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SEPARATION OF SOURCE AND SITE EFFECTS USING WAVELET TRANSFORM COEFFICIENTS

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SUMMARY

We attemp to separate source, propagation path and site effects from the earthquake records in time-frequency domain using the wavelet expansions. We understand that nonstationary waveforms of the strong motion records are composed of direct S-waves and the lateral-arrival waves such as reflected, refracted wave and surface wave in propagation-path. We factorize small events records observed on different soil conditions to minimum-phase function and all-pass function. Then we take into account that the minimum-phase function consists of path effect and the all-pass function consists of source and site effects. Modified all-pass function that the propagation-path effects are removed shall be separated to source and site effects. If we assume the source effect following the omega square scaling model, we reconstruct the source time function using this wavelet spectrum. Then we obtain the site time function subtracting the source time wavelet expansions from observed wavelet expansions.

INTRODUCTION

Several authors have studied separation to source, propagation path, and site effects for ground motions based on observed records using the spectral inversion method[Andrews, D. J., 1982; Iwata and Irikura, 1986; Takemura, et al., 1991; Kato et al., 1992; Tai et al., 1992; Kowada et al., 1998]. They evaluated source, path and site characteristics in terms of Fourier spectral amplitude. Fourier spectrum ignores the nonstationary phase characteristics of ground motions. In this paper, we attempt to separate those three effects from the earthquake records in time-frequency domain using wavelet expansions. Seismic waves are separated into amplitude-dependent parts, called minimum-phase functions(MPS), and other parts , called all-pass function(AP) [Izumi et al., 1988a; Izumi et al., 1988b; Katsukura et al., 1989; Katsukura et al., 1991]. This method is called factorization by means of deconvolution using FFT(Fast Fourier Transform). Wavelet expansions will present us many effective knowledge about nonstationary phase characteristics of observed data [Yamada and Okitani, 1990; Sasaki et al., 1992; Sasaki and Maeda, 1993; Miyawaki and Toki, 1995; En and Takami, 1998]. We try to separate observed earthquake data to three large effects, source, path and site effects using the method of factorization and wavelet expansions. We show an example of the separation using the data from an aftershock records of 1995, Hyougoken-Nanbu Earthquake.

2. SEPARATION OF SOURCE, PATH, AND SITE EFFECTS

2.1 Method

2.1.1 Factorization

We follow Izumi's method and factorize observed ground motions to MPS and AP function by means of deconvolution using FFT. x(t) is time history of observed ground motion and X(f) is Fourier spectrum of observed ground motion. Deconvolution of X(f) and x(t) is expressed as

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$$x(t) = x_{MPS}(t) * x_{AP}(t) \tag{1}$$

$$X(f) = X_{MPS}(f) \cdot X_{AP}(f) \tag{2}$$

Here, f is frequency, t is time, dot is product, and * is convolution. MPS shows the minimum-phase function and AP shows the all-pass function. Both functions satisfy the condition of causality. This factorization is only weighted in phase. The amplitude of both factors are arbitrarily given like

$$|X(f)| = |X_{MPS}(f)| \text{ , } |X_{AP}(f)| = 1.0 \tag{3}$$

We assume a different amplitude relation as shown below. The observed spectrum can be expressed as

$$X(f) = S(f) \cdot P(f) \cdot G(f) \tag{4}$$

Here S(f) is source spectrum, P(f) is propagation path spectrum constrained by frequency dependent Q-value, and G(f) is site amplification spectrum. In those three effects in X(f), we can assume that P(f) is related to the minimum-phase function. Then equation(4) is factorized as follows.

$$X(f) = [P(f)]_{\text{minimum phase}} \cdot [S(f) \cdot G(f)]_{\text{all pass}} \tag{5}$$

Here we determine the path effects based on the minimum-phase function.

