STRESS ANALYSIS OF PRESTRESSED CONCRETE LIQUID STORAGE TANKS SUBJECTED TO STRONG GROUND MOTION

Hideaki NAKAMURA[1], Shingo MORIKAWA[2], Sumio HAMADA[3] And Ayaho MIYAMOTO[4]

SUMMARY

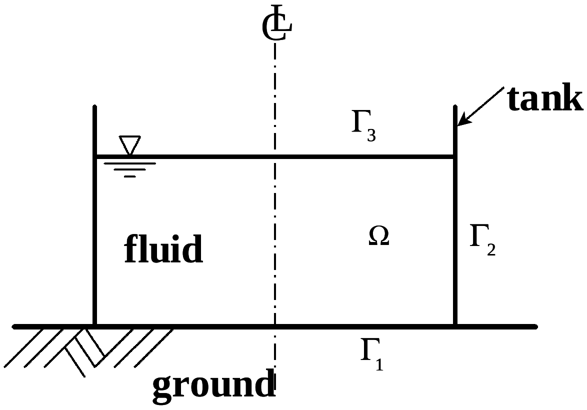
Liquid storage tanks are important structures which requires aseismic design. In the aseismic design of concrete liquid storage tanks an assumption that the tank wall is a rigid body has conventionally been made. However, since the size of liquid storage tanks becomes larger, the dynamic rigidity comparatively decreases and it is necessary to include deformations of the tank walls in the design of tanks. In the linear or nonlinear analysis of a liquid-tank-ground system, which is subjected to seismic excitations, the effect of hydrodynamic pressure has to be taken into account. One of the most powerful tools for analyzing this effect is the finite element method. As the finite element method can only treat finite regions when using it, artificial boundaries must be introduced into the ground system because they extend semi-infinitely. To perform the analyses accurately, the energy absorption at the boundaries due to wave propagation through the boundaries has to be taken into account.

In this study, a numerical method was developed for axi-symmetric analysis of tank structures including hydrodynamic and foundation interaction effects. The finite element method was newly developed for the internal liquid based on the velocity potential theory. In order to absorb the radiation wave energy through the boundary, a viscous boundary has been employed. Validity and utility of the program are demonstrated by some test calculations including those for a tank made of concrete.

INTRODUCTION

Liquid storage tanks are important structures which require aseismic design. In the aseismic design of concrete liquid storage tanks an assumption that the tank wall is a rigid body has conventionally been made. However, since the size of liquid storage tanks becomes larger, the dynamic rigidity comparatively decreases and it is necessary to include deformations of the tank walls in the design of tanks. In the linear or nonlinear analysis of a liquid-tank-ground system, which is subjected to seismic excitations, the effect of hydrodynamic pressure has to be taken into account. One of the most powerful tools for analyzing this effect is the finite element method. As the finite element method can only treat finite regions when using it, artificial boundaries must be introduced into the ground system because they extend semi infinitely. To perform the analyses accurately, the energy absorption at the boundaries due to wave propagation through the boundaries has to be taken into account. In this study, a numerical method was developed for axi-symmetric analysis of tank structures including hydrodynamic and foundation interaction effects. The finite element method was newly developed for the internal liquid based on the velocity potential theory. In order to absorb the radiation wave energy through the boundary, a viscous boundary has been employed. Validity and utility of the program are demonstrated by some test calculations including those for a tank made of concrete.

[1] *Dept of Computer Science & Systems Eng, Yamaguchi University, JapanE-mail: nakamura@design.csse.yamaguchi-u.ac.jp*
[2] *General Manager, Asano Koji Co., Ltd., Tokyo, Japan*
[3] *Department of Civil Engineering, Yamaguchi University, Yamaguchi, Japan*
[4] *Department of Computer Science & Systems Engineering, Yamaguchi University, Yamaguchi, Japan*



