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INFLUENCE OF DIFFERENT HYSTERETIC BEHAVIOURS ON SEISMIC RESPONSE OF SDOF SYSTEMS

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SUMMARY

A parametric study on the inelastic dynamic response of single degree of freedom (SDoF) systems characterised by different hysteretic behaviours is presented in this paper. Based on the review of existing related works, some peculiar factors characterising the cyclic response of typical structural components (i.e. non-linearity, hardening, pinching, strength degradation) have been preliminarily selected. Numerical analyses, devoted to investigate the main parameters ruling the fatigue life of the system, namely kinematic ductility and hysteretic energy, have been performed. Results, obtained as average from a number of natural records, allow the phenomenological aspects having a major impact on the seismic performance of the system to be identified.

INTRODUCTION

As it is well known, the great success of SDoF models relies mainly on two aspects: it is the simplest conceivable structural system and it can be used for predicting linear elastic behaviour of MDoF structures according to response spectrum concept. This kind of schematisation has been widely used for studying the inelastic response of structures as well, even if with little theoretical and experimental background. Most of recent studies are focused on the possibility to extend effective methods for dynamic analysis in the linear range to the case of non-linear structures and this is still referred to SDoF system theory. However, the extension to non-linear problems needs some difficult questions to be solved. In fact, the number of parameters defining the response of a linear SDoF system is small and defined uniquely. On the contrary, the type of hysteretic behaviour characterising the restoring force conditions non-linear SDoF system response. Besides, in non-linear case there is an additional difficulty related to viscous damping modelling. In fact, in linear systems viscous damping is usually used as an equivalent source of non-linearity, in order to globally account for all dissipating sources of the structural system (material damping, friction damping, material non-linearity). But if material non-linear response is considered explicitly, viscous damping should be referred to other sources of energy dissipation only, about which there is the largest uncertainty.

It is the author opinion that modelling of SDoF systems for predicting seismic response of MDoF structures is still questionable, it being therefore worthy of further investigations. It has to be clearly understood what is the relation between SDoF and MDoF systems (as in the undamped linear case) in order to correctly and uniquely define the rules for choosing the equivalent appropriate SDoF system, by which obtain the desired information. However, as a first step for the fully understanding of the problem, the modelling parameters for both damping and hysteretic behaviour affecting the seismic non-linear response of SDoF systems should be identified. Then such parameters should be accounted for when using SDoF system as analytical tool for investigating the earthquake damage potential of actual MDoF structures.

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PREVIOUS STUDIES

First important studies concerning with seismic response of SDoF systems date back to the last fifties [Housner, 1959]. Ten years later Newmark and Hall [1969] published a fundamental work on linear elastic response spectra. Since that date a large amount of research effort has been spent for the evaluation of seismic response of linear SDoF systems, with particular attention to the influence of input motion, in some cases reflecting the influence of site conditions. It is interesting to observe that the study of inelastic SDoF systems developed contemporary to the elastic case. In fact, in 1969 Veletsos presented first studies on IRS (inelastic response spectra). In 1975 Murakami and Penzien computed probabilistic non-linear constant strength response spectra for SDoF systems with four types of hysteretic behaviour and subjected to 100 artificially generated earthquakes. In 1979 Riddel and Newmark computed constant ductility IRS of 10 recorded earthquake ground motions considering the effects of both damping and hysteretic behaviour. More recently, Minami and Osawa (1988) conducted parametric studies on elastic-plastic response spectra for different hysteretic models classified according to their strain energy-absorbing capacities. By analysing obtained results it seems that fundamental inelastic response parameters, namely kinematic and cumulated ductility, are slightly influenced by hysteretic assumptions for models belonging to the same group. While significant differences can be observed when radically changing the type of dissipative behaviour (for example going from a bilinear full-dissipative model to a partial-dissipative pinching-type one). In 1992 Krawinkler and Nassar studied average IRS of bilinear and stiffness degrading SDoF systems. They concluded that IRS are only slightly modified by the type of hysteretic model considered. Extensive parametric studies were conducted by Fajfar et al. [1989, 1992, 1994] in almost ten years of research activity. The influence of both damping and hysteretic modelling on four types of interrelated constant ductility non-linear response spectra (strength, displacement, input and hysteretic energy) was considered. Their relevant results show again that hysteretic model influence significantly inelastic response only when changing radically the type of dissipative behaviour (with or without pinching of hysteretic cycles), while the influence is slight in case of substantial similar shape of hysteretic cycles. Besides an important influence of damping modelling was observed. These conclusions are drawn in comparison with results of influence of input earthquake motions, which is more significant. Cosenza and Manfredi [1994] carried out a study on the influence of stiffness and strength degradation on constant ductility strength reduction factor spectra. It was concluded that damage phenomena are somewhat influencing the design strength, but this is strictly related with the type of input motion considered. However, this influence seems to be slight when compared with the one concerned with other factors, e.g. the adopted collapse criterion.