$$X_Q(f) = [P(f)]_{\text{minimum phase}} \tag{6}$$

$$\mathrm{Re}[P(f)] = \exp(-\pi f R / QsVs) \tag{7}$$

$$\mathrm{Im}[P(f)] = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\mathrm{Re}[P(f)]}{2\pi f - y} dy \tag{8}$$

That is, $X_Q(f)$ is given as a complex function consisting of a real part, equation (7) and a imaginary part, equation(8), i.e. the Hilbert transform of equation(7) to satisfy the condition of causality. Using this complex function, we can express source and site effects as

$$[S(f) \cdot G(f)]_{\text{all pass}} = \frac{X(f)}{X_Q(f)} \tag{9}$$

2.1.2 Wavelet expansions

Then we have the problem how we can separate S(f) and G(f). We put $X(f)/X_Q(f)$ as $X_{il}(f)$. Subindex i shows the source and l shows the site. $x_{il}(t)$ is given as the inverse Fourier transform of $X_{il}(f)$ that the propagation-path effects are removed from the observed records. For simplicity of explanation for separating S(f) and G(f), first we take into account one-layer structural model on a bed rock. The bed rock motion is assumed to be approximately equal to the source effect because the propagation-path effects are already removed from the records. Then $x_{il}(t)$ is expressed as

$$x_{il}(t) = \gamma\left[s_i(t) + \sum_m \beta^m s_i\left(t - \frac{2mH}{V}\right) + \sum_l \beta^{m-1} s_i\left(t - \frac{2mH}{V}\right)\right] \tag{10}$$

where $s_i(t)$ is incident wave from the bed rock to the layer, γ is transmit coefficient, H is layer thickness and V is velocity of layer. We can generalize equation(10) to the following equation for more complex layer problem.

$$x_{il}(t) = \gamma_l s_i(t) + s_i(t) * g_{il}(t) = \gamma_l s_i(t) + g_l(t) \tag{11}$$

where $g_{il}(t)$ is layer response. This means that $x_{il}(t)$ is expressed as a sum of the source term $s_i(t)$ and site term $g_l(t)$. Now we transform $x_{il}(t)$ into the wavelet expansions using Meyer' analyzing wavelet $\psi(t)$.We follow the calculation technique in the papers[Sasaki et al., 1992; Yamaguchi et al., 1990; Yamada and Okitani, 1990].

$$x_{i1}(t) = \sum_{j}\sum_{k} \alpha_{jk}\psi(t) \tag{12}$$

When we assume the source effect follows the omega square scaling model, the wavelet spectrum WT(f) is evaluated using the Fourier spectrum S(f). We modify the omega square model using high cut filter $P_m(f_{max})$[Boore, 1983].

$$\mathrm{WT}(f_j) = \sum_{j} \left| \ S(f_j) \ \right|^2 \qquad , \ S(f_j) = \frac{(2\pi f_j)^2 M_0}{1 + (f_j / f_c)^2} P_m(f_{max}) \tag{13}$$

Here, fc is corner frequency and Mo is seismic moment. We define the ratio, the wavelet spectrum following the omega square model to the observed wavelet spectrum of $x_{il}(t)$, that is

$$W_{ratio}(f_j) = \mathrm{WT}(f_j) / \sum_{j}\sum_{k} \left|\alpha_{jk}\right|^2 \tag{14}$$

Then we can separate the source term $s_i(t)$ following the omega square model using the wavelet expansions as

$$s_i(t) = \sum_{j} \sqrt{W_{ratio}(f_j)} \sum_{k} \alpha_{jk}\psi(t) \tag{15}$$

Then we can evaluate the site term $g_l(t)$ subtracting $s_i(t)$ from $x_{il}(t)$

$$g_l(t) = x_{il}(t) - s_i(t) = \sum_{j}\sum_{k} \alpha_{jk}\psi(t) - \sum_{j} \sqrt{W_{rati}(f_j)} \sum_{k} \alpha_{jk}\psi(t) \tag{16}$$

