PERFORMANCE OF STATICALLY INDETERMINATE STRESS IN THE ULTIMATE STATE

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SUMMARY

Statically indeterminate stress of frame member in the service State was calculated commonly by elastic stiffness analysis. Similarly, member stress under seismic loading was calculated by elastic or linear stiffness analyses in a normal rise building. Ultimate strength design of the members was carried out due to sum of these stresses using load factors for the ultimate state. Also, statically indeterminate stress of each member accompanied by dry-shrinkage, creep, axial deformation and deflection due to prestressing of tendons were calculated based on elastic stiffness. However, such a design to sum the stresses is very severe in high-rise building to consider economical structural design. Thereupon, five story prestressed concrete frame building was planed to study performance of the statically indeterminate stress in the ultimate state by nonlinear analysis. The member stress of the frame under service loading was first calculated using elastic stiffness. To investigate member stress and ductility factor of each frame member at seismic loading stage was carried out using two cases of incremental load analyses under statically indeterminate stress and unloading. From the results of two cases of analyses, it's clarified that statically indeterminate stress in the ends of a member by service loads is not remained favorably in the ultimate state. Performance of the statically indeterminate stress in each stage of seismic loading is discussed.

INTRODUCTION

This study was carried out by Working Group (chairman: H.Watanabe) for evaluation on structural characteristics and capacity of the member in Joint Coordinating Committee (chairman: S.Okamoto) on High-rise Prestressed Concrete Building organized by Building Research Institute, Ministry of Construction. Statically indeterminate stress of frame member in the service state was calculated commonly by elastic analysis. Similarly, member stress under seismic loading was calculated by elastic or linear stiffness analyses in a normal rise building. Then, ultimate strength design of the prestressed concrete members was carried out due to sum of these stresses using load factors for the ultimate state. Also, statically indeterminate stress of each member accompanied by dry-shrinkage, creep, axial deformation and deflection due to prestressing of tendons were calculated based on elastic stiffness. However, such a design to sum the stresses is very severe in high-rise

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building to consider economical structural design. A cause to arise the statically indeterminate stress is riginated to restrain the rotational angle, displacement, and axial deformation at the free ends of the member. There, a value of the statically indeterminate stress is varied accompanied by fixing condition at the end of a member. Excessive flexural moment corresponding to the ultimate flexural strength may be undergone at the both ends of a member in the ultimate state by seismic lateral load, the rotational stiffness at the end of member was reduced significantly. Also, the statically indeterminate stress at the end of a member undergone at the initial elastic stage was reduced accompanied by reducing level of rotational stiffness at the end of a member. The reduce of rotational stiffness from flexural cracking to yielding in the member undergone the anti-symmetric moment was represented generally as a relation factor α_y at yielding. A common relation factor α_y at yielding in the prestressed concrete member is distributed from 0.25 to 0.35. The relation factor α_y at yielding is shown as Fig.1.