In the framework of the European Research Project Copernicus-RECOS [Mazzolani, 1999] the authors developed a mathematical model able to consider most of the relevant aspects of the mechanical behaviour of beam-to-column steel joints in framed structures, as experimentally stated. In particular, the proposed model take into account non linearity and kinematic hardening of the monotonic behaviour, cyclic hardening and cyclic damage of mechanical properties, and, if it is the case, pinching of hysteretic cycles. In this paper the model is applied to study inelastic dynamic response of SDoF systems.

EQUATION OF MOTION

The well-known equation of motion of SDoF systems writes:

$$m\ddot{x} + b\dot{x} + F_s = -ma_g \tag{1}$$

where m is the mass, b the viscous damping coefficient, F_s the restoring strain-related force, x the displacement of the mass relative to the ground, and a_g the time-dependent ground acceleration. With the following positions:

$$\mu = \frac{x}{x_y}\,; \quad \varphi = \frac{F_s(x)}{k_0 x_y}\,; \quad \omega_0 = \sqrt{\frac{k_0}{m}}\,; \quad \nu = \frac{b}{2\sqrt{k_0 m}}\,; \quad PGA = max[\,a_{g,max}; abs(\,a_{g,min}\,)] \tag{2}$$

x_y being a conventional yield displacement, k_0 the initial stiffness of the system and PGA the peak ground acceleration, equation (1) can be rewritten as:

$$\ddot{\mu} + 2\nu\omega_0\dot{\mu} + \omega_0^2\varphi = -\frac{k_0}{m}\frac{mPGA}{k_0 x_y}\frac{a_g}{PGA}\,. \tag{3}$$

By introducing the 'resistance level' R^* defined as follows:

$$R^* = \frac{mPGA}{k_0 x_y} = \frac{mPGA}{F_y} \tag{4}$$

equation of motion finally writes:

$$\ddot{\mu} + 2\nu\omega_0\dot{\mu} + \omega_0^2\varphi = -R^*\omega_0^2\frac{a_g}{PGA} \tag{5}$$

For a given hysteretic model (i.e. relationship $\varphi = \varphi(\mu)$), a fixed value of ω_0, ν, R^* and a chosen non-dimensional ground acceleration time-history (a_g/PGA), the step-by-step numerical integration of equation (5) has been performed by means of the linear acceleration Newmark's method [1959]. Thus we obtained ductility (μ_{max}) and non-dimensional hysteretic energy demand ($e_h = E_h/R_y x_y$, E_h being the actual hysteretic energy), defined as the maximum values reached throughout the whole deformation history.

HYSTERETIC MODELS

Considered models can be grouped in two different types according to their dissipative capacities: 1) full-dissipative-type models (without pinching) and 2) partial-dissipative-type models (with pinching). To group 1 belong classical elasto-plastic bilinear model (EPB) and a new specifically developed hysteretic fully non-linear model (EPNL), whose behaviours are qualitatively shown in figures 1.a and 1.b, respectively. In particular, the fully non-linear model was based on the generalised force-deformation relationship proposed by Richard and Abbott [1975]. It should be noted that EPNL model may also account for strength degradation, due to both repeated inelastic excursions and softening branches [Della Corte et al. 1999]. To group 2 belong classical elasto-plastic bilinear with slackness model (EPBS) and a new developed fully non-linear with pinching model (EPNLP) [De Matteis and Landolfo, 1999], whose behaviours are qualitatively shown in figures 2.a and 2.b, respectively. Performed parametric study is addressed to evaluate the effect of non-linearity, hardening ratio, cyclic damage and pinching on plastic engagement, measured through kinematic ductility μ and non-dimensional hysteretic energy e_h.

