DESIGN METHOD OF CONTROLLING THE MAXIMUM RESPONSE DISPLACEMENTS OF THE BUILDING

Takao NISHIKAWA[1], Cheng WU[2] And Kazushige YAMAMURA[3]

SUMMARY

This paper presents the design method, especially for high-rise reinforced concrete building, of the distribution of the stiffness and yield strength of each floor along height for the response which designates the response deformation of elasticity and elasto-plasticity of the design building, when some ground shaking intensity and spectral characteristics are assumed.

By assuming each floor weight of the building and period from the empirical formula in a past, the proposal for following calculation methods or design is carried out.

1) The method for calculating the stiffness of each floor when the maximum response displacement of the each floor sets optionally, by assuming the participation function of the building most upper floor, has been presented.

2) Under the assumption that simultaneously each floors yields, the calculation method of yield strength of each floor from that the designated response ductility factor of each floor can be obtained, has been also presented.

To show the validity of this proposed procedure, linear and non-linear response analyses were conducted.

INTRODUCTION

The Building Standards Law was revised in 1998 in Japan, which tends to shift to design method of the performance regulation type. The way is opened from design method of the conventional specification type to design method of the performance regulation type, and it seems to totally shift to the system of design method of the performance regulation type in the soon close future. In the conventional design method, the designer became a system which inevitably satisfied a lowest demand, if it is designed, by conforming to the standard, and it was often, while it not generally ask it, and while the final performance of the structure is consequentially non-clear.

However, in the design method of the performance regulation type, the designer grasps the behavior for external force of the structure variously, and it becomes the occurrence of the necessity of explaining the performance. And, it has been proven that the seismic damage of the structure that the idea of performance design was designed by high-rise buildings and base-isolated structure, etc. adopting to some extent, was very slight in Hyogo-ken Nanbu Earthquake, 1995. In the design method of future performance regulation type, earthquake vibration intensity of construction planned site is rationally assumed, and it is required that it is designed in order to estimate the quantitative response for it, and they are client or base of the mutual agreement with the society, and it is safe for the response. Though the research on the evaluation of the performance of the structure would be carried out in great numbers at present, the proposal of the concrete design method is required which would be able to realize the required performance. For those purposes, in this paper, the method to determine the stiffness and strength distribution along height, which can control the maximum displacement response of structure, is proposed.

[1] *Graduate School of Engineering, Tokyo Metropolitan University, Tokyo, JapanEmail: tanishi@arch.metro-u.ac.jp*
[2] *WING CORPORATION, Kanagawa, Japan Email: cwu@tky3.3web.ne.jp*
[3] *Graduate School of Engineering, Tokyo Metropolitan University, Tokyo, Japan Email: kyamamur@arch.metro-u.ac.jp*

ELASTIC STIFFNESS AND YIELD STRENGTH OF EACH STORY

In this analysis, building is replaced to lumped mass shear model with story stiffness K_i and story mass m_i, where K_i : stiffness of i th story, m_i : mass of i th story.

The design method of elastic stiffness

It is assumed possible to get roughly the maximum value of response displacement of each story, if the participation function is given, by equation (1).

$$\left|y_i\right|_{\max} = \sqrt{\sum_{s=1}^{n}\left|{}_s\beta\cdot{}_s u_i\cdot{}_s D\right|^2} \tag{1}$$

where y_i : relative response displacement to the base, ${}_s\beta\cdot{}_s u_i$: participation function of the s th order and ${}_s D$: displacement response spectrum of s th order.

It is possible to show the relationship between vibration mode, elastic stiffness, mass and frequency in undamped oscillation equation (2).

$$\left(\mathbf{K}-\omega^2\cdot\mathbf{M}\right)\mathbf{u}=\mathbf{0} \tag{2}$$

where $\mathbf{K}$: stiffness matrix, $\mathbf{M}$: mass matrix and ω : circular frequency.

