individuals decrease at this stage, the number of individuals returns to the origin number in the selection procedure. These procedures are repeated up to a predetermined generation.

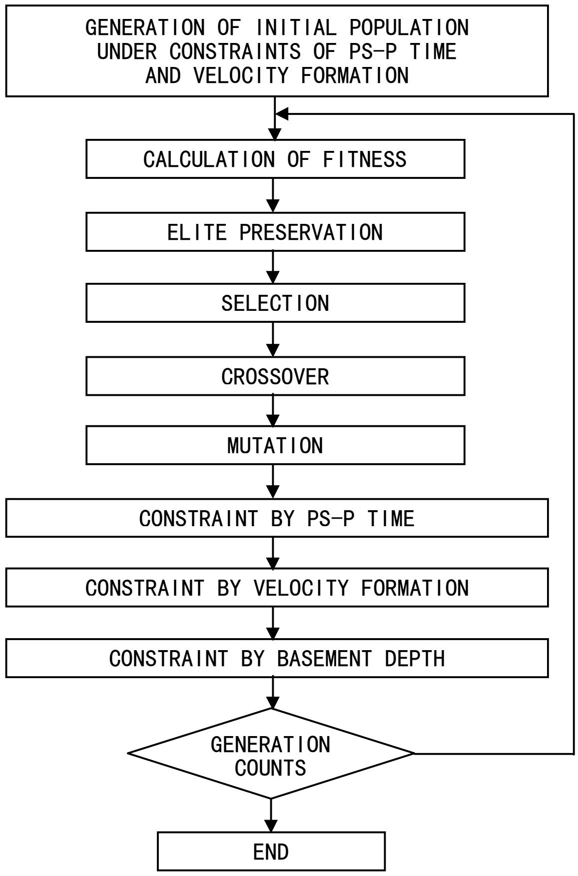


Figure 1: Procedure of the inversion

Table 1: Structural model A

No.	Thickness (m)	Vs (m/s)	Vp (m/s)	ρ (g/cm^3)
1	400	800	2000	2.1
2	600	1500	2800	2.3
3	∞	3000	5500	2.5

Table 2: Structural model B

No.	Thickness (m)	Vs (m/s)	Vp (m/s)	ρ (g/cm^3)
1	2.2	94	320	1.7
2	3.4	170	970	1.7
3	3.1	215	970	1.9
4	6.8	215	1550	1.9
5	8.3	170	1550	1.8
6	5.0	225	1650	1.8
7	9.0	340	1650	1.9
8	8.7	320	1650	1.8
9	28.5	490	1700	2.0
10	325	800	2000	2.1
11	600	1500	2800	2.3
12	∞	3000	5500	2.5

3. INVERSION OF SYNTHETIC DATA

We tested the modified genetic inversion method mentioned above using synthetic spectral ratio of horizontal to vertical component. The two structural models shown in Tables 1 and 2 were used for calculating the synthetic spectral ratio. A three-layered model is assumed in the model A in Table 1. The top two layers correspond to the Tertiary sediments and the basement with S-wave velocity of 3.0km/s, the uppermost of the earth's crust. The depth of the basement is 1.0km. The PS-P time calculated from the model becomes 0.50sec on the assumption of P-wave incident angle of 50 degrees. On the other hand, a twelve-layered model is assumed in the model B in Table 2. This model was constructed replacing the portion up to a depth of 75m in the model A by nine layers with low S-wave velocities. Those nine layers correspond to the Quaternary sediments. The PS-P time of this model is 0.66 sec. The spectral ratios calculated from the models A and B assuming the P-wave incident angle of 50 degrees are shown in Figure 2. The solid line indicates the spectral ratio from the model A and the dotted line, that from the model B. Both of the spectral ratios are smoothed by a Parzen window with a bandwidth of 0.2Hz because the same smoothing is used when estimating observed spectral ratios. Comparing both of the spectral ratios, there is a remarkable difference between them in the range longer than 1Hz.

