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STOCHASTIC IDENTIFICATION OF EARTHQUAKE WAVE ENTITIES

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SUMMARY

In recent years, little progress has been observed in modelling earthquake acceleration as a realistic multi-directional structural loading process. In commonly applied earthquake load models, the paradigm of three independent one-dimensional stationary stochastic models is still preserved, which is contradictory to the existence of different types of body and surface (guided) waves. The is because methods are still missing which identify the different wave types in observed strong motion records in order to adequately analyse and describe these wave packages as stochastic sub-processes of a realistic 3D stochastic non-stationary acceleration vector process.

A promising method to capture the 3D characteristics of the seismic process and to represent the stochastic correlation between the three components in a transparent and easy-to-interpret way is the method of stochastic principal axes originally introduced by Penzien and Watabe in 1975. Application as a wave identification method in subsequent years lead to the result that the on-set of the S-wave could quite reliably be determined, but whether the duration nor other wave types, because only the principal variance and the vertical of the two angles describing the time-dependent principal axis had been useful for interpretation, while the important horizontal angle was very strongly fluctuating with irregular jumps between +/– 90 degree.

A correction method is suggested that copes with the periodicity problem of the horizontal angle which was detected to be the cause of these irregular jumps. The corrected horizontal angle now shows a quite smooth variation with time in long time intervals, but still variations of more than hundred degrees may occur within few tenths of a second between these intervals. A stability analysis concerning the two parameters of the moving window technique applied to cope with the time-dependency of the seismic process revealed that the occurrence of these strong changes depends on the window length, whereas the underlying Eigenvalue problem does not. An extension of the correction method is under development in order to cope with these findings.

For a subset of the 1994 Northridge earthquake records, the expected behaviour of the direction of the stochastic principal axis caused by body waves radiating from a moving rupture front sweeping over the fault is revealed by the corrected horizontal angle in the first part of the course. Also the stronger variations arising in the second part are in a reasonable accordance with those of the vertical angle, as it should be for surface and scattered waves. In a combined analysis of the two angles, the different time intervals are quite reliably recognisable. Therefore, wave types that are dominating the accelerogram are identifiable and can be separated by appropriate methods.

INTRODUCTION

Seismic strong ground motion is a three dimensional, highly complex process. Because of the heterogeneity of the structure and the uncertainty of material properties of the earth's upper crust, strong ground motion is frequently modelled as a stochastic process, and stochastic quantities are examined in order to predict the possible load on structures and buildings. Common stochastic models are dealing with significant simplifications of the ground acceleration process, especially the process is assumed to be stochastic stationary in wide sense. This allows to use the methods developed in this well-known field. Further, most tools only

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apply to one-dimensional processes. Therefore, only one component of the 3D process is used for predictions. Unfortunately, several earthquakes have proved this to be inadequate, e.g. Northridge 1994

The main non-stationary characteristics of earthquake acceleration from the structural engineering point of view is the dominance of different kinds of particle movement and shares of energy during certain phases of an earthquake, caused by the different behaviour, velocity, location and time of origin of several kinds of waves occurring at the oscillation of a solid body, which sum up to the whole process. In a non-stationary stochastic model, these phases should be separately dealt with, resulting in more simple stochastic sub-models [Scherer, 1994]. Further, every wave has a transient character because its source is active only for a short time and its energy will be damped and finally absorbed by the body of the earth. In order to introduce these non-stationary characteristics into stochastic or other models, methods to identify those wave dominance phases from recorded accelerograms are indispensable.

TARGET PROPERTIES FOR IDENTIFICATION

One distinctive feature of wave types is the different spreading velocity of longitudinal or primary (P-) and transversal or secondary (S-) waves, causing an S-wave of the same origin to arrive later than the corresponding P-wave. Further, waves during an earthquake have more than one origin. While the main source of all is the earthquake fault, emitting so called direct waves, abrupt changes of material density cause reflections as well as P-S, S-P and other wave conversions anywhere along the propagation path. Finally, Love and Rayleigh waves may be generated by body waves incident to the surface or layer interfaces beyond the critical angle. The occurrence of those waves is of course a while after that of direct waves. Usually, waves of different type and origin differ in the share of energy and the frequency content, too, and they overlay each other. There are several time domain methods that try to use these features for identification, f. i. RMS acceleration time histories, Husid's plot of the cumulative Areas-Intensity, stochastic principal axes [Kubo & Penzien, 1979; Bond, 1980] and evolutionary power spectra [Kameda & Sugito, 1984; Scherer, 1993].