The response displacement can be expressed by participation function, elastic stiffness, proper period, mass, displacement response spectrum, if it is developed, by utilizing vibration mode term of equation (1) for the relationship between vibration mode and elasticity stiffness of equation (2). It is possible here to require elastic stiffness from the designed response displacement, if it makes participation function, proper period, mass and displacement response spectrum known.

To begin with, equation (1) which is the simplified formula of elastic response displacement maximum value in each story is converted like equation (3).

$$\left.\begin{aligned}
y_n^2 &= \left({}_1\beta\cdot{}_1u_n\cdot{}_1D\right)^2+\left({}_2\beta\cdot{}_2u_n\cdot{}_2D\right)^2+\cdots+\left({}_n\beta\cdot{}_nu_n\cdot{}_nD\right)^2\\
&\vdots\\
y_i^2 &= \left({}_1\beta\cdot{}_1u_i\cdot{}_1D\right)^2+\left({}_2\beta\cdot{}_2u_i\cdot{}_2D\right)^2+\cdots+\left({}_n\beta\cdot{}_nu_i\cdot{}_nD\right)^2\\
&\vdots\\
y_1^2 &= \left({}_1\beta\cdot{}_1u_1\cdot{}_1D\right)^2+\left({}_2\beta\cdot{}_2u_1\cdot{}_2D\right)^2+\cdots+\left({}_n\beta\cdot{}_nu_1\cdot{}_nD\right)^2
\end{aligned}\right\} \tag{3}$$

where suffix i with the right means the story of the structure, and suffix s with the left means the degree of the mode. In the meantime, equation (2) which is the undamped oscillation equation is converted like equation (4).

$$\left(k_n-{}_s\omega^2\cdot m_n\right){}_su_n-k_n\cdot{}_su_{n-1}=0 \tag{4-a}$$
$$-k_i\cdot{}_su_{i-1}+\left(k_i+k_{i+1}-{}_s\omega^2\cdot m_i\right){}_su_i-k_{i+1}\cdot{}_su_{i+1}=0 \qquad i=2,3,\cdots,n-1 \tag{4-b}$$
$$\left(k_1+k_2-{}_s\omega^2\cdot m_1\right){}_su_1-k_2\cdot{}_su_2=0 \tag{4-c}$$

From equation (4-a), vibration mode ${}_su_{n-1}$ of the $(n-1)$th story is expressed by vibration mode ${}_su_n$ of the n th story as following.

$${}_su_{n-1}=\frac{k_n-m_n\cdot{}_s\omega^2}{k_n}{}_su_n \tag{5}$$

From equation (4-b) and equation (5), vibration mode ${}_su_{n-2}$ of the $(n-2)$th story is expressed by vibration mode ${}_su_n$ of the n th story as following.

$$ {}_s u_{n-2} = \frac{\left(k_{n-1} + k_n - m_{n-1} \cdot {}_s\omega^2\right)\left(k_n - m_n \cdot {}_s\omega^2\right) - k_n^2}{k_{n-1} \cdot k_n} {}_s u_n \tag{6} $$

It will become that vibration mode ${}_s u_{n-3}$ of the $(n-3)$th story is similarly expressed from equation (4-b) and equation (6) by vibration mode ${}_s u_n$ of the n th story.

$$ {}_s u_{n-3} = \left\{ \frac{\left(k_{n-2} + k_{n-1} - m_{n-2} \cdot {}_s\omega^2\right)\left(k_{n-1} + k_n - m_{n-1} \cdot {}_s\omega^2\right)\left(k_n - m_n \cdot {}_s\omega^2\right)}{k_{n-2} \cdot k_{n-1} \cdot k_n} - \frac{\left(k_{n-2} + k_{n-1} - m_{n-2} \cdot {}_s\omega^2\right)k_n^2 + \left(k_n - m_n \cdot {}_s\omega^2\right)k_{n-1}^2}{k_{n-2} \cdot k_{n-1} \cdot k_n} \right\} {}_s u_n \tag{7} $$

And, the vibration mode becomes the participation function ${}_s\beta \cdot {}_s u$, when it puts on the participation factor in both sides of equation (5) to (7). It is possible to express participation function of each story in the participation function of most upper story and stiffness of each story, if such operation is carried out one after another to the first story. The response displacement of each story can be shown by stiffness and participation function of most upper story, when this relation is utilized for participation function term of equation (3). That is to say, it is possible to require from the response displacement on the assumption of elastic stiffness of each story by making mass, circular frequency (proper period), displacement response spectrum and participation function of most upper story known.