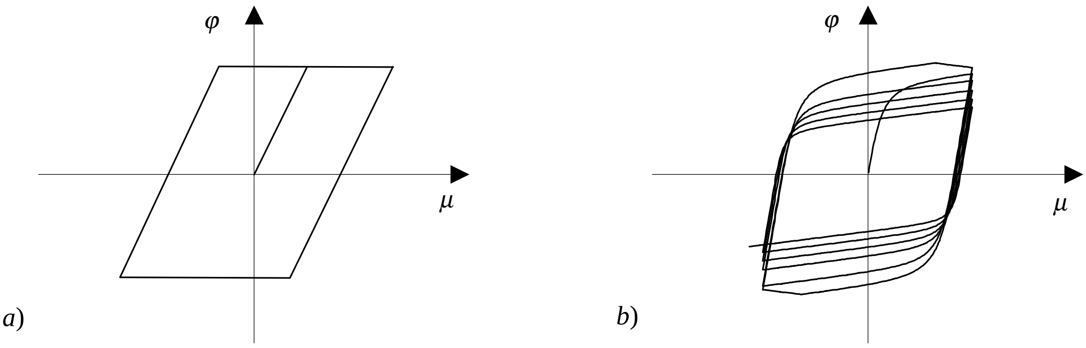

Figure 1. ***a*) Elasto-plastic bilinear (EPB) and *b*) Elasto-plastic fully non-linear model (EPNL).**

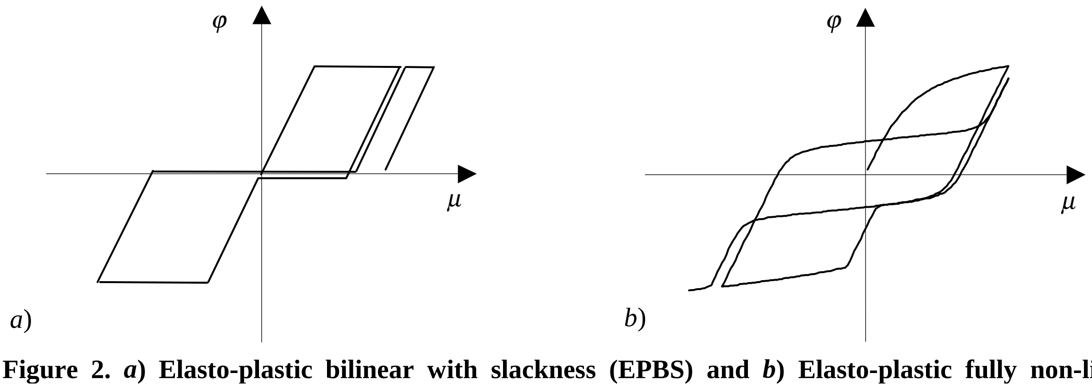

Figure 2. ***a*) Elasto-plastic bilinear with slackness (EPBS) and *b*) Elasto-plastic fully non-linear with pinching model (EPNLP).**

RESULT FORMAT

For the presentation of results, the following parameter R has been introduced:

$$R = R^* \frac{S_{a,el}(T, \nu = 0.05)}{PGA} = \frac{mPGA}{F_y} \frac{S_{a,el}(T, \nu = 0.05)}{PGA} = \frac{mS_{a,el}(T, \nu = 0.05)}{F_y} \tag{6}$$

in which $S_{a,el}$ is the pseudo-acceleration of the equivalent elastic system of period $T = T_0$ (= $2\pi/\omega_0$) and damping ratio ν = 5%. In other words, R is the ratio of maximum elastic force that would stress the system ($mS_{a,el}$) and its actual strength (F_y). Five values of the reduction factor R were assumed in the parametric studies, namely 1, 2, 4, 6 and 8. However, only some results related to R = 1 (elastic system) and R = 6 (high inelastic deformation demand) are presented in the following.