According to the scenario of Earthquake wave propagation described above and discarding special cases of topography or upper crust structure, the following course of events should be reflected in most accelerograms, i.e., three component acceleration records. The first part is dominated by direct P-waves, because they have both earliest origin and highest velocity. With increasing epicentral distance, the first arrival of S-waves is delayed. With its arrival, the direct S-wave dominates the principal direction because its share of energy is usually the greatest of all wave types. When the phase of direct P-waves has finished, the characteristics of the S-wave should be visible very clearly. As the energy of the direct S-wave decays, indirect waves converted at discontinuities of layered rock along the propagation path determine the state, alone or together with surface waves.

Special, but nevertheless quite frequent constellations could disturb this course of events, depending on the epicentral distance, near fault and subsurface topography. Near the epicentre of an earthquake, the share of energy of the direct P-wave could come up to that of the direct S-wave and the offset between their arrivals may be very small. Surface waves may occur very early even during the direct S-waves endure. A moving rupture process causes a Doppler effect and changing angle of incidence along with the fault at certain sites near epicentre. At greater distance, where the P-phase should be generally longer because of the increasing runtime difference to the S-wave, PS waves converted in the bedrock near the recording site could even precede the direct S-wave and prematurely disturb the P-wave characteristics. Guided waves may completely change the scenario because they may travel in bedrock over long distances at very high velocity.

The most important differences for identification of all types of body and surface waves are the characteristics of particle movement. Particles are moved in parallel to the spreading direction by longitudinal waves, but perpendicularly to by transversal waves of either polarisation. Surface waves roll particles within a plane parallel or perpendicularly to the surface plane. If the share of energy of one type of wave is significantly larger than the others, it dominates the adjustment of the principal plane or direction of acceleration at this time. If this dominance is strong enough over a certain period, the principal direction should remain at a certain state or behaviour for this time, and show significant changes, if there is a strong change of shares in the present mix of waves in the accelerogram. Being able to visualise the behaviour of this principal direction would be a means to profoundly analyse the occurrence of waves and to identify the phases of their dominance.

In the context of wave identification, it is not sufficient to visualise the actual direction of particle movement with time, i.e., the orientation of the principal axes, which may be very oscillatory, but the course of its time-dependent principal direction or plane of acceleration, a task commonly performed by a technique with subsequent time windows moving over the record. Of course, the shape of curves computed via moving time windows depends on the choice of moving window parameters, which will be discussed below.

TIME-DEPENDENT STOCHASTIC PRINCIPAL AXES METHOD

Earthquake accelerograms are delivered as representations of the three-dimensional acceleration vector in a Cartesian co-ordinate system, generally with axes parallel to east-west, north-south and vertical direction. This representation is arbitrary regarding to the physical meaning and therefore for a realistic stochastic non-stationary load model of the three-dimensional vector process constituted by one- and two- dimensional particle movements of body and surface waves, respectively. Several attempts to improve this representation have been undertaken in the past, with limited success, however. Stochastic principal axes first introduced by [Kubo & Penzien, 1976] have been found to be the most promising one, even if recent application for identification purposes by [Bond, 1980] had some drawbacks which considerably restricted its use. In the following, a correction method is developed to overcome most of these disadvantages.