Γ1:Boundary between fluid and tank bottom

Γ2:Boundary between fluid and tank wall

Γ3:Boundary at the free liquid surface

Ω: Fluid

Figure 1: Fluid, Tank and Ground Domain

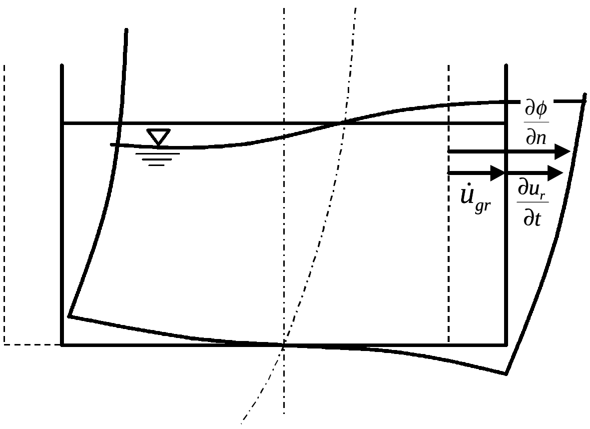


Figure 2: Boundary condition at the tank wall

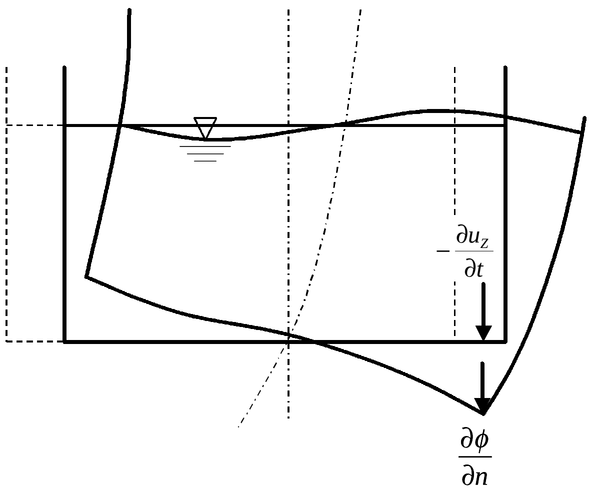


Figure 3: Boundary condition at the tank bottom

EQUATION OF MOTION

Equations governing liquid motion:
According to linear potential flow theory, a scalar velocity potential $\phi(r,\theta,z,t)$ can be defined within the region of an irrotational, inviscid fluid. In the case of incompressibility the function ϕ must satisfy the so-called Laplace equation which can be written in terms of cylindrical coordinates as

$$\frac{1}{r}\frac{\partial \phi}{\partial r}+\frac{\partial^2 \phi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}+\frac{\partial^2 \phi}{\partial z^2}=0 \qquad (1)$$

The function ϕ must be finite at every point within the liquid region. At the liquid boundary the following conditions have to be satisfied:

1) The boundary condition at the tank wall:
The liquid adjacent to the tank wall must move radially with a velocity equal to that of the tank wall. This is the first coupling condition between liquid and tank wall.

$$\frac{\partial \phi}{\partial r}=\dot{u}_{gn}+\frac{\partial u_r}{\partial t} \quad \text{boundary } \Gamma_2(r=R) \qquad (2)$$

2) The boundary condition at the tank bottom:

The liquid adjacent to the tank bottom must move vertically with a velocity equal to that of the tank bottom.

$$\frac{\partial \phi}{\partial z} = \frac{\partial u_z}{\partial t} \qquad \text{boundary } \Gamma_1 (z = 0) \tag{3}$$

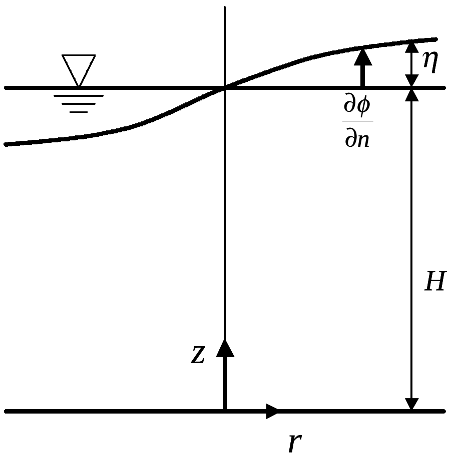


Figure 4: Boundary condition at the free liquid surface

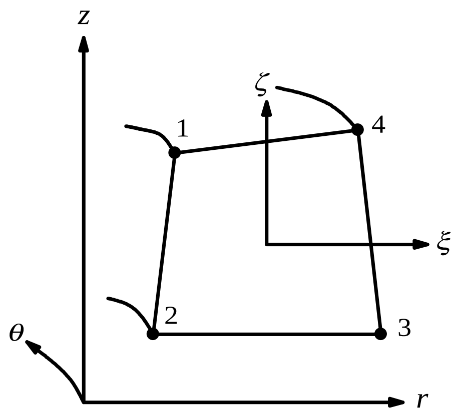


Figure 5: Iso-parametric ring element

3) The boundary at the free liquid surface
The motion equation at the free liquid surface can be expressed as