The search areas in the inversion for the models A and B are shown in Tables 3 and 4, respectively. The Quaternary sediments are considered also as the unknown parameters in the inversion for the model B, however a four-layer model was assumed for the Quaternary sediments by the following two reasons. One is that it is very difficult to identify the number of layers for the Quaternary sediments in a general case, the other is that a high resolution cannot be expected for the Quaternary sediments. The search area for the incident angle was set from 40 to 60 degrees. The population size was set at 4000, the crossover probability at 0.7 and the mutation probability at 0.03. All the unknown parameters were coded with twelve-bit binary strings. We carried out ten inversions with different initial random numbers and determined a final optimal model by averaging the models obtained from each inversion. The number of generations in each inversion was set at 50.

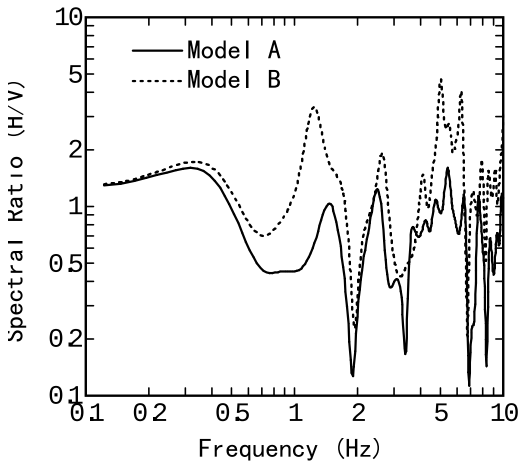


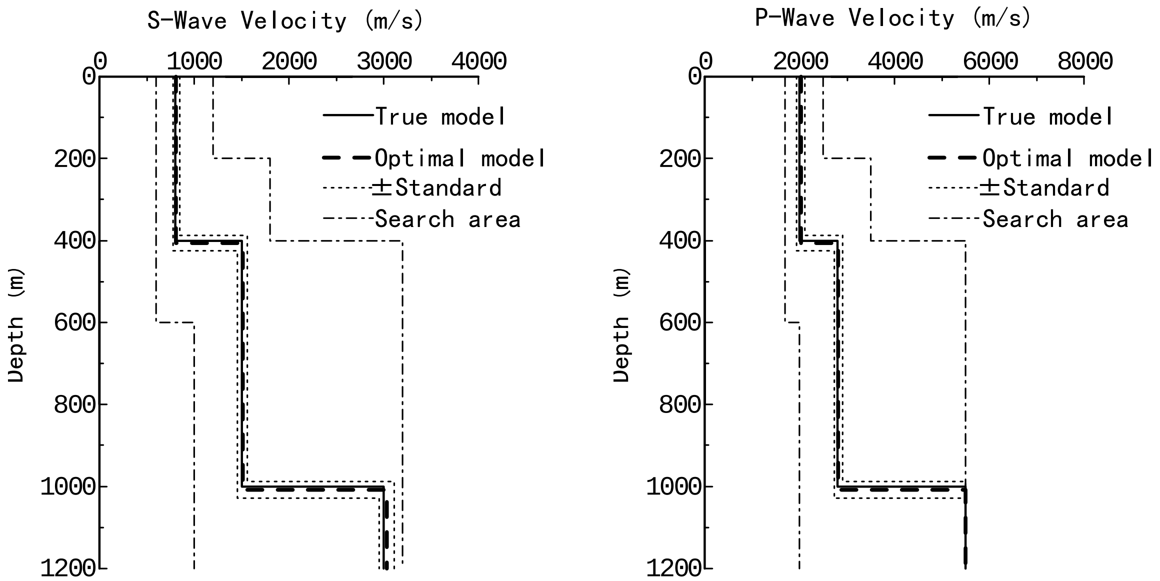
Figure 2: Spectral ratio calculated from models A and B

Table 3: Search areas for model A inversion

No	Thickness (m)	Vs (m/s)	Vp (m/s)
1	200– 600	600–1200	1700–2500
2	200–1200	1000–1800	2200–3500
3	∞	2500–3000	5500