These above equations regarding to participation function, stiffness, mass, frequency put into equation (3), and then we can get the relationship between the maximum displacement and story stiffness, if m_i, ω are assumed to be constants. Although the maximum displacement is determined from upper most participation function and response displacement spectrum, maximum displacements of another story can be set optionally. After this procedure, story stiffness K_i can be obtained from modified equation (3).

The design method of yield strength

The calculation formula of yield strength by which response ductility factor of each story becomes designates value is developed by the assumption that maximum potential energy of each story of elastic system is equal to that of the elasto-plastic system (figure 1). The maximum potential energy in elastic system is shown in equation (8).

$$ S_e = \frac{1}{2} Q_e \cdot \delta_e \tag{8} $$

where Q_e: maximum elastic response shear force, δ_e: maximum elastic response displacement.

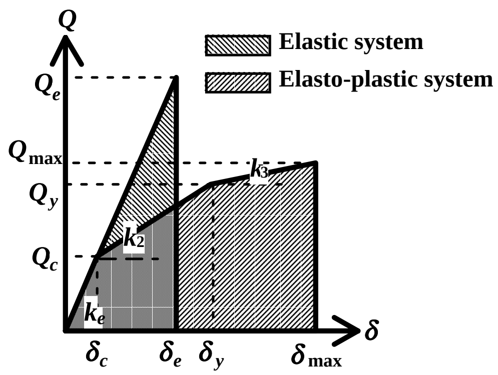

Figure 1: The maximum potential energy

In the meantime, the maximum potential energy in elasto-plastic system with 3 polygonal line hysteretic characteristics is shown in equation (9).

$$ S_p = \frac{1}{2} Q_c \cdot \delta_c + \frac{1}{2}\left(Q_c + Q_y\right)\left(\delta_y - \delta_c\right) + \frac{1}{2}\left(Q_{\max} + Q_y\right)\left(\delta_{\max} - \delta_y\right) \tag{9} $$

where Q_c: shear force at cracking, δ_c: displacement at cracking, Q_y: yield strength, δ_y: yield displacement, $Q_{\max}$: maximum response shear force and $\delta_{\max}$: maximum response displacement.

The maximum displacement, yield displacement and cracking displacement are shown by the following equation with ductility factor μ, Q_c and elastic stiffness k_e.

$$\delta_{\max} = \delta_y \cdot \mu \qquad \delta_c = Q_c / k_e \tag{10}$$

In addition, equation (9) is developed like equation (11), if equation (10) is substituted in equation (9).

$$S_p = \frac{1}{2}\frac{\beta \cdot Q_y^2}{\alpha \cdot k_e} - \frac{1}{2}\frac{\beta \cdot Q_y^2}{k_e} - \frac{1}{2}\frac{Q_y^2}{\alpha \cdot k_e} + \frac{\mu \cdot Q_y^2}{\alpha \cdot k_e} + \frac{(\mu - 1)^2}{2}\frac{\eta \cdot Q_y^2}{\alpha^2 \cdot k_e} \tag{11}$$

where $\alpha = Q_y / (\delta_y \cdot k_e)$, $\eta = k_3 / k_e$, $\beta = Q_c / Q_y$.