$$\left(\frac{P}{\rho_w}\right)_{z=H+\eta} = \left(\frac{\partial \phi}{\partial t}\right)_{z=H+\eta} - \frac{1}{2}\left(\dot{u}_\ell^2 + \dot{v}_\ell^2 + \dot{w}_\ell^2\right)_{z=H+\eta} - g\eta \tag{4}$$

where, ρ_w is the density of liquid and η is the height of the free liquid surface and g is the gravity acceleration. Considering the pressure at the free surface equal to the atmospheric pressure and neglecting infinitesimal terms, the boundary condition at the free liquid surface can be expressed as

$$\frac{\partial \phi}{\partial z} = -\frac{1}{g}\frac{\partial^2 \phi}{\partial t^2} \tag{5}$$

Finite element procedure

The domain for analysis can be discretized using iso-parametric ring element, in which the velocity potential is written under the following form:

$$\phi(r, \theta, z; t) = \Phi(r, z; t)\cos m\theta \tag{6}$$

where m is the coefficient of the Fourier's expansion.
The standard discretization processes using Garlerkin procedures result in a form.

$$\int_V \delta\phi\left(\frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}\right)dv = 0 \tag{7}$$

Substituting Eq.(6) into Eq.(7), using the formula of integration by parts, and rearranging leads to

$$\rho_w \int_V \left(\frac{\partial \delta\Phi}{\partial r}\frac{\partial \Phi}{\partial r} + \frac{\partial \delta\Phi}{\partial z}\frac{\partial \delta\Phi}{\partial z}\right)\cos^2 m\theta dv + \rho_w \frac{m^2}{r^2}\int_V \delta\Phi \cdot \Phi \cos^2 m\theta dv = \rho_w \int_S \delta\Phi \frac{\partial \Phi}{\partial n}\cos^2 m\theta ds \tag{8}$$

It will be noted that the right part of Eq. (8) must be determined by boundary condition. By using the shape function, the velocity potential can be expressed as

$$\Phi = [N]\{\Phi\} \tag{9}$$

in which

$[N] = \{N_1, N_2, \cdots\cdots, N_n\}$, $\{\Phi\} = \{\Phi_1, \Phi_2, \cdots\cdots, \Phi_n\}^T$, n :number of nodal points consisting the element

Substituting Eq. (9) into equation (8), and using the formula of $\int_0^{2\pi} \cos^2 m\theta d\theta = \pi$, Eq.(8) can be expressed as

$$\pi\rho_w \int_{-1}^{1}\int_{-1}^{1} [B]^T [B] r|J| d\xi d\eta \{\Phi\} + \pi\rho_w \frac{m^2}{r^2} \int_{-1}^{1}\int_{-1}^{1} [N]^T [N] r|J| d\xi d\eta \{\Phi\} \tag{10}$$

$$= -\frac{\rho_w}{g}\pi \int_{-1}^{1} [N]^T [N] r|J| d\xi \{\ddot{\Phi}\} \qquad \text{(at the free liquid surface)}$$

$$-\pi\rho_w \int_{-1}^{1} [N]^T [N]^* r|J| d\xi \{\dot{u}_b\} \qquad \text{(at the tank bottom)}$$

$$+\pi\rho_w \int_{-1}^{1} [N]^T [N]^* r|J| d\eta \{\dot{u}_r + \dot{u}_g\} \qquad \text{(at the tank wall)} \tag{11}$$

in which, $[N]^* = [n_r, n_\theta, n_z][\tilde{N}]$, $[\tilde{N}] = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & \cdots\cdots & N_n & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \cdots\cdots & 0 & N_n & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & \cdots\cdots & 0 & 0 & N_n \end{bmatrix}$