In order to cope with the non-stationarity of the process concerning time-dependent acceleration intensity and frequency content, which is the main focus of interest, the moving window technique is applied, assuming that the non-stationary process can be approximated by a process which is stochastic stationary in wide sense within short time intervals. Stochastic principal axes are defined by the Eigenvectors of the Covariance matrix of the components of a record, while the Eigenvalues can be thought of as a stochastically generalised energy of the acceleration process. A representation of the accelerogram in the Co-ordinate system of the Eigenvectors of the Covariance matrix would consist of components which are statistically independent within those intervals. For a given time t_0 and window length L, the cross-covariances that build the covariance matrix are

$$Cov_{i,j}(t_0, \tau, L) = \int_{t_0-\frac{L}{2}}^{t_0+\frac{L}{2}} a_i(t') \cdot a_j(t'+\tau)\, dt' \quad \text{for } i, j = x, y, z \tag{1}$$

In this work, the time delay τ will be restricted to zero. The choice of the window length L is very important. On the one hand, it must be small enough in order to resolve the non-stationarity, on the other hand, it must be wide enough in order to allow replacement of the mean value over the ensemble by the mean value over time.

The Eigenvector corresponding to the maximum absolute Eigenvalue, referenced here as „the principal axis“, has been visualised by [Kubo & Penzien, 1976; Bond, 1980] according to Figure 1(a) via two angles defined as

$$\theta = arctan\frac{y}{x} \qquad \varphi = \left| arctan\left(\frac{x}{z \cdot cos\,\theta}\right)\right| \tag{1}$$

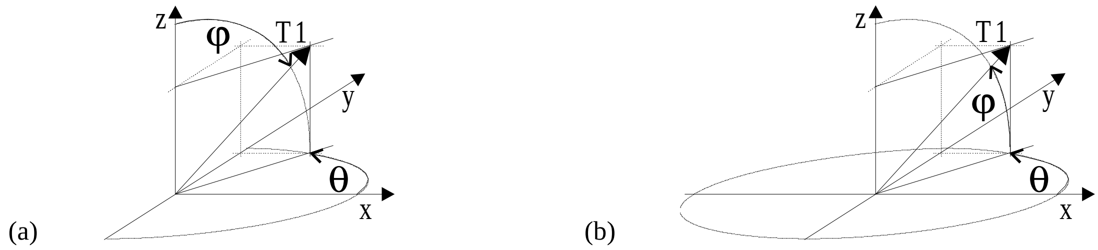

Figure 1Angles describing the principal axis defined by equations (1) [Bond, 1980] (a) and (2) (b)

where (x,y,z) correspond to the Co-ordinates of the Eigenvector in the Cartesian East, North and vertical system, respectively. Plots of these angles are shown as an example in Figure 2(a) .

A similar approach has been to characterise the course of the mean acceleration vector within moving time windows called deterministic principal axis. The same angles as in the stochastic approach, called deterministic angles β_1 and β_2, are represented in Figure 2(b) together with the absolute value of the mean acceleration for the same accelerogram. Deterministic principal axes are strongly influenced by short-term, second order effects and therefore are less appropriate to identify the governing dominant effects of the non-stationarity of the seismic load process. Therefore, the analysis has been focused on stochastic principal axes.

As an important fact, body waves are refracted towards decreasing material density, causing their principal propagation direction to rise up towards the Earth’s surface. Taking in mind the oscillation characteristics of the different wave types and regarding the subsurface as a horizontally layered medium, coincidence of the principal direction of acceleration with the normal of the earth’s surface indicates dominance of P-waves of either direct or indirect origin as well as of the vertical portions of Rayleigh waves, while strong deviation is caused by S-waves and respective types of surface waves.