Analyses have been carried out considering 9 acceleration time histories, recorded on rigid and medium soil conditions, corresponding to earthquakes registered in different world Regions. All accelerograms have been scaled to the same *PGA* and then amplified by means of parameter R in order to consider increasing plastic engagement.

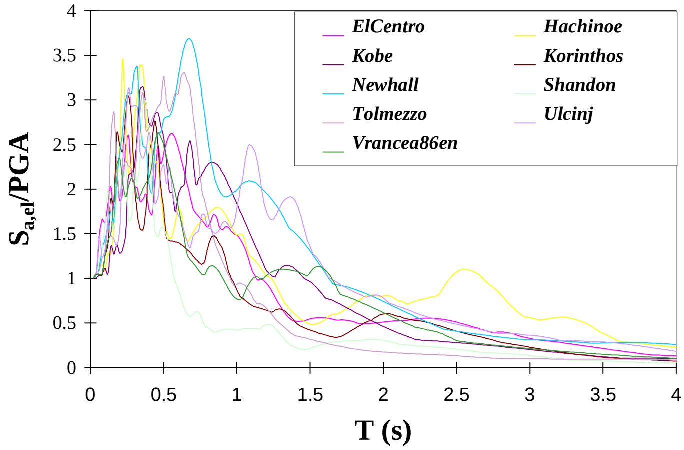

Figure 3. Normalised linear elastic pseudo-acceleration spectra for considered earthquake records (ν = 5%).

For every value of R, non-linear ductility and non-dimensional hysteretic energy demand spectra have been computed as mean values among all the earthquakes. The range of variation for the initial period of vibration ($T = T_0$) was assumed equal to 0.4 – 3 s, with time step equal to 0.2 s. Results are presented by varying the values of several parameters of the above hysteretic models. The meaning of these parameters and their influence on plastic engagement are discussed in the following.

OUTCOMES

Figures 4 and 5 refer to full-dissipative systems (EPB and EPNL) without damage of mechanical properties and in absence of strain hardening. Considered parameter, named n in the figures, regulates the sharpness of transition from elastic to fully plastic behaviour in EPNL model. Small values of n indicate strong non-linear behaviour also for small deformation amplitudes, while for high values of this parameter the model approaches the same behaviour of EPB model.

As it was expected, figure 4.a shows that there is no plastic engagement for R = 1 and that displacements of non-linear systems are slightly smaller than displacements concerned with EPB model. This last observation is justified considering the hysteretic energy dissipated by non-linear system as confirmed by energy demand graphs (figure 4.b). For high plastic engagement (figures 5.a and 5.b), the influence of shape parameter is even less important, allowing EPB model to be used for the evaluation of plastic engagement.

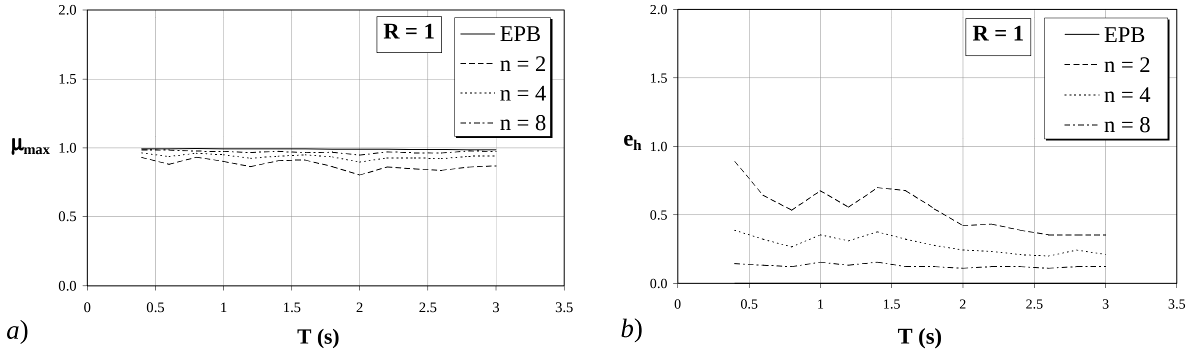

Figure 4. Ductility (*a*) and non-dimensional hysteretic energy (*b*) demand for full-dissipative type models characterised by various degrees of non-linearity (*n*), for *R* = 1.