S_e of equation (8) and S_p of equation (11) is equal, because that the maximum potential energy of each story of elastic system is equal to maximum potential energy of the elasto-plastic system is assumed. Then, the following equation is deduced.

$$Q_y^2 = \frac{Q_e \cdot \delta_e \cdot \alpha^2 \cdot k_e}{\alpha(2\mu - 1) - \alpha^2 \cdot \beta + \alpha \cdot \beta + \eta(\mu - 1)^2} \tag{12}$$

From equation (12), it is possible to get the required story yield shear force Q_y, if maximum response shear force Q_e, maximum response displacement δ_e of elastic system, ratio α between the initial stiffness and yield stiffness, ratio η between initial stiffness and stiffness after yielding, and ratio β between yield shear force and cracking shear force were given respectively.

THE EXAMINATION OF PROPOSED METHOD ACCORDING TO THE RESPONSE ANALYSIS

The outline of the analytical model

The analytical model is the 30-floor high-rise reinforced concrete building with equal weight of 158.4*tf* (1.1tf/m^2, 24$m \times$ 6m) in each floor and has equal story height of 3m (total height becomes 90m).

Though in the calculation formula of elastic stiffness presented in the section 2, the effect of vibration mode of all degrees could be taken in, it was regarded as obtaining to some extent good accuracy on the engineering considering the effect of the vibration mode the third or more, and the effect of the vibration mode the third was considered. Two analytical models are considered to calculate the elastic stiffness. One is the case in which only the first mode is considered (model-1), the other is the case in which the first three modes are considered (model-2) in order to examine the effect of modes after secondary.

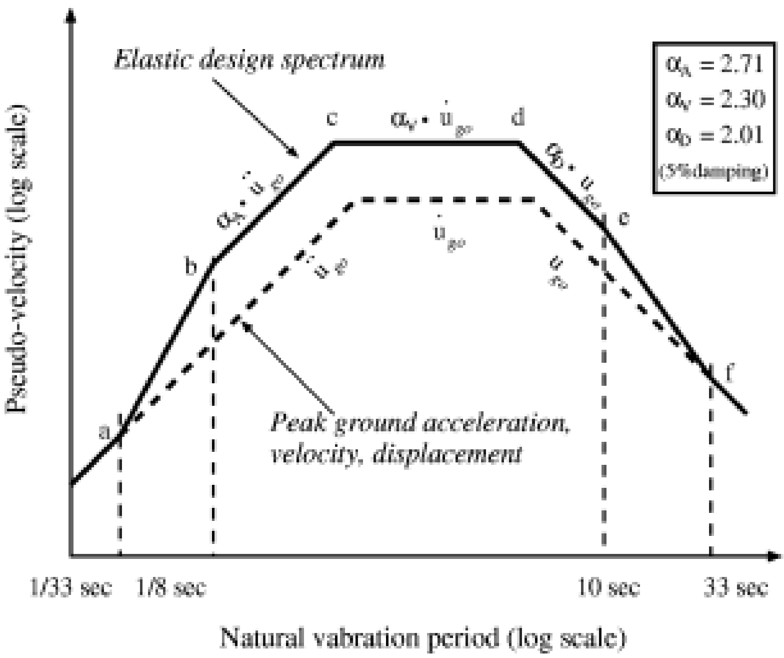

Figure 2: Elastic design response spectrum

Assumption of the response displacement of each story

The maximum value of the elastic response displacement of most upper story is determined from equation (3). The response spectrum ${}_sD$ is obtained from elastic design response spectrum proposed by Newmark as shown in figure 2 in which the maximum velocity of ground motion is assumed to be 50*cm/sec*. And the damping factor is 5% of critical used. The participation function of most upper story is given from equation (A1) and natural periods of each mode are also calculated by equation (A2) (see appendix). These equations were derived from the statistical analysis data about more than 20 high-rise buildings constructed in Japan. The calculated elastic response displacement of the upper story is 31.1*cm* (model-1) and 31.7*cm* (model-2) respectively. Both models indicate almost equal response displacement at the top floor.