2.2 Results of separation

2.2.1 Data set

Small earthquake records that we use have been observed in Kinki area. We show an example of the separation using the data from an aftershock record of 1995,Hyogoken-Nanbu Earthquake. We show the epicenter and locations of observed points in Figure 1. This small event happened at 23:15, 25/January, 1995. JMA magnitude is 5.1. Depth is 14.8km. The ground motions were recorded at 11 array stations, KBU, MOT, TYN, ABN, AMA, FKS, MRG, YAE, TDO, SKI, and CHY. The observed points of KBU and CHY are on bed rock, MOT,TYN,ABN,TDO, and SKI on hard soils, AMA, FKS, MRG, and YAE on soft soils. We use the transverse component of the records for the separation analysis, because SH-wave have predominant power at this direction.

2.2.2 Propagation-path, source, and site effects

Using the earthquake records x(t) at all of the observed sites, we factorize x(t) to the minimum-phase and the all-pass functions (see equation(5)). Then we calculate the propagation path effect $X_Q(f)$ given by equation(6) assuming Q-value $Qs=37.0f^{0.84,}$ [Kowada et al., 1998].

We determine source and site effects using equation(9) at 11 array sites. The results are shown in Figure 2.1-2.3 in time-frequency domain. In Figure 2.1, we show the envelope, and wavelet expansions coefficients. In Figure 2.2, we show the inverse wavelet transform, called one of deconvolution. In Figure 2.3, we show the Fourier spectrum(solid line) and the wavelet spectrum(●) of the modified all-pass function given by equation(9).

In Figure 2.2, we find that the waveforms of direct-S wave at 0.-1.0sec at each site differs one another, because it might be affected by site effect. The seismogram at KBU(rock site) has remarkable later-arrival wave about 1sec after the onset(t=0) which has clear peak about 4 Hz from the wavelet expansion. Such later phase might be reflected or refracted waves from some heterogeneous geological configulations, composing the site effect. The seismograms at FKS and ABN also have similar later-arrival waves with different predominant frequencies, low frequency at FKS and high frequency at ABN.

Source time function and site time function

We consider the separation of source and site using the wavelet expansion coefficients assuming source spectrum follows omega square model. First we get the wavelet expansions coefficient with the omega square scaling model using equation(13). We use 2.5Hz for fc and 7.0Hz for fmax from the spectral shape at rock site, KBU. Then calculate wavelet spectrum ratio using equation(14). We determine source time function and site time function using equation(15) and (16) at all array sites. We show the source time function in Figure 3. and the site time function in Figure 4. We find that the source functions have nearly the same waveform for the first 0.8 seconds in Fig.3. However, they remain small later-arrival waves, suggesting that the separation is not complete yet. On the other hand, the site time functions show different characteristics on each site.

3. CONCLUSIONS

We proposed a new separation method of source, propagation-path, and site effects from the earthquake records in time-frequency domain using the wavelet expansions. The applicability of this method was examined using the observed data from an aftershock of the 1995 Hyogo-ken-Nanbu earthquake.

(1)We can remove attenuation effect from the observed records using facterization method and get the modified all-pass function consists of source and site effects. Factorization is very effective method to modify the earthquake records that satisfy the condition of causality.

(2)We can determine the source time function using the wavelet expansions coefficients following omega square scaling model. Then the source time function from the observed data on different soil conditions is nearly same as the waveform. This means that the separation based on omega square scaling model is efective. However we can not delete later-arrival wave completely.

(3)We can determine the site time function subtracting the source time function from modified all-pass function of the observed records. The results of the site time function show different characteristics on each site that has different soil condition.