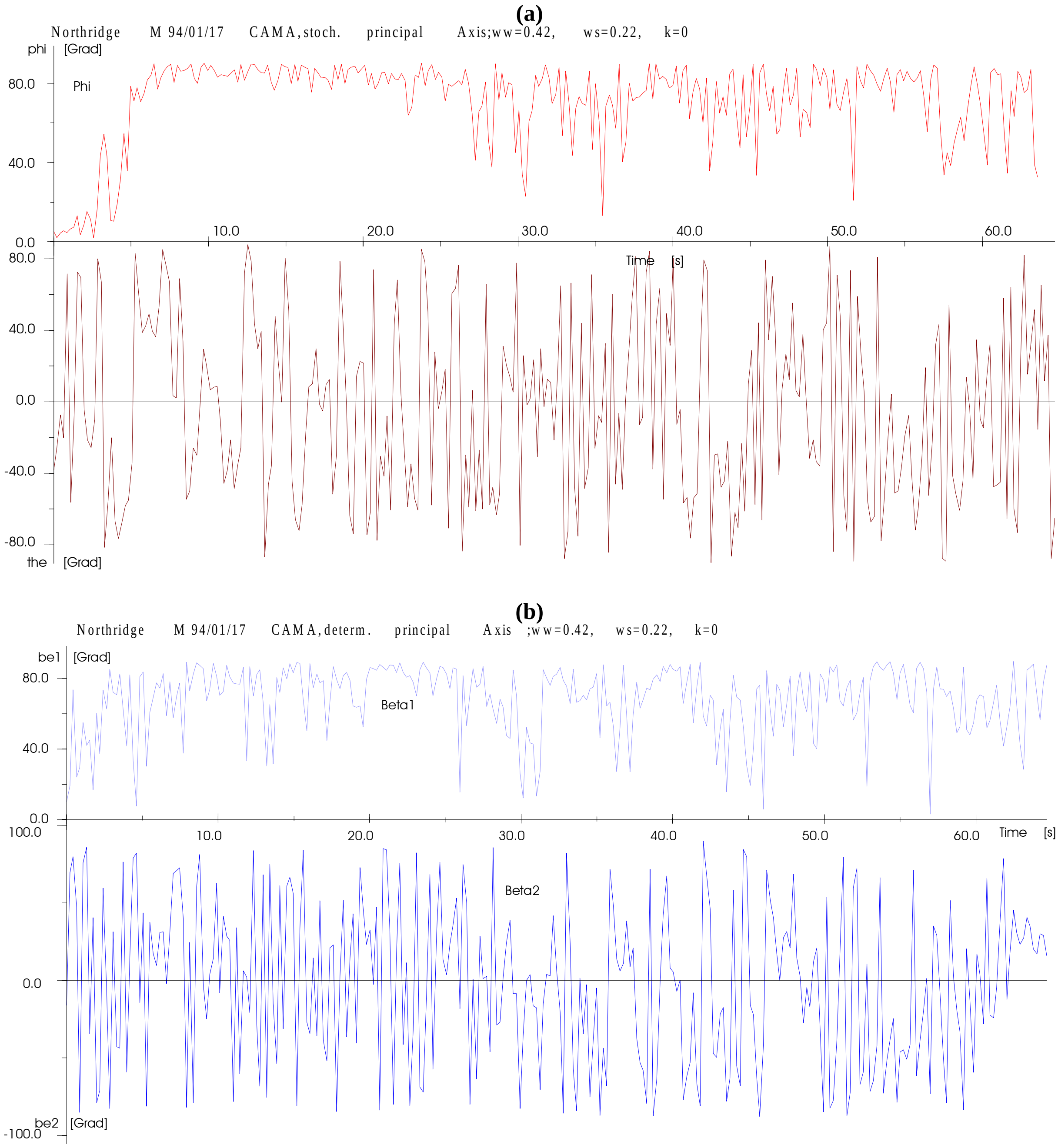

Figure 2Angles φ, θ of the Stochastic (above) and β_1, β_2 of the Deterministic Principal Axis (below), CSMIP record of the Northridge 1994 Earthquake at Camarillo, window length 0.4s at every 0.2s

Now, this deviation is expressed by the value of the angles φ or β_1, defined as the deviation of the respective principal axis from the surface normal, making them indicators of dominance interchange between P- and S-waves. Calling in mind the scenario of wave propagation described above, low values of these angles correspond to P- or Rayleigh waves, while high values will indicate dominance of S-waves or other surface waves.

In our analysis, the stochastic principal axes have been found generally more stable and therefore better to interpret as the deterministic ones in terms of the course of the vertical angle. An example which clearly demonstrates the features of the angle φ and β_1 is shown in Figures 2(a) and 2(b), respectively. However, neither polarisation characteristics of S-waves nor acceleration direction of Love waves or Rayleigh lateral wave parts can be significantly expressed by this angle.

The second angle, which is defined in the surface plane of the earth (see Figure 1) should show some kind of these characteristics, but could never be used up to now because its course usually shows a very strong oscillatory behaviour as to be seen in Figure 2. The reason for these strong oscillations has now been carefully investigated and a method to reduce them considerably in order to reveal the true course of the horizontal angle was developed.