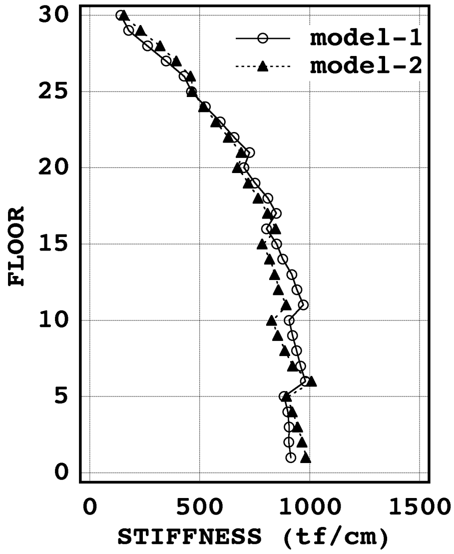

Figure 3: The calculated stiffness of each story

Since the maximum displacement distribution of the building along height becomes a curve generally, the maximum value of elastic response displacement of each story is assumed relative displacement of every 5 story decreasing 10% from the bottom to the top. The calculated stiffness of each story is shown in figure 3. The effect of the vibration modes after secondary is a little in these analytical case.

The eigenvalue analysis is carried out using calculated stiffness as tabulated in table 1. Periods agree with setting value (see table A1) of each case well.

Table 1: The natural period of models

Mode	1st	2nd	3rd
Model-1	1.6722	0.6243	0.3830
Model-2	1.6722	0.6300	0.3830

The design yield strength of each story

The story yield shear forces are calculated from equation (12) when the required response ductility factor μ is given. The other parameters β, α, η are set in this paper to be 1/3, 1/4, 1/100 respectively. The specified value of μ is assumed to be 2 in each story. Elastic stiffness k_e is calculated in the previous section 3-2, and also the elastic maximum response shear forces of each floor can be calculated from the elastic design spectrum and participation factor etc. like the calculation of the maximum displacements of each story. The calculation result of yield shear forces is shown in figure 4. The maximum velocity of ground motion is assumed to be 50*cm/sec*. same as in elastic case.

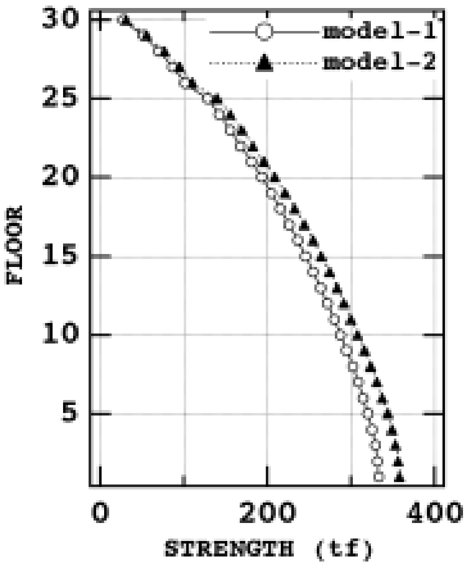

Figure 4: The calculated yield strength of each story

The elasto-plastic response analysis

In the case of elastic and elasto-plastic earthquake response analysis of mass system, it will be set as an analysis condition as following.

1) Elasto-plastic hysteretic behavior of the model is

made to be a case of mass system elasto-plastic earthquake response analysis with the Degrading-Trilinear model (figure 5). In the model, δ_c and δ_y are determined by the equation of $\delta_c = Q_c / k_e$ and $\delta_y = 4Q_y / k_e$.

2) The damping is to be the elastic stiffness proportional. The damping factor is made to be 5% for first natural frequency.
3) The input seismic waves are El Centro NS, Taft EW and Hachinohe NS which standardized the maximum input velocity at 50kine. It becomes about 300gal for El Centro NS and Taft EW, and 400gal for Hachinohe NS when it is converted into the acceleration.