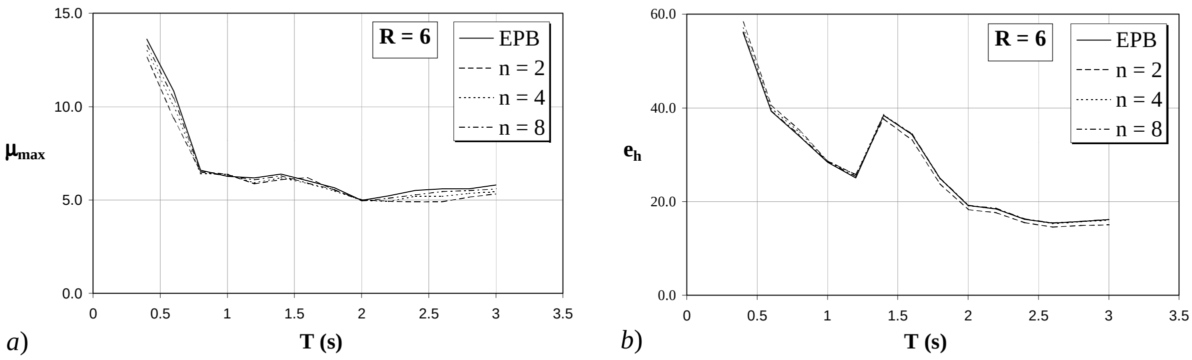

Figure 5. Ductility (*a*) and non-dimensional hysteretic energy (*b*) demand for full-dissipative type models characterised by various degrees of non-linearity (*n*), for *R* = 6.

Figure 6 illustrates effects of kinematic strain hardening on the response of EPNL model (n = 4). Hardening ratio h, which is defined as the ratio of post-elastic to initial stiffness, equal to 0, 5, 10 and 20% have been considered.

Generally, a decrease of ductility demand with increasing values of h can be observed. Hysteretic energy is however increasing with h probably owing to the compensation effect due to wider hysteretic cycles for higher hardening. Besides, it can be noticed that the influence of strain hardening is reduced as far as the period of vibration increases, this being partially due to the shape of the single spectrum characterised by higher gradient for lower periods of vibration. It is also interesting to observe that there are only slight differences in ductility demand when changing the value of h from 0.05 to 0.20 but differences are much more pronounced when going from h = 0 to h > 0. This means that the value chosen for the kinematic hardening ratio is not so important as the assumption of h different from zero itself.

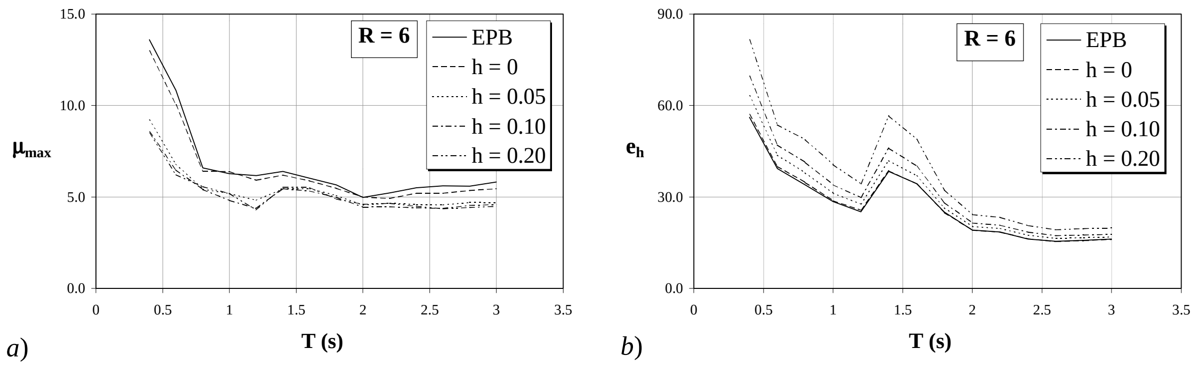

Figure 6. Ductility (*a*) and non-dimensional hysteretic energy *(b*) demand for full-dissipative type models characterised by various degrees of kinematic hardening (*h*), for *R* = 6.

