SEISMIC VERIFICATION OF UPDATED FINITE ELEMENT MODELS OF CABLE-STAYED BRIDGES

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SUMMARY

The work presented in this paper summarises some of the major concepts and difficulties associated with Finite Element updating. The approach outlines optimal measurment techniques for sensor location and the develops the theory of sensitivity based model updating. This leads to a comparison of sensitivity and ill-conditioning and how they influence the updating process. Examples of the application of some of the techniques to Cable Stayed Bridges are included. Also practical advice on redundancy and choice of measurement data is provided.

INTRODUCTION

The commercial and aesthetic attraction of Cable Stayed Bridges (CSBs) for long span applications is compelling. However, as with any large scale construction in a potentially hostile environnment the dynamic behaviour becomes a critical design constraint. For increased span bridges the aeroelastic behaviour must be fully understood. This implies not only accurate modelling of the mass, stiffness and damping distribution, but also the aerodynamic and seismic loads, giving rise to forced response. Clearly any tools which are able to predict the dynamic response would be of benefit to designers. The approach adopted here is to assess model updating strategies developed for the aerospace industry for the application of CSBs.

The work described below outlines the strategies normally adopted for rfinite element updating and how CSBs present particular challenges. The challenge for any model updating problem is to modify exiting finite element descriptions using measured response or modal data. Here we will only be considering the mass and stiffness distributions. Damping, both its characterisation and quantification remains a problematic area for all structures and is beyond the scope of this paper.

The goal of this paper is to present the challenges associated with model updating, both from a practical perspective and the theoretical basis.

OPTIMAL DATA REQUIREMENTS

In an ideal world we would have an infinite number of sensors and exciters an be able to measure all of the response properties. Unfortunately large civil engineering structures require measurements to be taken in-situ. Only a limited number of transducers can be used, and moving the sensors around the structure is usually inconvenient. Therefore, the problem of positioning the sensors in the best possible locations on the structure becomes important.

The problem of optimally locating sensors on a structure arises from the following considerations:

1. use a small number of sensors in order to minimise the cost of instrumentation and data processing;
2. obtain good estimates of the modal parameters from noisy data;

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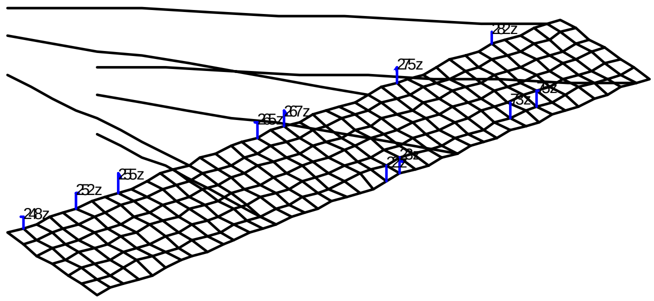
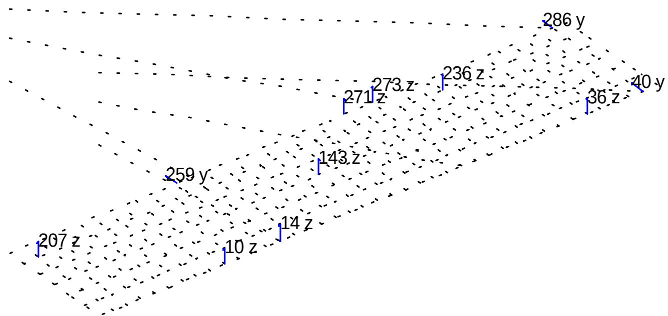
3. improve structural control by using valid models;
4. determine efficiently the structural properties and their changes for health monitoring of the structure; and
5. ensure the visibility of modeling errors.