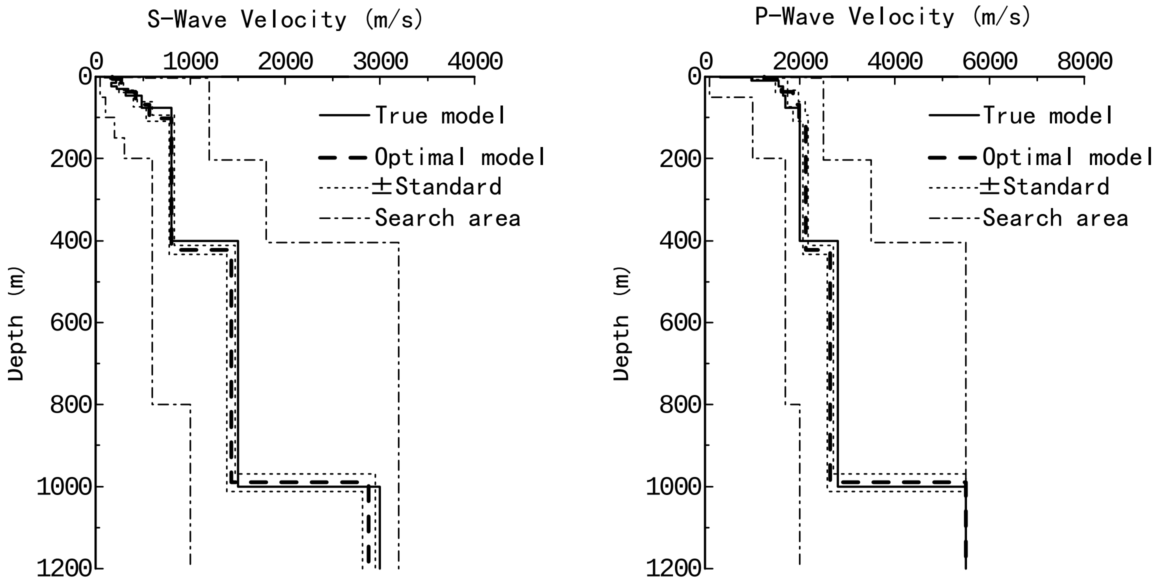
Table 4: Search areas for model B inversion

No	Thickness (m)	Vs (m/s)	Vp (m/s)
1	1–50	50–300	100–1800
2	1–50	100–300	1000–1800
3	1–50	200–500	1000–2200
4	1–50	300–700	1000–2200
5	200– 600	600–1200	1700–2500
6	200–1200	1000–1800	2000–3500
7	∞	2500–3000	5500



(a) S-wave velocity profile **(b) P-wave velocity profile**

Figure 3: Comparison of the optimal model from model A inversion with the true model



(a) S-wave velocity profile **(b) P-wave velocity profile**

Figure 4: Comparison of the optimal model from model B inversion with the true model