n_r, n_θ, n_z is normal component of the direction r, θ, z. $n_r = 0, n_\theta = 0, n_z = 1$ at the tank bottom and $n_r = 1, n_\theta = 0, n_z = 0$ at the tank wall. Matrices $[B]$ and $[J]$ are as follows:

$$[B] = [J]^{-1} \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \cdots\cdots & \frac{\partial N_n}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \cdots\cdots & \frac{\partial N_n}{\partial \eta} \end{bmatrix}, \quad [J] = \begin{bmatrix} \frac{\partial r}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial r}{\partial \eta} & \frac{\partial z}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} r_i & \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} z_i \\ \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} r_i & \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} z_i \end{bmatrix} \tag{12}$$

The equation of motion for the liquid-tank system can be expressed as

$$[M_\Phi]\{\ddot{\Phi}\} - [C_{\Phi T}]\{\dot{u}\} + [K_\Phi]\{\Phi\} = \{f_\Phi\} \tag{13}$$

in which

$$[M_\Phi] = \frac{\rho_w}{g}\pi \int_{-1}^{1} [N]^T [N] r|J| d\xi \qquad \text{(at the free liquid surface)} \tag{14}$$

$$[K_\Phi] = \pi\rho_w \int_{-1}^{1} [B]^T [B] r|J| d\xi d\eta + \pi\rho_w \frac{m^2}{r^2} \int_{-1}^{1}\int_{-1}^{1} [N]^T [N] r|J| d\xi d\eta \tag{15}$$

$$[C_{\Phi T}] = -\pi\rho_w \int_{-1}^{1} [N]^T [N]^* r|J| d\xi \quad \text{(at the tank bottom)}$$

$$+\pi\rho_w \int_{-1}^{1} [N]^T [N]^* r|J| d\eta \quad \text{(at the tank wall)} \tag{16}$$

$$\{f_\Phi\} = \pi\rho_w \int_{-1}^{1} [N]^T [N]^* r|J| d\eta \{I\} \dot{u}_g \quad \text{(at the tank wall)} \tag{17}$$

$$\{\dot{u}\} = \{\dot{u}_b\} + \{\dot{u}_r\} \tag{18}$$

And hydrodynamic pressure can be evaluated as

$$P_d = -\rho_w \frac{\partial \phi}{\partial t} \tag{19}$$

Therefore, the nodal force caused by hydrodynamic pressure can be expressed as

$$\{f_p\} = \left(\pi\rho_w \int_{-1}^{1} [N]^{*T} [N] r|J| d\xi + \pi\rho_w \int_{-1}^{1} [N]^{*T} [N] r|J| d\eta\right)\{\dot{\Phi}\} = -[C_{\Phi T}]^T \{\dot{\Phi}\} \tag{20}$$

Motion of equation for the tank-ground system introducing viscous boundary:

As the finite element method can only treat finite regions when using it, artificial boundaries must be introduced into the ground system because they extend semi-infinitely. To perform the analyses accurately, we need to take into account the energy absorption at the boundaries due to wave propagation through the boundaries In order to absorb the radiation wave energy through the boundary, a viscous boundary has been employed.
The motion of equation for the tank-ground system introducing viscous boundary can be expressed as

$$\begin{aligned}[M]\{\ddot{u}\} + ([C]+[C_B]+[C_S])\{\dot{u}\} + [K]\{u\} \\ = -[M]\{I\}\ddot{u}_g + \{f\} + [C_S]\{\dot{u}_f\} + [G_S]\{u_f\} + [G_{CS}]\{\dot{u}_f\}\end{aligned} \tag{21}$$

where

$[M],[C],[K]$: Mass, damping and stiffness matrices
$[C_B]$: Viscous boundary matrix for the base boundary
$[C_S]$: Viscous boundary matrix for the side boundary
$[G_S]$: Boundary stiffness matrix
$[G_{CS}]$: Boundary damping matrix
$\{u_f\}$: Displacement vector of the free field
$\{I\}$: Unit vector in the direction of applied load
$\ddot{u}_g$: Acceleration at the base layer

Motion of equation for the liquid-tank-ground system:

Using Eqs.(19) which expressed the equation of motion for the liquid-tank system and Eqs.(21) which expressed the equation of motion for the tank-ground system, the equation of motion for the liquid-tank-ground system can be expressed as

$$\begin{bmatrix} M & 0 \\ 0 & -M_\Phi \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ -\ddot{\Phi} \end{Bmatrix} + \begin{bmatrix} C_T & -C_{\Phi T}^T \\ -C_{\Phi T} & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ -\dot{\Phi} \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -K_\Phi \end{bmatrix} \begin{Bmatrix} u \\ -\Phi \end{Bmatrix} = \begin{Bmatrix} P_S \\ f_\Phi \end{Bmatrix} \tag{22}$$

where,

$$[C_T] = [C] + [C_B] + [C_S] \tag{23}$$

$$\{P_S\} = -[M]\{I\}\ddot{u}_g + \{f\} + [C_S]\{\dot{u}_f\} + [G_S]\{u_f\} + [G_{CS}]\{\dot{u}_f\} \tag{24}$$

The equation (22) can be solved numerically by means of a FORTRAN77 program which has been developed by the authors.

NUMERICAL EXAMPLES

Model for the liquid-tank system:

1) Analytical conditions
The PC cylindrical tank of the water-supply system is analyzed as an example of the liquid-tank system. The material properties of the content liquid and the tank are shown in Table-1. Various distributions of maximum hydrodynamic pressure are shown in Fig.6. Housner's theory gives two results which includes the convective

pressure and excludes the convective pressure. The conventional methods provide larger hydrodynamic pressure. In the analysis, El Centro NS compornent of the seismic wave (Imperial Valley earthquake in 1940) was applied for the input acceleration. The maximum acceleration is corrected to 196 gal corresponding to horizontal seismic coefficient 0.2. The horizontal seismic coefficient used by a simplified method is assumed to be 0.2. The analysis is carried out in the frequency domain at time interval of 0.02sec.

Table 1: The material properties of the content liquid and the tank

Capacity	37100m^3	Unit weight (Liquid) N/m^3	9806.65
Depth	15.6m		
Inside diameter	55.0m	Unit weight (Tank) N/m^3	24500
Thickness of wall	0.4m		
Height	19.0m	Young's Modulus (N/mm^2)	3.43×10^6
Slab thickness	2.0m		
		Poisson's Ratio	0.2