The problem can be addressed as follows: “given a limited set of sensors, where should they be located on a structure so that the data collected from those locations yields the best information about a structure's dynamics?”. In addition it is necessary not to duplicate or acquire redundant information. Several researchers have addressed this issue. . [Kammer 1991, 1992] presents the Effective Independence Method which ranks the candidate sensor locations according to their contribution to the linear independence of the target modal partitions. [Hemez et al. 1994] present another method called the Energy Matrix Rank Optimisation which positions sensors according to the strain energy distribution of the structure. A third method, called the Energy Optimisation Technique is presented by [Heo et al. 1997]. This method positions sensors according to the kinetic energy distribution of the structure. [Park et al. 1996] present a sensor placement technique which is based on an iterative scheme similar to the Effective Independence Method, while, [Larson et al. 1994] use the Effective Independence technique, the kinetic and average kinetic energy and the eigenvector product techniques to place sensors on a 8-bay truss which will identify the first five modes of vibration. [Baruh et al. 1990] present optimisation procedures in order to determine the most desirable locations of the sensors. The problem of optimally positioning sensors in lumped and distributed parameter systems is presented by [Shah et al. 1978], who position sensors for the system identification from time-domain input-output data. Finally, [Worden et al. 1995] deal with fault detection using neural networks and optimal sensor location using genetic algorithms.

Perhaps the most widely used is the Effective Independence Method which calculates an orthogonal projection based on the Fisher Information Index. Below is a comparison between Kammer’s method and Heo’s Energy Optimisation Technique.



(a). Effective Independence method (11 sensors)



(b). Energy Optimisation Technique (11 sensors)

Figure 1 Sensor locations

Although both methods indicate optimal measurement locations around the perimeter of the deck, their distribution is far from consistent. It should be borne in mind that the objective of modal testing is to extract the independent modes of vibration within the measured frequency range. In order to compare the results, a Modal

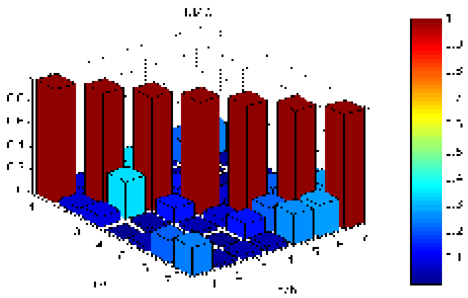
Analysis tool, called the Modal Assurance Criterion (MAC) will be used. This criterion is an effective way of comparing two sets of dynamic data and measuring their correlation. The MAC uses the orthogonality properties of the modeshapes to compare either two modes from the same test or from different tests. One can compare analytical with experimental or analytical with analytical or experimental with experimental data. Suppose that one has two sets of modeshapes:

$[\phi^a]$ --- of order $n \times m_a$ and $[\phi^e]$ --- of order $n \times m_e$

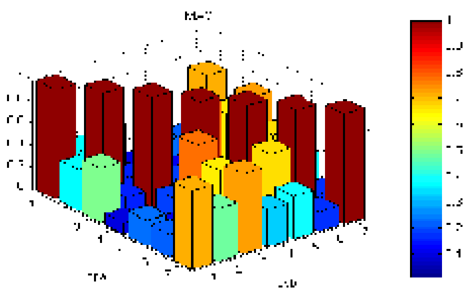
where, m_a and m_e are the number of modes in each set and n is the number of coordinates included. The MAC is given by:

$$MAC\left(\{\phi^a\}_i, \{\phi^e\}_j\right) = \frac{\left|\{\phi^a\}_i^T \{\phi^e\}_j^*\right|^2}{\{\phi^a\}_i^T \{\phi^a\}_i^* \{\phi^e\}_j^T \{\phi^e\}_j^*} \tag{1}$$

The results shown in figure 2 the MAC plot for the modal data applied to the measured data using only the eleven sensor locations indicated by the Effective Independence Method and the Energy Optimisation Technique.



(a). Effective Independence Method (11 sensors)



(b). Energy Optimisation Technique (11 sensors)

Figure 2 – MAC Plots for Sensor Locations

If the correlated modeshapes are identical, the leading diagonal of the MAC matrix will indicate this by having values close to one. If two modes are orthogonal a MAC value of zero is calculated. Figures 2a,b show the MAC plots for the two sensor arrangements. If one compares the two plots in Figure 2 it is clear that the Effective Independence Method gives better results than the Energy Optimisation technique, since the off-diagonal values of the MAC in Figure 2a are much smaller than the corresponding ones in Figure 2b.
Since the aim of the sensor placement work is to find the least number of sensors that will render the modal partitions independent, one can conclude that in this particular exercise the Effective Independence Method gives

the best sensor arrangement. The conclusion of this section of the paper is that sensor location on the basis of convenience can significantly reduce the inherent information in the measured data set. If time allows, the sensor location should be based on application of the Effective Independence Method using modes from an FE model.