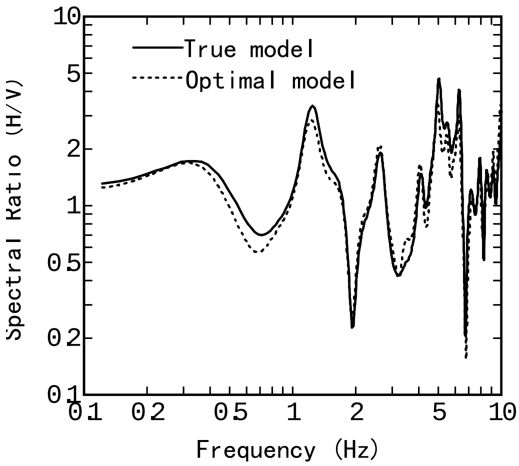


Figure 5: Comparison of spectral ratios in model B inversion

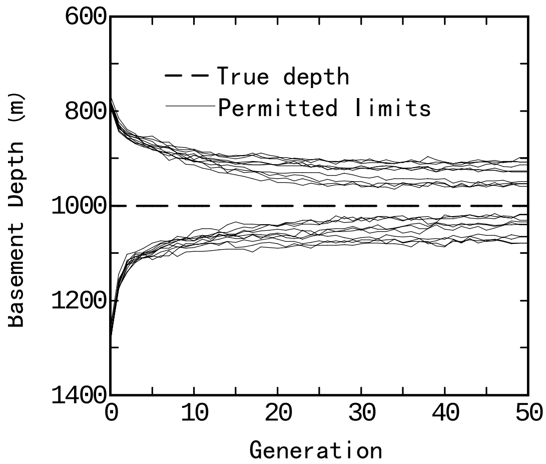


Figure 6: Permitted limits of basement depth in model B inversion

The comparison of the optimal model obtained from the inversion for model A with the true model is shown in Figure 3. Both the S-wave velocity and P-wave velocity profiles of the optimal model are in fine agreement with the true profiles and the standard deviation of ten inversions is very small. The inverted incident angle was 49 degrees almost equal to the true angle. The same comparison in the inversion for model B is shown in Figure 4. The spectral ratio calculated from the model B is affected much by the Quaternary sediments as indicated above, but a fairly good agreement is obtained. The inverted incident angle was 55 degrees in this case. The fitness of spectral ratios in the inversion for model B is shown in Figure 5. The fitness is considerably good. Figure 6 shows the resultant permitted limits from the constraint of the basement depth in the inversion for the model B. The permitted limits in each inversion covers the true basement depth of 1km at each generation. This shows that the constraint acts effectively, however it should be noted that the constraint of the basement depth becomes effective after the two constraints of the PS-P time and the velocity formation are executed.

4. INVERSION OF OBSERVED DATA

We applied the modified GA inversion method to the observed spectral ratio at Tomioka, Fukushima Pref., Japan where a detailed velocity structure to the basement has been obtained by PS-logging [e.g.,Omote,1984]. Tomioka site is located on the Tertiary rock and the basement with S-wave velocity of 2.8km/s and P-wave velocity of

5.5km/s is confirmed at a depth of 920m. Records of 27 earthquakes with good S/N ratios were selected for the inversion. The list is shown in Table 5. Many of the earthquakes occurred offshore in Fukushima Pref. Mj by the Japan Meteorological Agency is in the range of 4.0 to 6.7. The average and standard deviation of the spectral ratios of horizontal to vertical component in the P-wave part obtained from the 27 earthquake records are shown in Figure 7. The spectral ratios were estimated using radial and vertical spectra smoothed by a Parzen window with a bandwidth of 0.2Hz. The spectral ratios in each earthquake have common characteristics that are expressed well by the averaged spectral ratio. The standard deviation is comparatively large in the range lower than 0.3Hz where noises might mixed with the observed. The averaged spectral ratio in the range from 0.8 to 10Hz was inverted in this study because it has been indicated by Kobayashi et al. (1998) that the observed spectral ratios near 0.5Hz at Tomioka site have effects on the deeper crust structures.

A four-layered model was assumed in the inversion of the observed data at Tomioka site. The search areas of the parameters to be optimized are shown in Table 6. The search areas of the S- and P-wave velocities for each layer cover the velocity distribution reported for the Tertiary sedimentary layers enough. The search areas of the thickness of each layer have a sufficiently wide range to the PS-logging model. The search area of P-wave incident angle was set from 40 to 70 degrees. The PS-P time at Tomioka site has been obtained for 0.56sec by Kobayashi et al. (1998). This time was used for the constraint of the PS-P time. We used the same GA conditions as those in the inversion of the synthetic data. The theoretical spectral ratio compared with the observed one was smoothed by the same Parzen window mentioned above.

The comparison of the inverted optimal model with the PS-logging model is shown in Figure 8 and the comparison of the theoretical spectral ratio calculated from the optimal model with the observed one, in Figure 9. Both the S-wave velocity and P-wave velocity profiles of the optimal model are in good agreement with the PS-logging profiles beyond our expectation, furthermore, the standard deviation of ten inversions is very small as well as in the inversion of the synthetic data. The theoretical spectral ratio also is consistent with the observed one. The inverted incident angle of P-wave was estimated as 57 degrees. Though the validity of this incident angle cannot be certified, it is inferred to be approximately adequate for the earthquakes used in this inversion. Figure 10 shows the resultant permitted limits from the constraint of the basement depth. The middle of the permitted limits at each generation corresponds to the layer of the weathering basement with P-wave velocity of about 4.5km/s, therefore the constraint acts effectively also in the inversion of observed data.