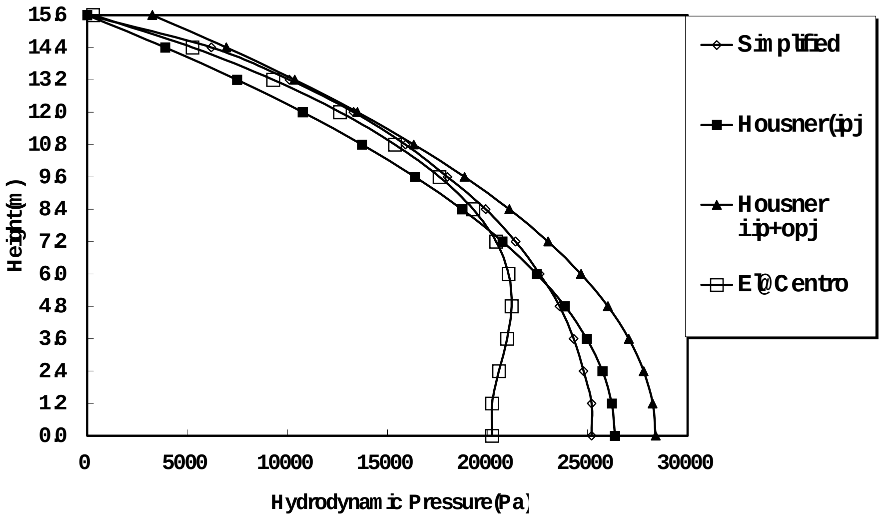


Figure 6: Distribution of Maximum Hydrodynamic Pressures

2) Analytical results

The hydrodynamic pressure at the tank wall is shown in Fig.6 in order to compare the hydrodynamic pressure calculated by this analysis and the conventional method. The result indicates the maximum value of the time history of the hydrodynamic pressure. The hydrodynamic pressure obtained by a simplified method is similar to the hydrodynamic pressure obtained by the Housner's method. The hydrodynamic pressure obtained by the finite element method is smaller than the conventional method in the lower part of the tank wall. Because the tank wall is assumed to be rigid in the conventional method, the hydrodynamic pressure which acts on the lower part of the tank wall is large. In the analysis by the finite element method, the tank wall is assumed to be elastic. Therefore, the deformation pressure is caused by the deformation of the tank wall, and the hydrodynamic pressure in the middle part of the tank wall with large deformation becomes maximum. Moreover, the hydrodynamic pressure does not necessarily become maximum in the bottom part of the tank wall like a rigid tank, because the deformation of the bottom part of the tank wall is smaller than the deformation of the middle part of that wall.

Model for the liquid-tank-ground system:

1) Analytical conditions

The material property of the ground is indicated in Table-3, and the finite element mesh is shown in Fig.7.

2) Analytical results

The hydrodynamic pressure distribution by the analysis including the ground and not including it is shown in Fig.8. The hydrodynamic pressure has the growing tendency in the analysis including the ground compared with the analysis not including it. It is thought that the reason why the hydrodynamic pressure grew is that the velocity of tank wall quickened including the ground. The influence of the ground cannot be considered to use a past conventional method. The hydrodynamic pressure might grow by the influence of the ground. Therefore, it is necessary to consider the interaction of liquid-tank-ground to obtain more detailed hydrodynamic pressure distribution.

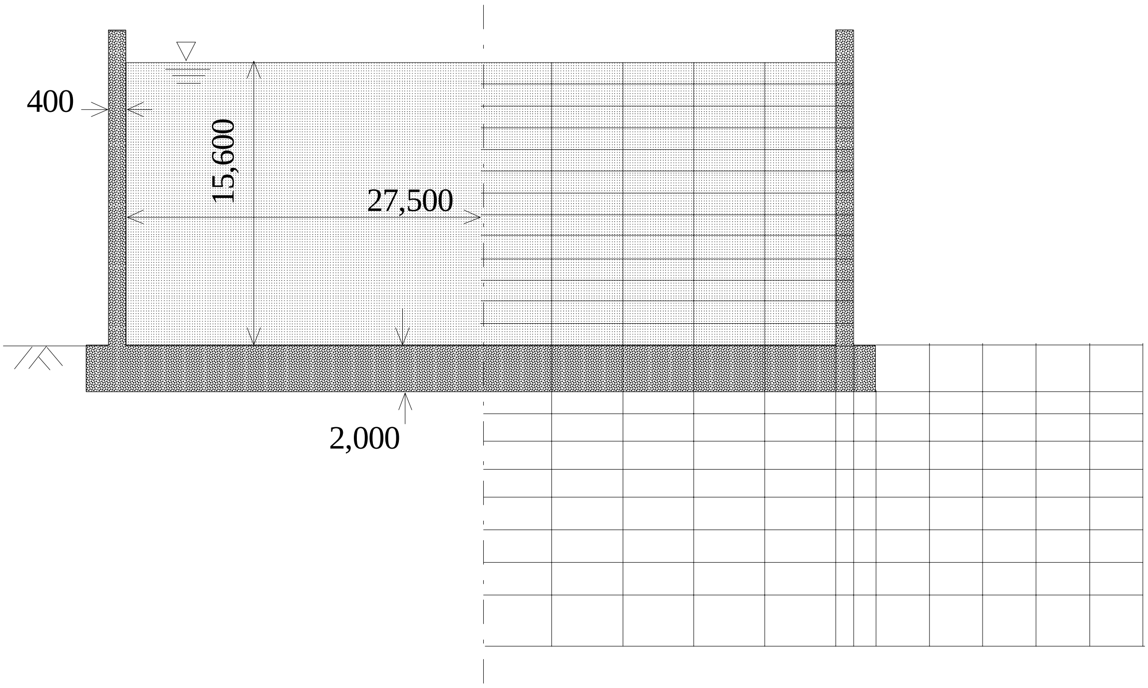


Figure 7: The discretization of the liquid-tank-ground system by the finite elements