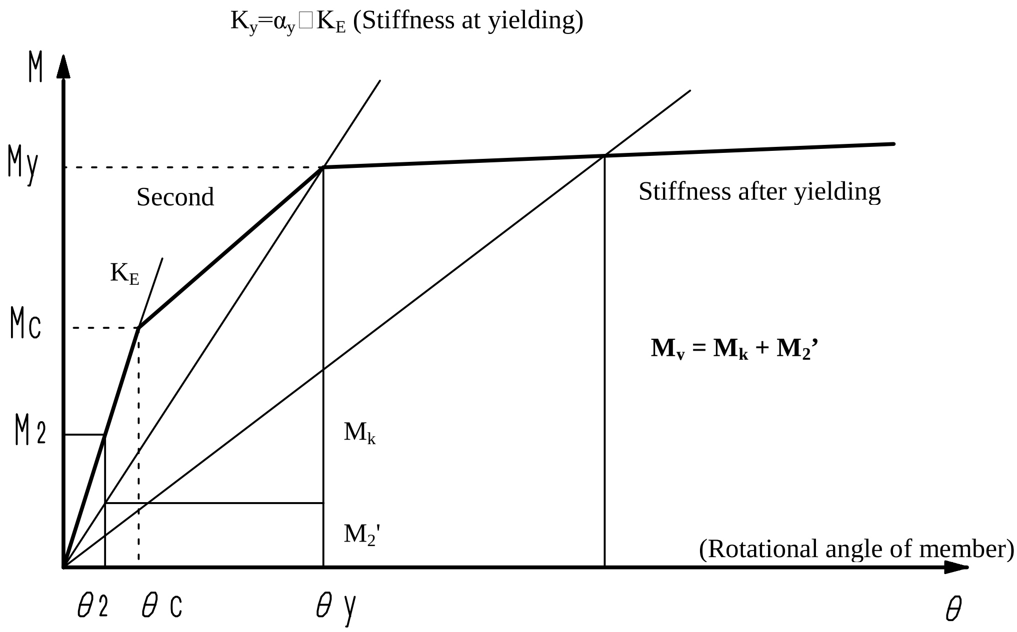

Fig.1 Relation Factor α_y at Yielding

where,
K_E: Initial elastic stiffness,
M_y: Flexural yield moment,
M_c: Flexural cracking moment,
M_k: Flexural moment by seismic lateral loading,
θ_2: Rotational angle restrained by fixing condition,
M_2': α_y□M_2 (equivalent statically indeterminate moment at yielding)

The relation factor α_y at yielding is used commonly as varying model of the rotational member stiffness in the nonlinear frame analysis. Therefore, from the nonlinear rotational stiffness model of member, equivalent statically indeterminate stress could be estimated favorably. Beside, the rotational angle restrained at the initial elastic stage is not varied favorably. The performance of statically indeterminate stress accompanied by varying of the rotational member stiffness is below;

1) In the initial elastic stage, all member stresses was summed up involving stress by seismic lateral loading completely. The statically indeterminate stress is not varied.
2) In the stage after flexural cracking to flexural yielding, The initial statically indeterminate stress is reduced to be the ratio of equivalent rotational stiffness to initial rotational stiffness. Also, the effective statically indeterminate stress M_{2e} may be assumed to be α_y□M_2 (about 0.25 to $0.35 \times M_2$) favorably at yielding.
3) The statically indeterminate stress was reduced significantly in the ultimate state after yielding, and the effective stress M_{2e} is assumed as $(\mu/\alpha_y)\ M_2$ favorably.
where, μ: member ductility factor (θ/θ_y), θ: rotational angle of member, and θ_y: rotational angle at yielding.

From the above estimation, the performance of statically indeterminate stress in each state may be assumed using the nonlinear characteristics of member favorably. Therefore, in the cases that the elastic statically indeterminate stress was equal of less than about 0.3 times the yield flexural strength at the end of the normal member, the effective stress was assumed to be about 0.1 times the yield flexural strength. Such a effective statically indeterminate stress could be taken no account favorably at the ultimate state. Thereupon, five story prestressed concrete frame building was planed to study performance of the statically indeterminate stress in the ultimate state by nonlinear analysis. The member stress of the frame under service loading was first calculated using elastic stiffness. To investigate member stress and ductility factor of each frame member at seismic loading stage was carried out using two cases of incremental load analyses under service loading and unloading. From the results of two analyses, it's clarified that statically indeterminate stress in the ends of member by service loads is not remained favorably in the ultimate state. performance of statically indeterminate stress in each stage of seismic loading are discussed

ANALYTICAL MODEL FRAME

A simple 5-story cast-in-situ partially prestressed concrete one span frame is designed to examine the performance of statically indeterminate stress in the ultimate state. A span is 15m. Total frame height is 21m with story height of 5m in first story and uniform height of 4m in second to fifth stories. The structural frame is shown in Fig.2, and the column and beam sections in Fig.3.

Materials Used

Compressive strength of cast-in-situ normal concrete is 30 N/mm2 for columns and girders. Yield strength of longitudinal reinforcement is 345 N/mm^2 in columns and beams, and PC tendons in beams is 1,580 N/mm^2. High strength deformed PC bar of yield strength equal to 1,280 N/mm^2 are used as lateral reinforcement in columns and beams.