Influence of strength degradation on ductility and non-dimensional hysteretic energy demand is illustrated in figure 7, where n = 4 and h = 0 have been considered for EPNL model. Damage of strength has been simulated adopting the following degradation rule:

$$F_{y,dam} = F_y\left(1-\beta\frac{E_h}{F_y x_u}\right) \tag{7}$$

$F_{y,\mathrm{dam}}$ being the damaged value of initial resistance F_y, x_u a conventional ultimate deformation and β an empirical coefficient. This degradation law was found to be effective in simulating a number of experimental cyclic deformation histories concerned with steel beam-to-column joints [Della Corte et al., 1999]. Appropriate values for parameter β were found to be similar to mean values of the β parameter of the well known Park and Ang damage model [1985]. Values for parameter β equal to 0.025, 0.05 and 0.15 have been therefore assumed, in order to consider a low, medium and high rate of strength degradation, respectively. Besides, for all cases, a fixed ultimate ductility $\mu_{\mathrm{ult}} = x_{\mathrm{ult}}/ x_y$ equal to 10 has been adopted.

From figure 7 it can be observed that the influence of strength degradation on both ductility and non-dimensional hysteretic energy demand is quite limited. Only for high rate of strength degradation (β = 0.15) differences in prediction may be occasionally significant.

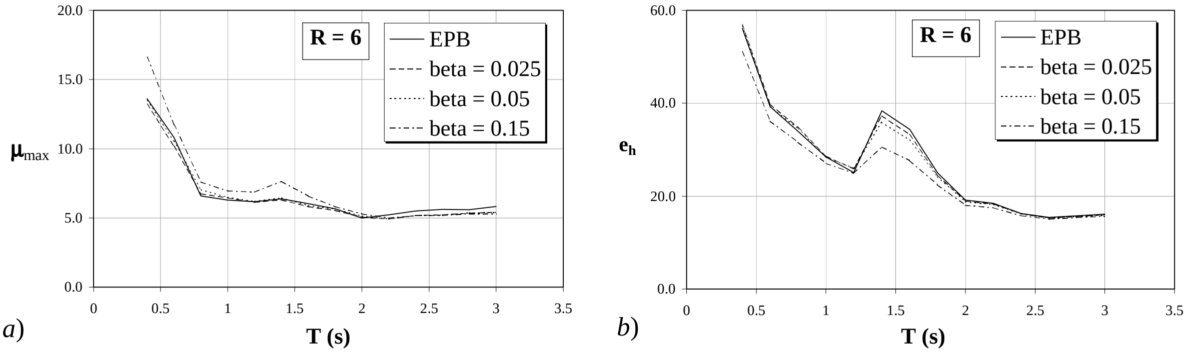

Figure 7. Ductility (*a*) and non-dimensional hysteretic energy (*b*) demand for full-dissipative type models characterised by various degrees of strength degradation (β), for *R* = 6.

Figure 8 shows the effect of softening branches in the hysteretic restoring force-deformation relationship (EPNL model with n = 4, h = 0, β = 0). When a softening branch is activated strength degradation verifies during that deformation excursion. The minimum level of strength reached along this decreasing branch was assumed to be the new strength for successive deformation excursions [Della Corte et al., 1999]. Two values of ductility activating softening branches have been considered, namely μ_{inst} = 3 and μ_{inst} = 5.

An increment in ductility demand with decreasing values of μ_{inst} can be observed in figure 8.a, while figure 8.b shows the decrease of non-dimensional hysteretic energy with decreasing values of μ_{inst}, which may be due to the compensation effect of increasing ductility and smaller size of hysteretic cycles. It can be noticed that any influence for long initial periods of vibration vanishes and all curves in figure 8.a approach the same value of ductility demand, practically coincident with the value of *R*.