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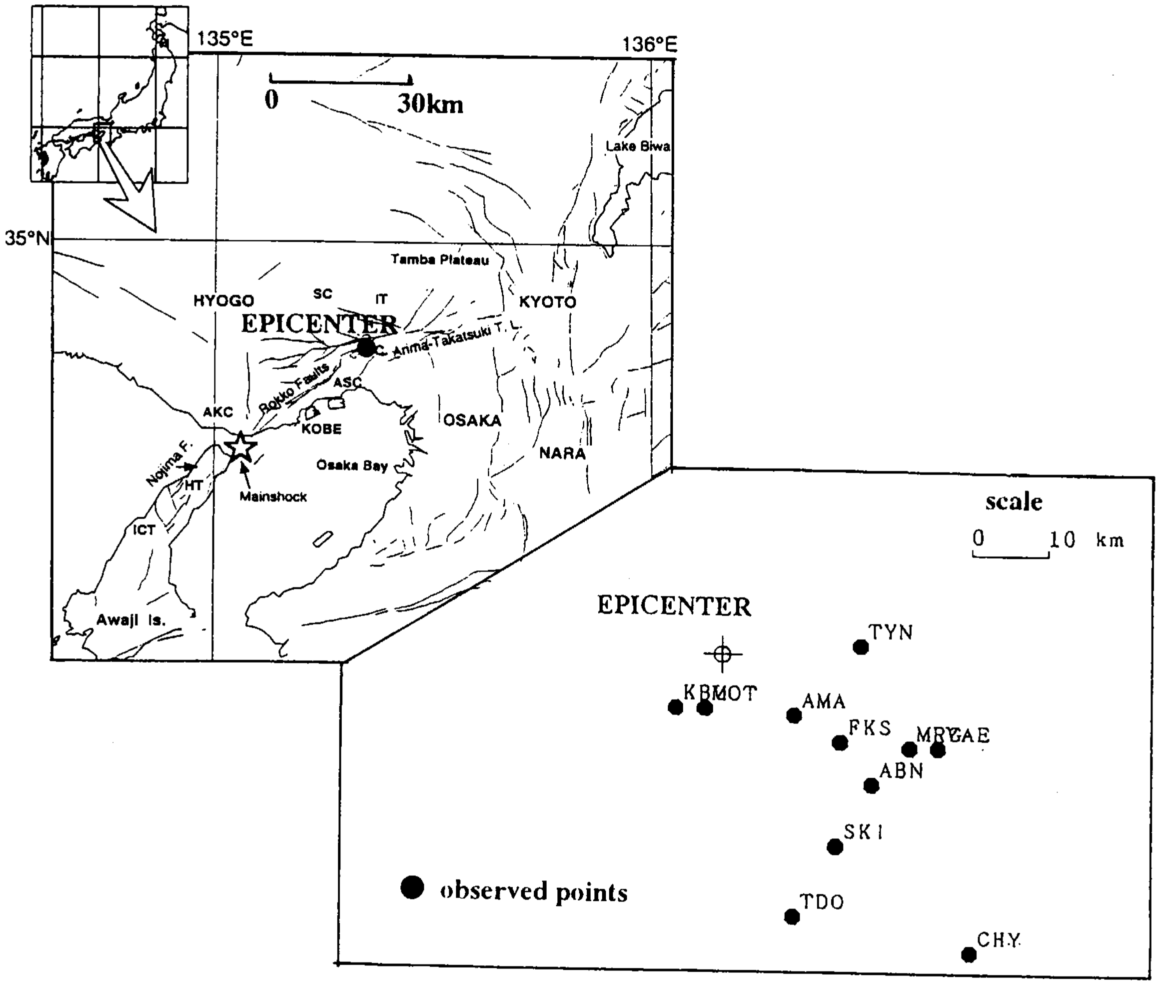

Figure l. Epicenter and locations of observed point.