IMPROVED DEFINITION OF ANGLES φ AND θ

As mentioned above, earthquake accelerograms are usually recorded within a 3D Cartesian co-ordinate system with two horizontal axes aligned with the north direction and the vertical axis. The angles representing either the deterministic or the stochastic principal axes can be regarded as some kind of polar co-ordinates of the mean acceleration vector and the main Eigenvector of the covariance matrix in moving time windows, respectively. If these vectors are defined as $\mathbf{X} = (x, y, z)^T$, their polar co-ordinates are given as $\mathbf{X} = (r, \theta, \varphi)$, where

$$r = |\mathbf{X}| = \sqrt{x^2 + y^2 + z^2} \qquad \theta = \mathit{arctan}\frac{y}{x} \qquad \varphi = \mathit{arctan}\left(\frac{z}{x \cdot \mathit{cos}\theta}\right), \tag{2}$$

Note that in the following, compared to equations (1), the vertical angle φ will be measured from the horizontal surface plane towards the vertical axis as in equations (2), but remains restricted.

One reason for the strong variance in the course of the horizontal angle θ computed via (1) or (2) is the periodicity of the tangent function, whose singularities force the restriction of the image domain of the arctangent function to $[-\frac{\pi}{2}, +\frac{\pi}{2}]$ corresponding to a range of [–90°,+90°]. In computation of the arctangent, all values are determined to be within this principal interval modulo full tangent periods $k \cdot \pi$, corresponding to shifts of 180°. Therefore, if between two consecutive time windows the real axis of an S-wave acceleration turns from 89° to 92°, measured from the East, the angle θ computed via (1) shows a jump of 177° instead of 3° because the principal value of the tangent function refers to –88°. Many of those jumps are visible in every plot of that angle, e.g., in Figure 2. If it was possible to filter out these artificial oscillations one could expect to obtain a curve which is similar smooth like that of the vertical angle φ and therefore more suitable for interpretation.

Though the course of the real axis is not known, some reasonable consideration can be introduced. First, the direction is of interest, not its orientation. Therefore the horizontal angle could be rotated by 180° without loss of information. Further, the intention is identification of wave phases via principal axes of acceleration. While actual acceleration amplitudes of a single wave may vary at a very high rate, its principal direction is not likely to change strongly from one instant to another. Even if another wave of completely different characteristics and origin is arriving, the principal axis could change at most 90°. Thus, replacing every jump of θ of more than 90° by its complement to 180° according to equations (3) should generally purge its course from those artificial jumps, while jumps of about 90° which could correspond to wave dominance interchanges remain unchanged.

$$\Delta = \theta_i - \theta_{i-1};\ \mathtt{if}\ |\Delta| > 90,\ \mathtt{for}\ j = i:n,\ \theta_j = \theta_j - \mathtt{sign}(\Delta)\cdot 180,\ \mathtt{end} \tag{3}$$

Figure 3 demonstrates the effect of this correction for two sites in the near and middle field of the 1994 Northridge Earthquake. The corrected curve of θ is presented together with the vertical angle φ and the principal variance σ , i.e., the root of the maximum Eigenvalue, which represents the energy at the principal axis component. Note, the angles are not shown with the same scales.

Obviously the correction has a strong effect. Many of the jumps have considerably been shrunk by the correction, and no jump greater than 90° has remained. Nevertheless, θ can preserve a strongly changing behaviour, for the correction doesn't filter out changes over several consecutive time windows. By correction (3), θ now exceeds the ±90° bounds and great trends become visible, showing several and sometimes frequent rolling rotations. Do they reflect any of the earthquake-related wave propagation phenomena mentioned above?

In order to decide how to interpret more or less conspicuous changes in this curve, the stability of its shape with respect to parameters of the moving window technique used to introduce non-stationarity into the models needs to be proved. For the method of stochastic principal axes, this means stability of the Eigensystem of the time-dependent covariance matrix of the acceleration components .