FINITE ELEMENT UPDATING

We have established that the testing, acquisition and analysis of structures for the purposes of modal analysis is neither trivial or straightforward. However, adopting rigorous testing procedures and attention to detail we can confidently assume that our measured data - whether in the time, modal or frequency domain - provide a truer representation of the dynamics of the structure. It is on this basis that we are able to use these data for engineering applications such as product development and life-cycle costing.

An application of modal analysis which has attracted particular interest is Finite Element (FE) model updating. To date, several hundred papers have been published in the field originating from both research and industry. The enthusiasm for this topic is easily understood: analytical models require verification if they are to be used in anger.

Given the abundance of reconciliation methods, it is impossible to describe them all in detail. It should be pointed out that [Friswell and Mottershead, 1995] provides a rigorous and comprehensive guide to the field, which is essential reading for anyone who's main research is in this area. The aim of the overview presented here is to lay special emphasis on the methods that either have stood the test of time, or are of current interest and relevance. Before describing these methods in detail, it is appropriate to define the categories into which they fall:

(i) *'Comparison'* methods - these are used for a preliminary assessment of the compatibility of the FE and experimental models. They are limited to giving an indication of which modes correspond to each other and do not attempt to explain why, in a spatial sense, they might differ;

(ii) *'Location'* methods - as the name suggests, these aim to provide information as to where differences exist between the two models without describing whether they are caused by mass or stiffness irregularities; and finally,

(iii) *'Correlation'* methods - are the most sophisticated of the three approaches in that they attempt to apply localised perturbations to the mass and/or stiffness properties or to the elemental parameters of the FE model, the goal being to achieve a modal and spatial model which accurately represents the physical characteristics of the real structure.

The logical approach which is normally adopted is to apply comparison, location and correlation methods sequentially. This is based on the rationale that in order to use location techniques a prior assessment has to be made as to whether they are 'comparable' systems on consideration of their modal properties. Similarly, several correlation methods rely on accurate location of the regions of discrepancy between the FE and experimental models; others, however, incorporate their own idealised locations in order to achieve accurate model updating.

Modes or FRFs for Correlation?

Unfortunately it is impossible to give hard and fast rules as to whether modal or frequency domain data is more appropriate for correlation. Some of the issues which need to be considered when adopting a particular approach are:

(i) modal extraction is subject to errors and omission, a problem exacerbated by increased modal density at high frequencies;

(ii) although FRFs give more equations, the only new information occurs due to the residual terms outside the measurement bandwidth;

(iii) modal parameters need weighting between the natural frequencies and the modeshapes. Natural frequencies are more reliable and sensitive to parametric changes. However, excessive weighting towards natural frequencies will cause modeshape information to be lost thereby encouraging the updating problem to become ill-conditioned;

(iv) for FRFs the weighting is implicit and depends on the proximity of the chosen frequency points to resonance. Weighting is therefore less controllable; and

(v) FRFs are more appropriate for the updating of damping due to the variation of phase over the measured bandwidth [Lammens, Heylen and Sas, 1994]. However, this asset can be a problem for updating undamped models.

The arguments for the use of FRF data become stronger as updating applications become more ambitious. By increasing the number of updating parameters including damping, means that over determination of the problem to combat ill-conditioning and noise effects will dictate an increased data requirement, currently only available in the frequency domain. This inevitably will require measurements over a higher frequency bandwidth, making modal extraction problematic in regions of high modal density.

Theoretical development of updating in the Frequency Domain

Suppose that a structure is modelled by FE analysis and that by applying some unknown parametric changes an updated FE model can be derived that mimics the measured response of the structure. Then applying a single input force to both the updated and original FE models gives

$$[Z_U(\omega)]\{\alpha_X(\omega)\}=\{f\}=[Z_A(\omega)]\{\alpha_A(\omega)\} \tag{2}$$

Equation (2) can be rearranged in the form

$$[\Delta Z(\omega)]\{\alpha_X(\omega)\}=\{I\}_j-[Z_A(\omega)]\{\alpha_X(\omega)\} \tag{3}$$

where $[\Delta Z(\omega)]$ is the error in the dynamic stiffness matrix, $\{I\}_j$ denotes the j^{th} column of the identity matrix and *j* indicates the location of the input force. Whilst the relatively linear variation of the dynamic stiffness matrix with respect to the updating parameters makes for a stable updating problem [Larsson and Sas, 1992], the right hand side of equation (3) - often referred to as the *input error*- is inherently ill-conditioned [Lammens Heylen and Sas, 1993].