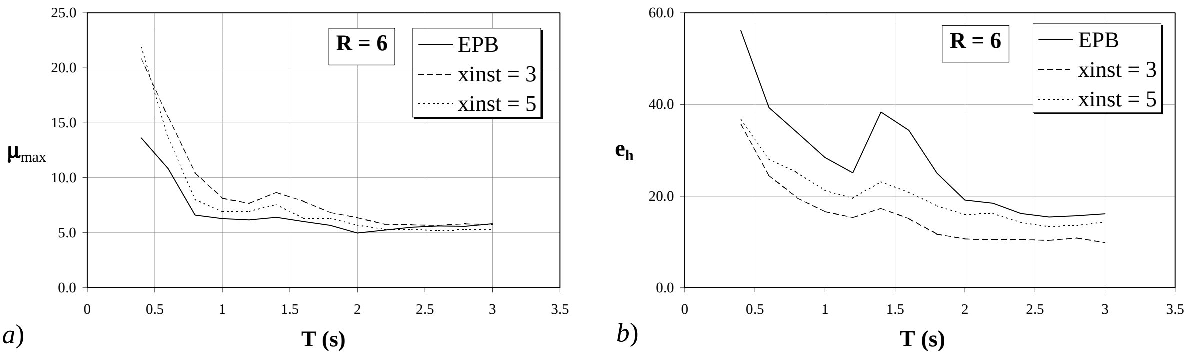

Figure 8. Ductility (*a*) and non-dimensional hysteretic energy (*b*) demand for full-dissipative type models characterised by various levels of softening ductility (x_{inst}), for *R* = 6.

Finally, figure 9 shows the variation of ductility and non-dimensional hysteretic energy demand going from full-dissipative (without pinching) to partial-dissipative (with pinching) type of hysteretic behaviour. The two examined models were characterised by the same envelope curve (same initial stiffness k_0, yielding resistance F_y, shape factor n, hardening ratio h) but they differ for the shape of the hysteretic cycles. In particular, the pinching-type model is characterised by the parameter F_{0p} representing the intersection of hysteretic curves with the force axis, normalised with respect to F_y. The analysed cases are referred to a constant value of this parameter, chosen equal to 0.2, which corresponds to a strong pinching effect.

It can be observed that ductility demand increases when going from EPNL to EPNLP model, while hysteretic energy demand decreases, due to compensation of increased ductility and reduced size of hysteretic cycles. Once again, the effect of pinching on ductility demand decreases for increasing values of the period of vibration.

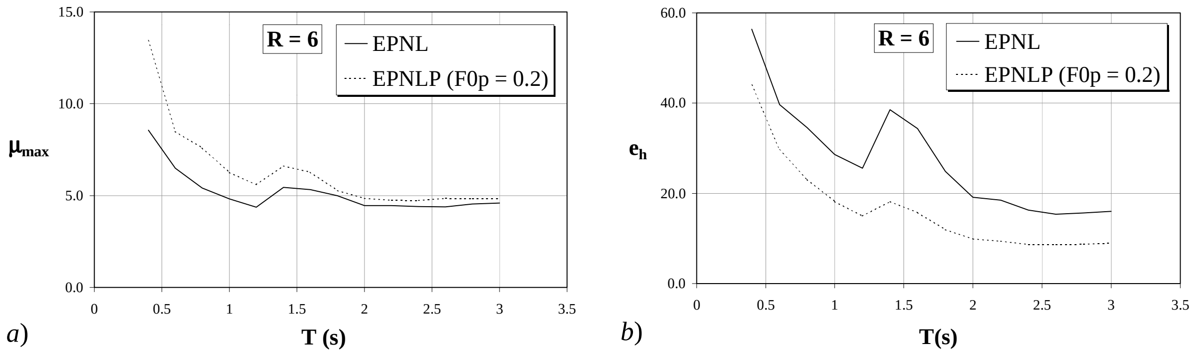

Figure 9. Ductility (*a*) and energy (*b*) demand for full-dissipative (without pinching) and partial dissipative (with pinching) type models, for *R* = 6.

It is interesting to observe from figures 10 and 11 the influence of the value chosen for viscous damping ratio in evaluating inelastic response spectra. Obviously, both ductility and non-dimensional hysteretic energy demand increase when decreasing values of damping ratios are considered. Differences are of the same order of magnitude than those due to the most influencing hysteretic modelling parameters.