DEPENDENCE OF STOCHASTIC PRINCIPAL AXES ON MOVING WINDOW PARAMETERS

The moving windows technique is governed by two parameters, length of the window and step width to shift the window forward. The step width S determines the sampling of the covariance values and therefore of the principal axis. The possible size of S is reasonably bound by the sampling rate Δt of the strong-motion record and the window length. However, for the range of the window length L no such simple bound exists. The range of L-values used in the past spread from 0.4 s [Scherer, 1984] applied for Friuli, Italy 1976 records, and 0.5 s [Bond, 1980] to 5 s [Kubo, Penzien, 1976 and 1979] applied for San Fernando, USA 1972 records. Windows are usually centred, requiring some kind of boundary correction at the beginning and the end of the accelerogram. Windows growing from half to full size L were chosen.

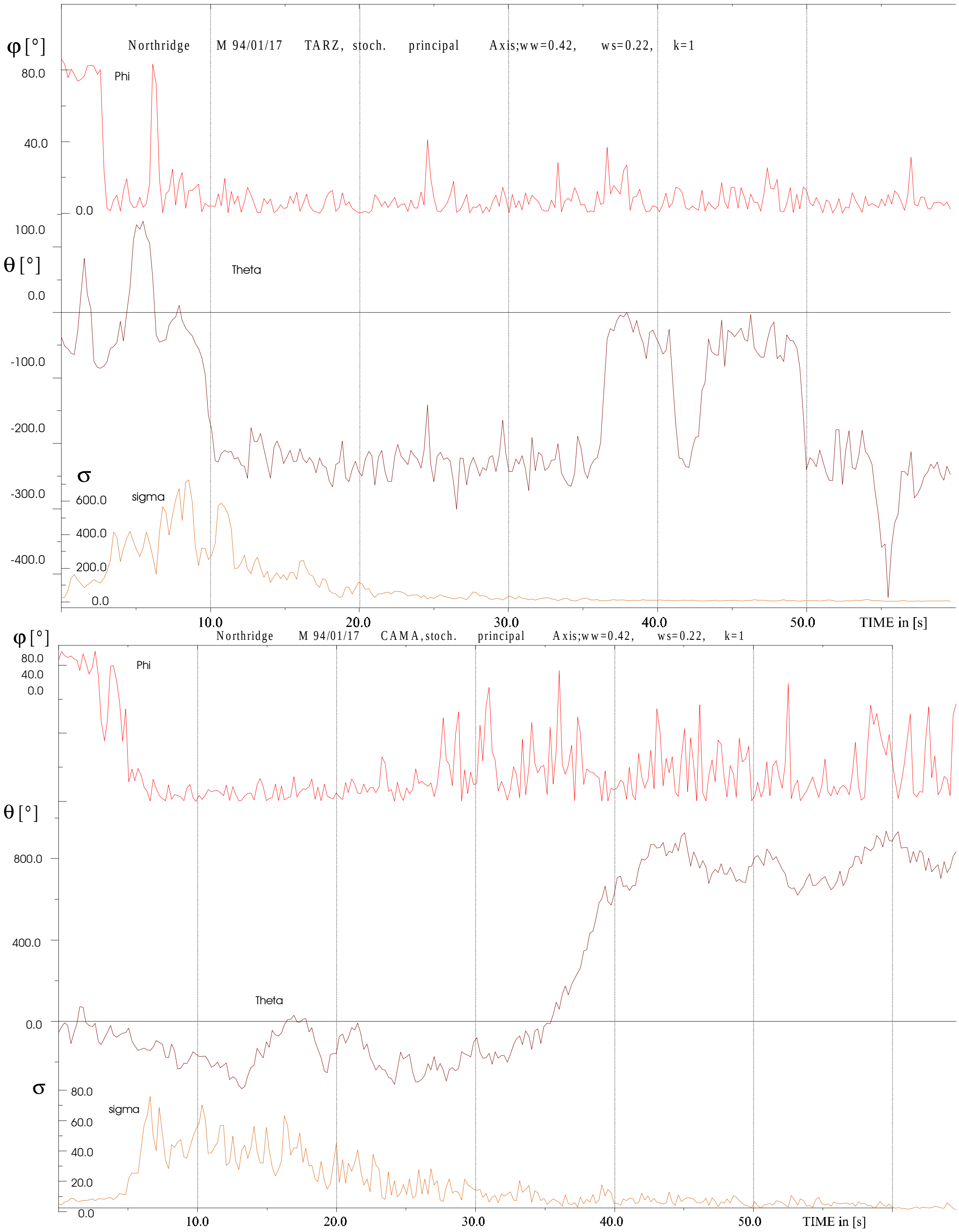

Figure 3Corrected horizontal angle θ, vertical angle φ and principal variance σ of the stochastic principal axis, 1994 Northridge Earthquake, Tarzana (above) and Camarillo (below), 5 km south and 50 km west epicentre, respectively, L=0.42, S=0.22

An increase in the step width S means nothing else than an artificial smoothing by omitting values of the target curve, i.e. loss of information. More than that, without the pre-application of a low-pass filter, aliasing may occur. Empirical studies with strong-motion records of the Northridge earthquake, sampled at a rate of 0.02 s showed that a step width S of 0.2 s is a good balance between a sufficient resolution of the target curves, computational efforts and negligible aliasing.