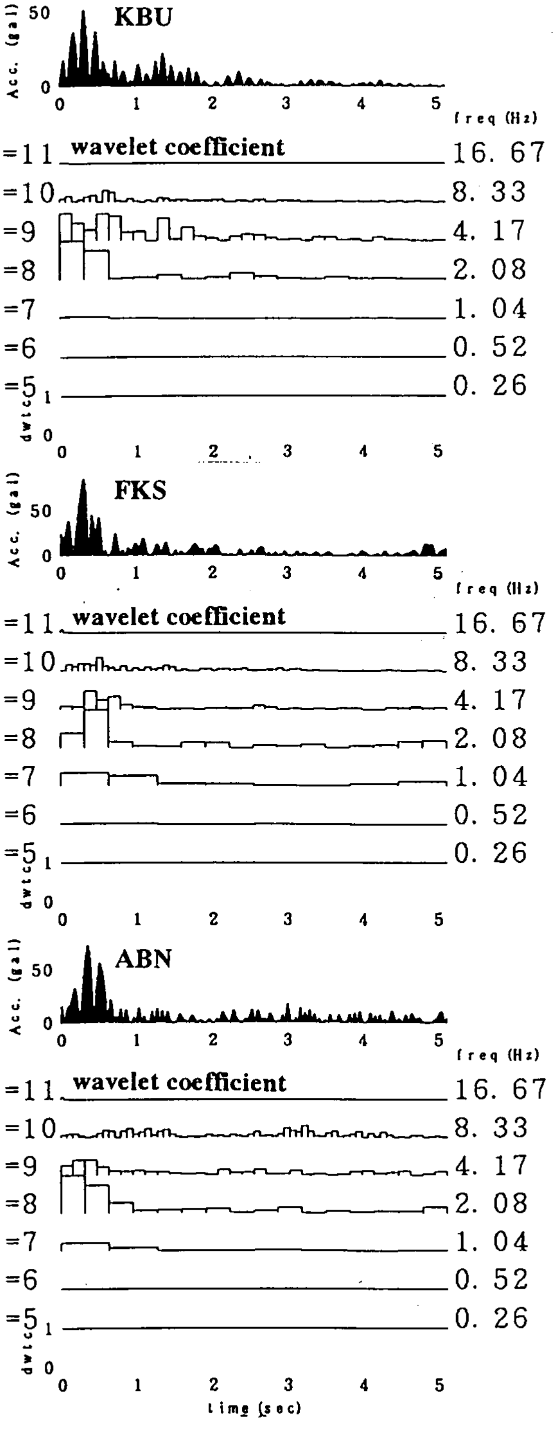

Figure 2.1 Envelope and wavelet coefficients.