Table 5: List of earthquakes used in the inversion at Tomioka site

No	Origin time	Mj	Depth (km)	Δ (km)
1	1982/08/28 21:12	4.9	60	32
2	1982/09/14 03:03	5.0	60	43
3	1983/07/02 07:03	5.8	54	50
4	1983/07/18 14:11	5.2	35	60
5	1983/09/02 12:05	5.2	49	74
6	1984/03/26 03:59	5.2	42	60
7	1984/05/01 10:15	5.2	46	59
8	1984/10/25 15:57	5.2	53	60
9	1984/12/19 04:35	5.3	44	55
10	1985/05/11 19:40	5.3	45	59
11	1985/07/29 04:33	5.5	90	28
12	1985/08/12 12:49	6.4	52	89
13	1986/10/14 06:17	5.7	53	36
14	1987/02/05 21:23	6.4	30	94
15	1987/02/06 22:16	6.7	35	90
16	1987/04/07 09:40	6.6	44	77
17	1987/04/20 19:09	4.9	49	62
18	1987/04/23 05:13	6.5	47	62
19	1987/04/24 13:32	4.6	52	53
20	1987/04/30 22:46	5.0	50	48
21	1987/09/24 13:55	5.8	41	83
22	1987/10/04 19:27	5.8	42	65
23	1988/01/26 05:20	5.6	34	62
24	1988/01/26 05:35	4.7	39	66
25	1988/03/03 16:36	4.5	48	49
26	1989/04/28 00:26	4.9	52	49
27	1990/12/27 11:01	4.0	52	43

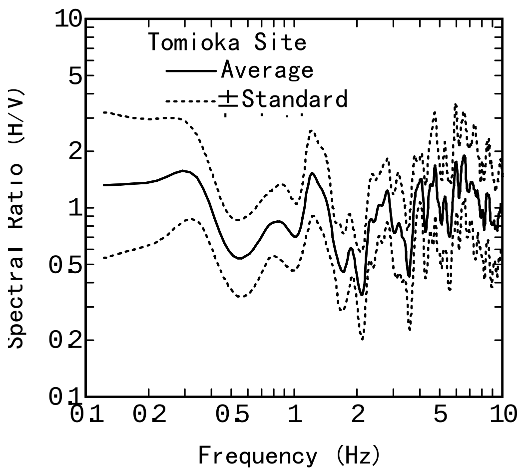
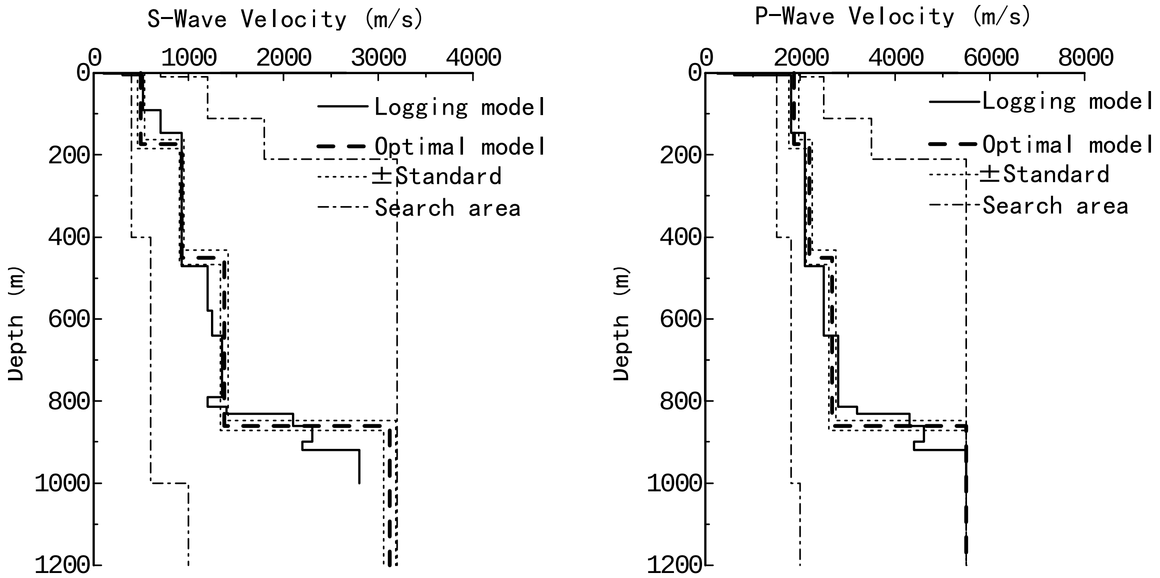