Varying window length L while keeping the step width S constant at 0.2 s showed that with increasing L the number of ±90° jumps in the uncorrected horizontal angle decreases and in general the curve smoothens, and that some variation at the beginning and at the end of the curve occurs. The first is due to the increasing overlap of windows and therefor similarity of consecutive covariance matrices, resulting in similar Eigensystems, which lines up with the theory of symmetric positively definite matrices. The latter arises from the growing and shrinking windows at the beginning and the end of the acceleration curve, respectively, and can be normalised.

The shape and scale of the course of the vertical angle φ as well as of the principal variance σ are generally preserved, which indicates that the Eigensystem is quite stable with respect to the window length. Frequently, the principal variance and the vertical angle indicate the arrival of direct S-waves by a strong increase of total energy and decrease of the vertical angle at the same time.

The course of the corrected horizontal angle is dependent on the window length in the following way. There are long segments in the curve with a similar course which is simply smoothened if L increases. Between those, the horizontal angle can change considerably in short-time intervals of some tenths of seconds showing some kind of rolling behaviour. The instant of occurrence of these changes is generally varying with L. This is in contradiction to the stability of the vertical angle φ, which describes the same vector. Most of the differences seem to be simple shifts of curve segments, as visible in Figure 4. These shifts occur at time instants where the horizontal angle changes by about 90° which leads to a correction to be applied or not. After discarding jumps that are not invariant with the window length, wave characteristics in the course of the angle θ can be investigated. At the time being, a heuristic discarding algorithm is under development.