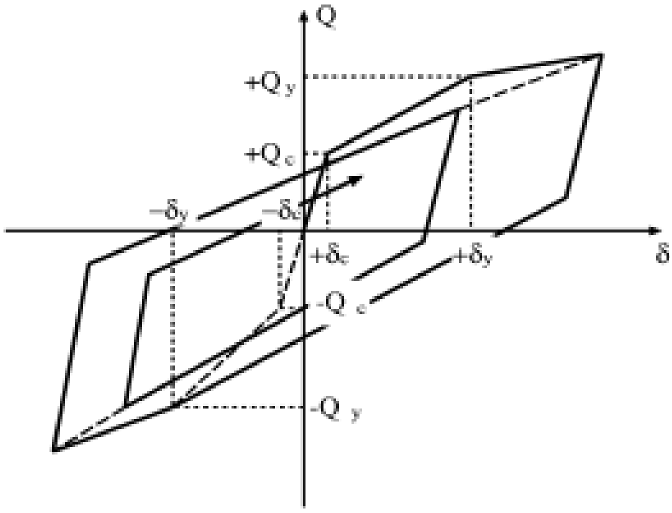

Figure 5: The elasto-plastic hysteretic model

Elastic and elasto-plastic response analytical results

In figure 6 to figure 11, mean maximum value of response relative displacement and ductility factor got from three seismic waves are shown compared to the specified value.

Except for a part of story, the difference between response displacement of each story and specified value is less than 10%. The standard deviation of each story is almost about 0.2cm on the dispersion by input earthquake vibration. Response ductility factor of each story becomes 2 approximately, and the difference between the response and specified value becomes less than 5%. From these facts, the validity of the proposed procedure will be proven.

CONCLUDING REMARKS

The 30-story reinforced concrete building was designed experimentally in order to examine the validity of the proposed calculation formula of elastic stiffness distribution and yield strength distribution along height presented, and the dynamic response analyses were carried out. It was designed by the method for proposing necessary elastic stiffness distribution and yield strength distribution along height which became specified earthquake response value. Using asked elastic stiffness and yield strength, the seismic waves were input, and elastic and elasto-plastic earthquake response analyses were carried out. Elastic and elasto-plastic response mean values for three kinds of input earthquakes were compared with the specified value, respectively. In addition, the standard deviation was calculated, and the dispersion of the response result was examined. The analytical result agrees well with the specified response displacement.

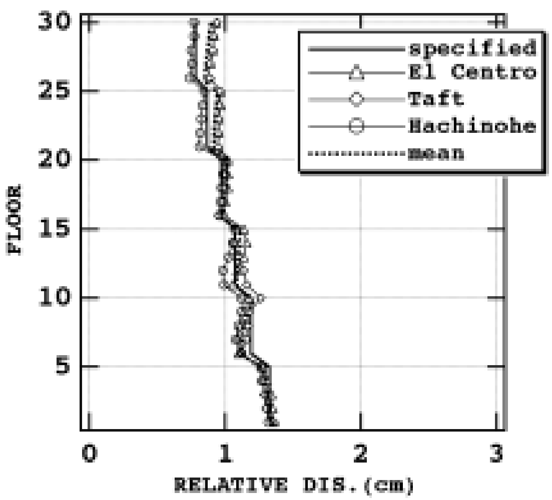

Figure 6: The elastic response displacement (model-1)

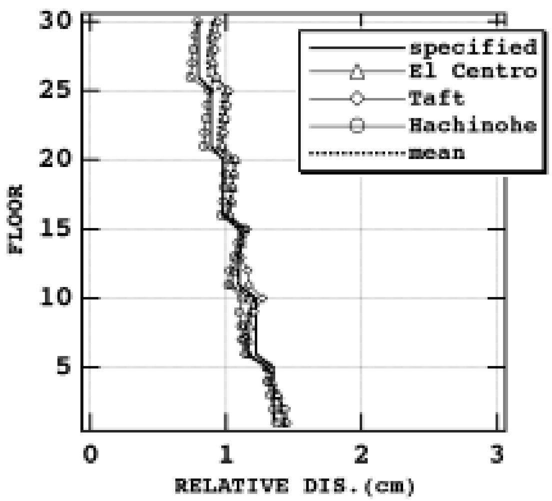

Figure 7: The elastic response displacement (model-2)