Figure 7: Observed spectral ratio at Tomioka site

Table 6: Search areas for the inversion of observed data at Tomioka site

No	Thickness (m)	Vs (m/s)	Vp (m/s)
1	10–400	400–700	1500–2000
2	100–600	600–1200	1800–2500
3	100–600	1000–1800	2000–3500
4	∞	2500–3200	5500



(a) S-wave velocity profile **(b) P-wave velocity profile**

Figure 8: Comparison of the optimal model with the PS-logging model at Tomioka site

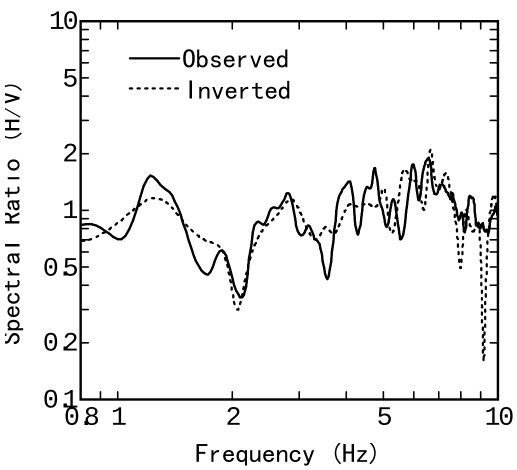


Figure 9: Comparison of spectral ratios in the inversion at Tomioka site

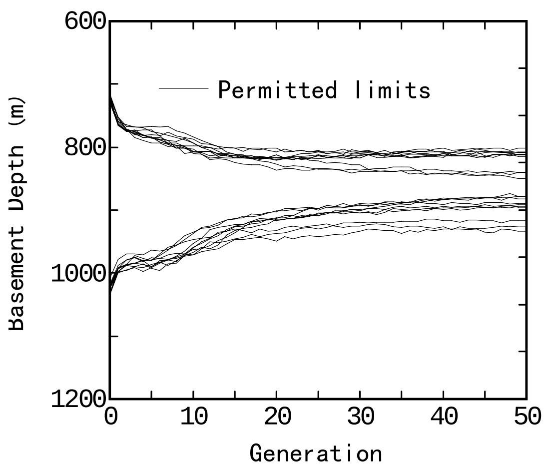


Figure 10: Permitted limits of basement depth in the inversion at Tomioka site

5. CONCLUSIONS

An inversion method of spectral ratios of horizontal to vertical component in P-wave part of earthquake ground motions was proposed in order to estimating deep underground velocity structures to the uppermost layer in the earth's crust. A genetic algorithm was applied as the basic algorithm of the inversion, however three kinds of constraints and the elite preservation were incorporated into the genetic algorithm in order to make powerful the search ability. First, the inversions of synthetic data were carried out in order to confirm the applicability of the inversion method for our purpose. As a result, it was found that the method is applicable also for structural models with soft Quaternary sediments and the modified genetic inversion algorithm acts very effectively. Next, the applicability of the method for observed data were investigated. The data used were observed at Tomioka, Fukushima Pref. Japan where a detailed PS-logging model to the uppermost layer in the earth’s crust has been obtained. As a result, it was found that the inverted optimal model gives fairly good agreement with the PS-logging model. The proposed inversion method is a very convenient method because the purpose can be achieved using records of small or medium-size earthquakes observed at ground surface. A strong point of this inversion method is that both the S-wave velocity and the P-wave velocity structures of deep sedimentary layers can be deduced with high accuracy. Consequently the proposed method can provide a schema that earthquake observation points turn into underground exploration points.

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