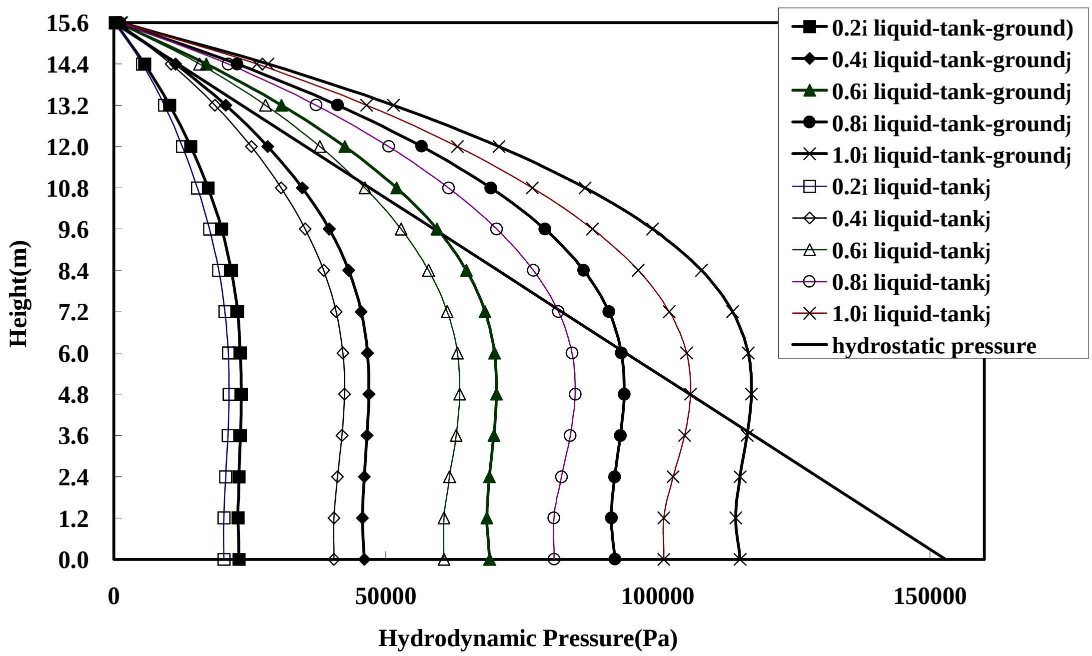


Figure 8: Distribution of Maximum Hydrodynamic Pressures

Table 2: The physical parameters of the ground

	V_{si} m/s	γ_t kg/m^3	H_i m	$4H_i / V_{si}$	T_G sec
Layer	333.3	2100.0	2.7	0.01	0.03
	321.5	2100.0	2.3	0.01	
	398.8	2100.0	3.7	0.01	
Base layer	750.0	2100.0	3.0	——	——

CONCLUSIONS

The conclusion of this study is summarized as follows.

1. It is possible to consider deformation of the tank wall and influence of the ground by introducing a velocity potential theory and a viscous boundary into the finite element analysis.
2. Since the tank wall was assumed to be rigid in a past conventional method, the influence which the deformation of the tank wall exerted on the hydrodynamic pressure was not able to be considered. At the middle part of the tank wall where the deformation of it is large, the hydrodynamic pressure is large because of the influence of the deformation pressure.
3. The hydrodynamic pressure caused at the tank wall is greatly influenced by the ground. It is necessary to consider the interaction of liquid-tank-ground to obtain more detailed hydrodynamic pressure distribution.

REFERENCES

1. Housner, G.W. (1957), "Dynamic pressure on Accelerated Fluid containers", *Bulletin of the seismological Society of America*, Vol.48, No.1, pp.15-35
2. Stiiss, U.W. (1981), *Proc. ASCE*, J.Eng. Mech. Div., Em4, pp.679
3. Shaaban, S.H. and Nash, W.A. (1976), "Finite element analysis of a seismically excited cylindrical storage tanks, ground supported and partially filled with liquid", *Report NSF/RA-760261*, University of Massachusetts,

Miura, F., Hamada,S. and Morikawa, s. (1986), "The viscous boundary for axisymmetric soil strucure interaction systems subjected to no-axisymmetric loads", Memoirs of the faculty of Engineering,