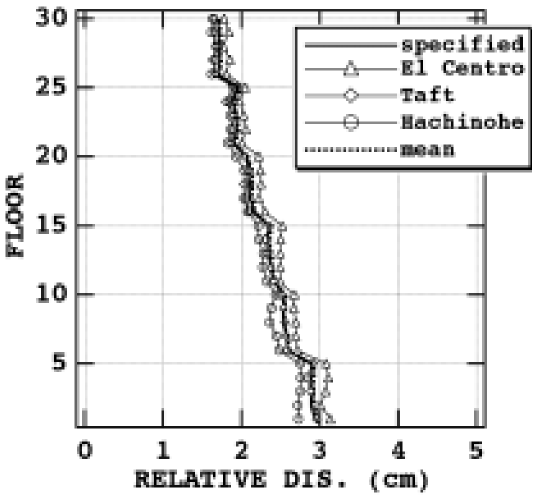

Figure 8: The elasto-plastic response displacement (model-1)

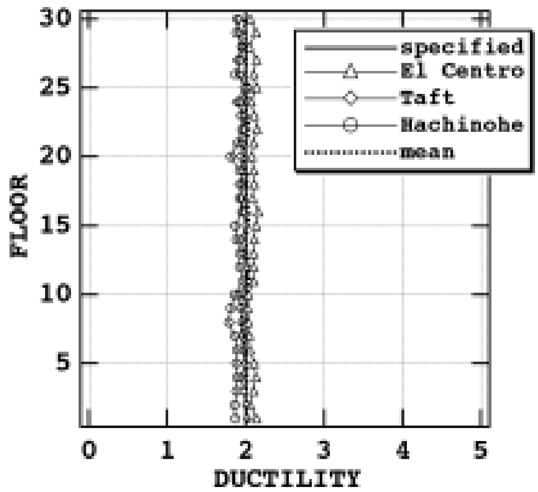

Figure 9: The elasto-plastic response ductility

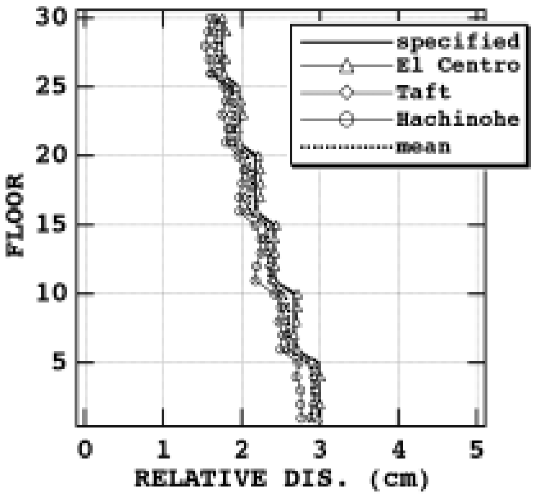

Figure 10: The elasto-plastic response displacement (model-2)

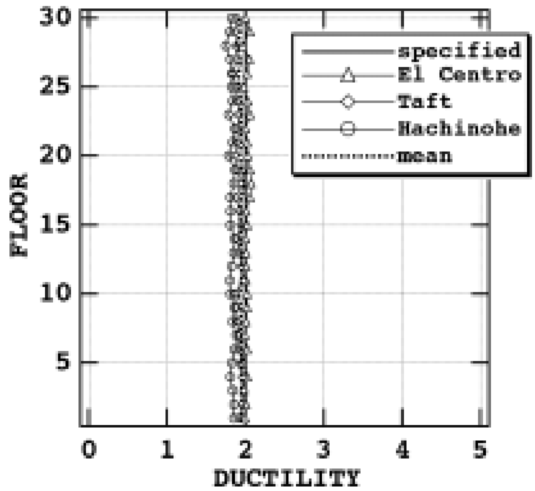

Figure 11: The elasto-plastic response ductility factor (model-2)