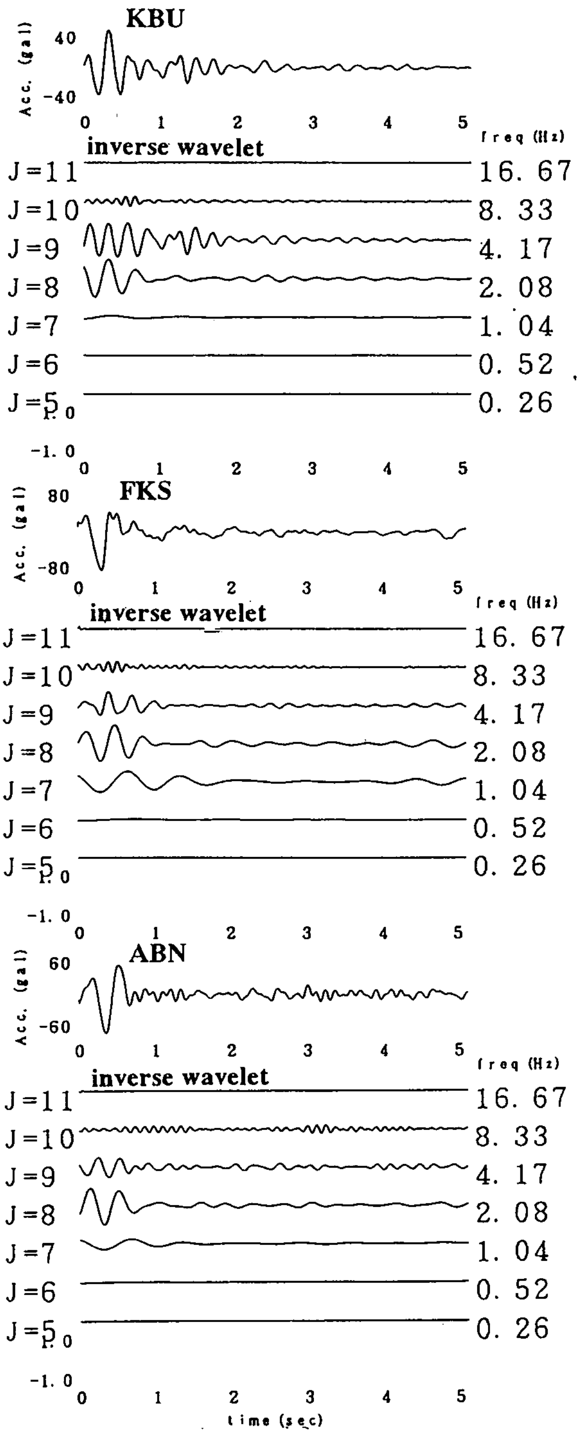

Figure 2.2 Inverse wavelet

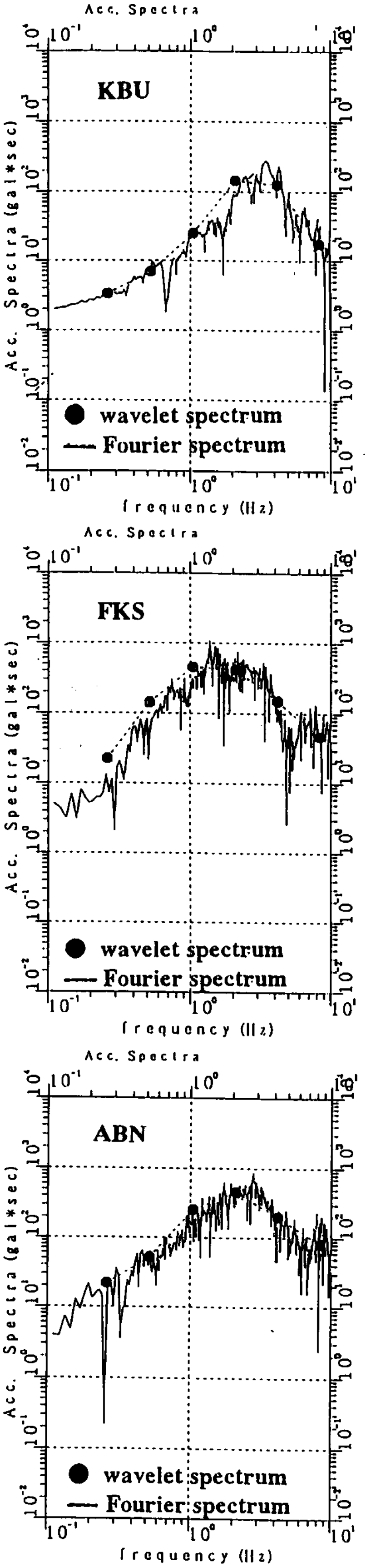

Figure 2.3 Wavelet spectrum
and Fourier spectrum

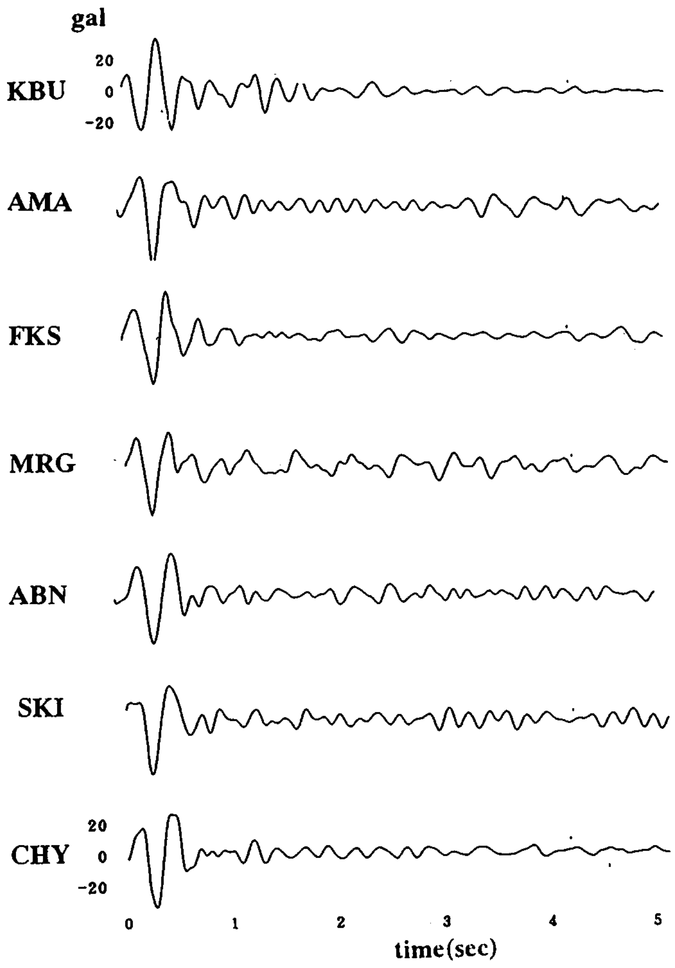