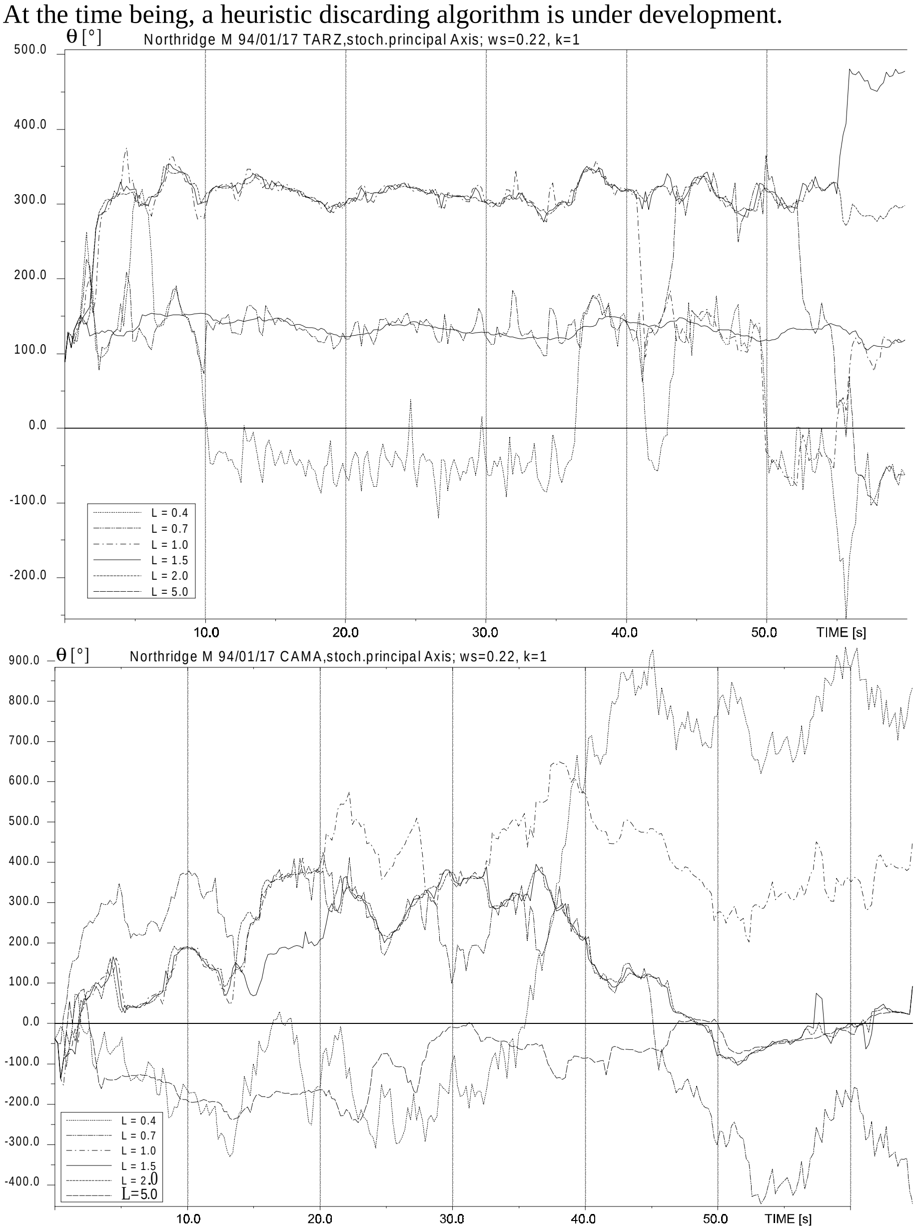

Figure 4Stability analysis of the horizontal angle θ corrected according to equation (3) with respect to window length L, Northridge 1994, Tarzana (above) and Camarillo (below)

The analysis of the strong motion records of Northridge showed an increased fluctuation in the course of the horizontal angle if the window length falls below 1.0 s. This agrees with the fact that the principal axis of acceleration cannot be recovered if the wave length is more than twice of the window length, and therefor dominant low frequency parts of the spectrum are not recognised. For window length of more than 5.0 s, changes in the course of the axis important for identification are smoothed out.

CONCLUSIONS

The horizontal angle was supposed to show especially characteristics of those waves, whose principal plane of acceleration is in parallel to the earth's surface, i.e. of S- and Love waves. Within the model of a horizontally layered subsurface topography, which was the basis for the scenario of wave arrivals developed above, the principal axis should not change more than a range corresponding to the extension of the fault plane seen from the recording site during incidence of direct S-waves. Tarzana (Figure 4) is an example. For L=1.0, 1.5, 2.0, the stochastic principal axis varies slowly around 150° with a bandwidth of variation of about 50° during the phase of direct S-waves, i.e. between about 3 to 12 s. This corresponds to the change of the direction between the moving active fault zone and the site. Other records at different distances from the epicentre show a similar balanced course of θ. However, this interpretation was not possible for all investigated sites. At Camarillo, for instance (Figure 4), only the horizontal angle estimated with a time window length of 5 s shows a smooth variation but still with a bandwidth of about 100°. At the time being it could not be decided whether these strong variations of the horizontal angle are due to physical reasons, i.e. complex wave trains and a much more complex sub-soil topography than the assumed horizontal layer system, or are exclusively due to estimation errors. As already mentioned , the series of curves in Figure 4 definitely shows that most of the strong variations are due to the insufficiency of the correction method (3). An improvement of the correction is currently under development, taking into account forecasting methods for the decision of the ±90° correction in order to overcome the leakage effect of the moving window technique, which most probably causes some of the ±90° jumps to be stretched over several windows and therefore deprived of the correction.

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