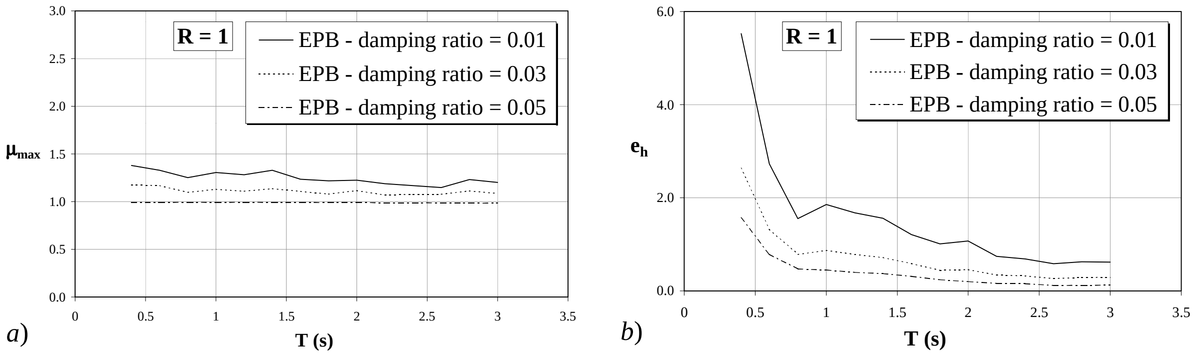

Figure 10. Ductility (*a*) and non-dimensional hysteretic energy (*b*) demand for EPB model characterised by different values of damping ratio (ν), for *R* = 1.

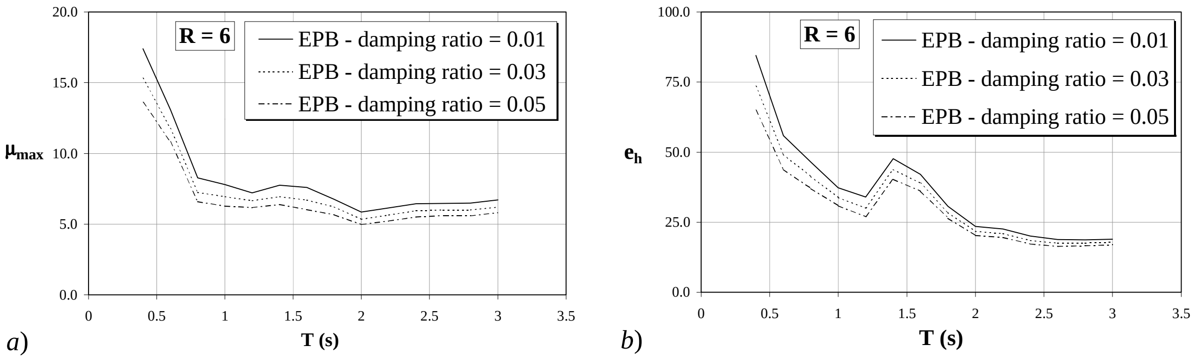

Figure 11. Ductility (*a*) and non-dimensional hysteretic energy (*b*) demand for EPB model characterised by different values of damping ratio (ν), for *R* = 6.

CONCLUSIONS

Effects of some modelling assumptions on ductility and non-dimensional hysteretic energy demand to SDoF systems have been evaluated in this paper. On the basis of obtained results it can be concluded that kinematic hardening, pinching of hysteretic cycle and strength degradation due to softening branches in the hysteretic response are the three phenomenological aspects that have a major impact on seismic demand to the system. On the contrary, the influence of fully non-linear behaviour is negligible, thus confirming the validity of simple elasto-plastic bilinear models. As far as strength degradation due to plastic fatigue is concerned a slight influence on plastic engagement has been observed. Moreover, the effect on ductility demand of most of parameters is period dependent, being slighter for longer periods of vibration. The influence of viscous damping ratio is of the same order of magnitude as the effect of the most influencing hysteretic parameters.

At the light of obtained results, it is apparent that the correct prediction of seismic response of SDoF systems is quite complex. On the other side most of parameters affecting the hysteretic response of the structural component have a limited influence on the global performance of the system. Therefore elastic-perfectly plastic models, with or without slackness, seem to be suitable models for simplified seismic analyses of SDoF systems, especially when used for obtaining information about the global performance required to MDoF systems. Nevertheless, the adoption of sophisticated hysteretic rules could be justified for non-linear global analyses of MDoF structures, by a detailed schematisation of all structural components, only one or few of these being possible responsible for the collapse of the whole.

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