REFERENCES

Hisatoku, T. and Kobori, T. (1970), "On the Optimum Aseismic Design Date for Multi-story Structures Based on Elasto-plastic Earthquake Response", *Proc. 3rd Eur. Symp. Earthquake Eng.*, Sofia

Nakamura, T. and Takewaki, I. (1985), "Optimum Building Design for Forced-mode Compliance", *Journal of Engineering Mechanics*, ASCE, Vol.111, No.9

Newmark, N.M. and Hall, W.J. (1982), *Earthquake Spectra and Design*, Earthquake Engineering Research Institute, Berkeley, Calif.

Pierson, B.L. (1968), "A Survey of Optimal Structural Design under Dynamic Constraints", *Int.J.Num.Meth.Eng.*

Prager, W. and Taylor, J.E.: "Problems of Optimal Structural Design", *J.Appl.Mech.*, Vol.35, 1968

Wu, C. and Nishikawa, T. (1998), "A Method of Elasto-plastic Seismic Response Constrained Design", *Proceedings of the 10th Earthquake Engineering Symposium*, Volume 3, pp.3051-3056. (In Japanese)

Wu, C. and Nishikawa, T. (1998), " Study on a method of elasto-plastic seismic response constrained design - For shear stiffness and strength distribution –", *Journal of Structural Engineering*, Vol.44B, pp.191-200. (In Japanese)

Wu. C, and Nishikawa, T. (1999), "Study on Method of Elasto-plastic Seismic Response Constrained Design",

Journal of Structural and Construction Engineering (Transactions of AIJ), No.515, pp.45-52. (In Japanese)

APPENDIX

The following are given at equation (A1) and (A2): Proper period to assuming first to third and participation function of each story.

$$\begin{cases} T_1 = 0.01858 \times H \\ T_2 = 0.00710 \times H \\ T_3 = 0.00439 \times H \end{cases} \quad \text{(A1)}$$

$$\begin{cases} {}_1\beta\{{}_1u_i\} = 0.04308 + 0.8974(1-\eta_i) + 1.4061(1-\eta_i)^2 - 0.6583(1-\eta_i)^3 \\ \qquad - 0.3094(1-\eta_i)^4 + 0.5170(1-\eta_i)^5 - 0.4776(1-\eta_i)^6 \\ {}_2\beta\{{}_2u_i\} = 0.05023 + 0.9846(1-\eta_i) + 1.6428(1-\eta_i)^2 - 2.6728(1-\eta_i)^3 \\ \qquad - 3.0931(1-\eta_i)^4 - 1.3107(1-\eta_i)^5 + 3.7098(1-\eta_i)^6 \\ {}_3\beta\{{}_3u_i\} = 0.03592 + 1.1730(1-\eta_i) + 2.6447(1-\eta_i)^2 - 13.034(1-\eta_i)^3 \\ \qquad - 12.076(1-\eta_i)^4 + 46.503(1-\eta_i)^5 - 24.646(1-\eta_i)^6 \end{cases}$$

(A2)

where H : building height (m), η_i : the parameter of the height direction.

$$\eta_i = \left(\sum_{k=i}^{N} W_k\right) \Big/ \left(\sum_{k=1}^{N} W_k\right)$$

where W_k : k th story weight, N : number of stories.

The estimation values in proper periods of the building are shown at table A1.

Table A1: The value of assuming natural period (unit: second)

Mode	1st	2nd	3rd
Natural Period	1.6722	0.6390	0.3951

Equation (A1), equation (A2) are regression formula based on architecture rating material of the building which received the structure rating with tall building in Building Center of Japan in about 4 years in September, 1985 to September, 1989, and it is applied to reinforced concrete construction tall building. Analytical datum numbers are 24's, and most is 21st floor to 25th floor.