An established remedy is to pre-multiply equation (3) by the analytical FRF matrix:

$$[\alpha_A(\omega)][\Delta Z]\{\alpha_X(\omega)\}=\{\alpha_A(\omega)\}_j-\{\alpha_X(\omega)\}=\{\Delta\alpha(\omega)\} \tag{4}$$

The right hand side of equation (4) is the disparity between analytical and experimental FRFs, or output error. Equation (4) is exact irrespective of the size or nature of the errors. To proceed towards a solution for the system error matrices we must assume a form for the errors. This is achieved by selecting a set of N_p design parameters to vary, $\{P\}$, to account for the discrepancy in response between experiment and analysis. It is convenient to non-dimensionalise the updating parameters as follows

$$p_i=\left(P_i-P_i^o\right)/P_i^o \tag{5}$$

where $\{P^o\}$ are the parameter values for the original FE model. The non-dimensionalised updating parameters - or p-values - represent the fractional changes in the design variables. The dynamic stiffness matrix for the updated FE model, $[Z_U]$, is a function of $\{p\}$ and can be expressed as a Taylor expansion about the dynamic stiffness matrix for the original FE model, $[Z_A]$, as follows:

$$[Z_U] = [Z_A] + [\Delta Z] = [Z_A] + \sum_{i=1}^{N_p} \frac{\partial [Z]}{\partial p_i} p_i + O\left(p_i^2\right) \tag{6}$$

Retaining only first order terms,

$$[\Delta Z] = \sum_{i=1}^{N_p} \frac{\partial [Z]}{\partial p_i} p_i \tag{7}$$

Substituting for $[\Delta Z]$ in (4)and rearranging gives

$$[S(\omega)]\{p\} = \{\Delta\alpha(\omega)\} \tag{8}$$

where

$$[S(\omega)] = [\alpha_A(\omega)] \left[\frac{\partial [Z]}{\partial p_1} \{\alpha_X(\omega)\} \;\vdots\; \cdots \;\vdots\; \frac{\partial [Z]}{\partial p_{N_p}} \{\alpha_X(\omega)\} \right] \tag{9}$$

Each row of the sensitivity matrix, $[S]$, defines the sensitivities of the response at a particular DoF to the set of p-values. Note that (8) is a set of N linear equations for N_p unknowns at a single excitation frequency, ω. Given N_f measured frequency points, N_f sets of equations can be combined to form an over-determined problem with $N_f \times N$ equations for N_p unknowns. Such a set of equations can be solved simultaneously in a least squares sense by application of Singular Value Decomposition [Maia, 1991]. Note also that the rows of equation (8) can be partitioned in the event of coordinate incompleteness so as to consider the response disparity at only master DoFs. However, the sensitivity matrix on the left hand side still requires measured data at every FE DoF if model reduction is to be avoided.

Sensitivity and Ill-Conditioning?

A set of linear equations of the form

$$[A]\{x\} = \{b\} \qquad (10) \tag{10}$$

is determined if $[A]$ has full rank, i.e. there are as many linearly independent rows as there are columns. However, the rank of $[A]$ is unclear if the independent rows are *nearly* linear combinations of each other. In this case, the solution is determined by almost parallel lines in multi-dimensional space and is therefore highly sensitive to noise on $\{b\}$. Therefore, the correct question to ask is not whether $[A]$ has full rank, but whether it is well conditioned.

All least squares methods yield a set of linear equations of the form of equation (10), and since $\{b\}$ is based on measured response data which are subject to noise contamination, the conditioning of $[A]$ is of crucial importance.