Figure 3. Source time function using the method of factorization and wavelet expansion

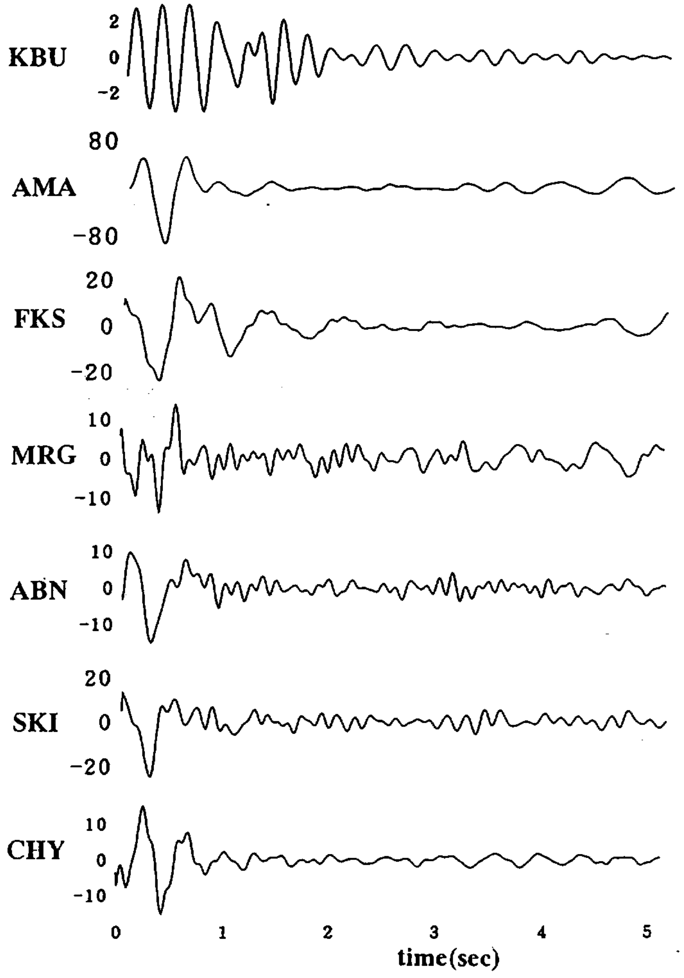

Figure 4. Site time function using the method of factorization and wavelet expansion