To demonstrate the conditioning of the updating problem it is instructive to consider briefly the simple simulated structure which is depicted in figure 3 Here, just two updating parameters are considered and the true solution is known to be $\{p_1, p_2\} = \{1,1\}$. In this - somewhat contrived - case, each DoF from each frequency point contributes a scalar equation of the form

$$ap_1 + bp_2 = c \tag{11}$$

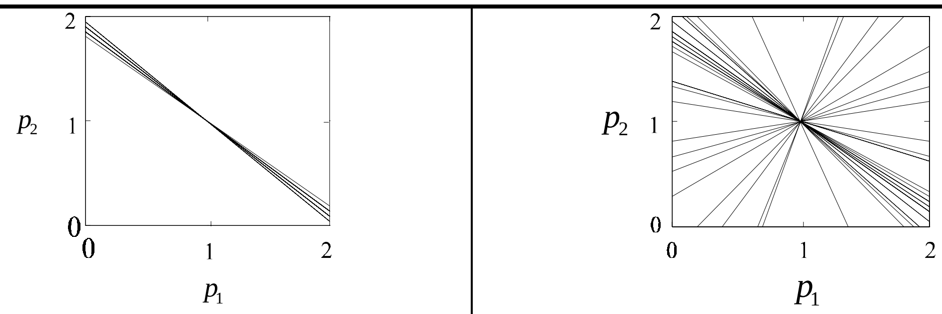
which can be plotted in $p_1 - p_2$ space. Figure 3 illustrates the updating equations (10) - the output error in the frequency domain - when two different frequency points are chosen, one close to resonance and the other away from resonance. Note that:

(i) all equations pass through the known solution $\{1,1\}$ because in this case the simulated *experimental* FRFs are noise-free. Any pair of linearly independent equations is sufficient to solve the problem;

(ii) the equations due to the frequency point near resonance are nearly parallel and in the interest of good conditioning cannot be relied upon to provide more than one independent relation between p_1 and p_2; and

(iii) the equations due to the frequency point away from resonance are more diverse.

Duplicity of equations is in general desirable to average out noise effects. However, it is essential that there are sufficient (in this case two) equations that are not nearly dependent. It is therefore of concern that frequencies near resonance can essentially contribute a single relation. The response sensitivities at frequency points close to and away from resonance will be examined in turn.



(a) - Near Resonance

(b) - Away from Resonance

Figure 3 - Updating Equations for Two p-values from Simulated Noise-Free FRFs

The sensitivity matrix from a single frequency close to analytical and experimental resonance results in a set of almost identical equations that are highly ill-conditioned. This implies that if only frequencies close to resonance are used and experimental and analytical natural frequencies are not too disparate, then the frequency range must cover as many modes as there are updating parameters to avoid ill-conditioning. Although selecting resonant frequencies alone is an unnatural choice for FRF updating, there is a real issue concerning the recommended balance between resonance and off-resonance frequency points. Should only off-resonance frequencies be used? To answer this question, the nature of the updating equations from off-resonance frequency points must be examined.

At off-resonance frequencies, the response is composed of a summation of all the modes of the structure, the adjacent modes usually being more dominant than others. The contribution from each mode is dependent on the excitation and measurement locations and so different DoFs react differently to the same change in updating parameters. For this reason, the equations at each off-resonance frequency point are diverse and therefore contain more information about the inter-dependencies of the unknown variables.

From figure 3(b) it is apparent that in this case most equations have a negative gradient indicating that changes in p_1 and p_2 have similar effects on the response. For a well conditioned problem, measurements are needed that can distinguish between changes in p_1 and changes in p_2. The equations with positive gradient are therefore of special interest since they represent responses for which p_1 and p_2 have opposing effects.

Now consider the response sensitivities obtained from the same off-resonance frequency point as a vector. In this way, the components of each vector represent the receptance sensitivities of a single DoF due to unit changes in p_1 and p_2 and the length of the vector indicates the magnitude of the sensitivity. It is usually observed that those DoFs where p_1 and p_2 have an opposite effect on the response are actually insensitive to either! This is a consistent observation over the whole frequency range. Inclusion of frequency points away from resonance potentially improves the conditioning of the updating problem due to the diversity in sensitivity ratios from DoF to DoF. However, those DoFs that are most able to dispel ill-conditioning are relatively insensitive to the updating parameters.

CONCLUDING REMARKS

The application of model updating strategies to large scale civil engineering structures has compelling attractions. In particular the prediction of aeroelastic and seismic response of Cable Stayed Bridges would be highly beneficial. This can only be achieved if the structural model is accurate. Model Updating strategies offer the prospect of being able to achieve this. However, it should be borne in mind that for the model updating techniques to work effectively accurate measured response or modal characteristics must be available along with a well defined finite element model. Given these provisos the application of the methods suggested in the paper offer the prospect of model updating as a viable inclusion in the design process.

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