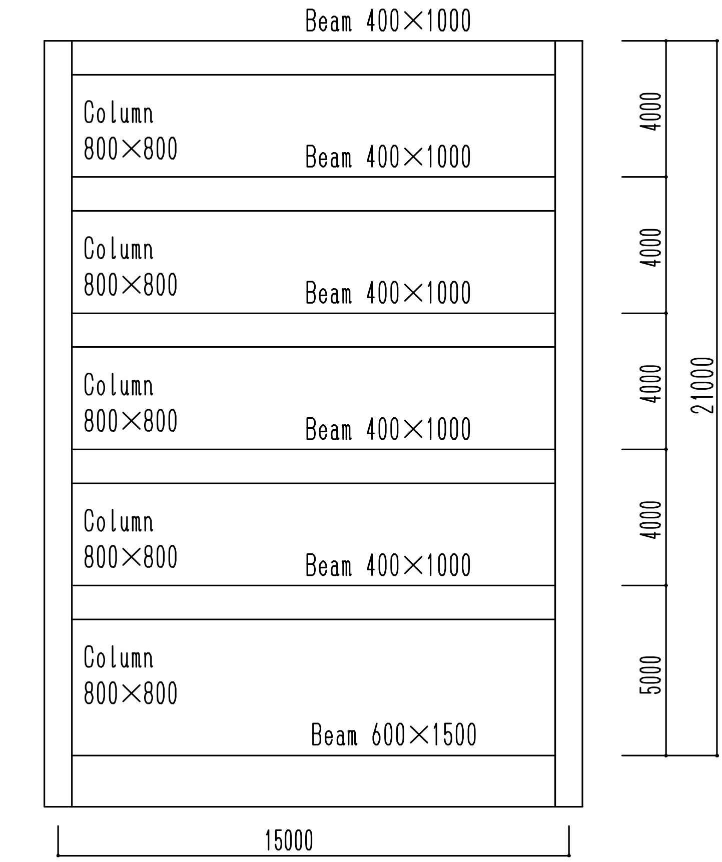

Fig.2 Structural Frame

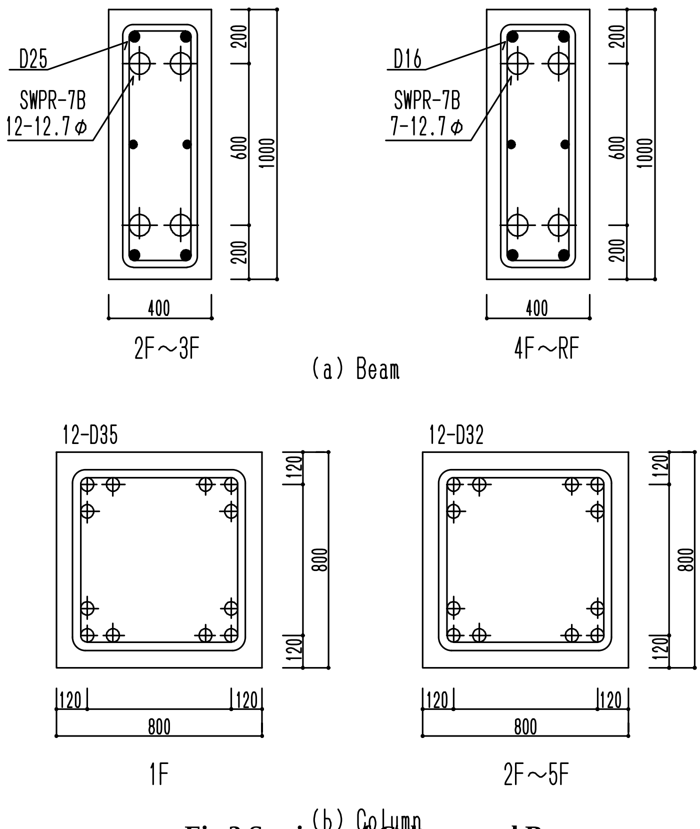

Fig.3 Sections of Column and Beam

Design Concept for Model Frame

Design stresses of vertical load and secondary stresses accompanied by prestressing was calculated by the elastic analysis due to the existing design practices. Structural design for the seismic lateral loading was carried out by incremental load analysis accordance with "Ultimate Strength Design Guidelines for Reinforced Concrete Buildings" (Japanese PRESSS Guidelines) developed as a part of U.S.-Japanese Coordinated PRESSS (Precast Seismic Structural Systems) project. The design concept for incremental load analysis are below;

1) Lateral load distribution factor was used as the inversely triangular distribution. Also, ten percentage of the base shear coefficient C_B was undergone at the top story.

2) Base shear coefficient C_B was used as 0.3 due to the Japanese Standard Building Low Enforcement Order.

3) Statically indeterminate stresses by vertical loads and prestressing was undergone at the end of member before incremental load analysis.

5) Structural yielding mechanism in the ultimate state was planned as a weak beam-strong column concept.

Seismic design criteria for model frame are shown in Table 1.

Table 1 Seismic Criteria for Model Frame

Base Shear Coefficient (C_B: ZR_tC_0)	Maximum Rotational Angle (Radian)	Ductility Factor of PC Beams	Ductility Factor of PC Columns
0.20	less than 1/200	no yield	no yield
0.30	less than 1/100	less than 1.0	no yield
more than 0.30	less than 1/50	less than 3.0	less than 2.0

Statically Indeterminate Stresses Calculated

The statically indeterminate stress by vertical loading in the ends of beam was not remained almost by deducting by anti stress accompanied by prestressing. However, that in the top and bottom of the column was remained fairly.

STUDY BY INCREMENTAL LOAD ANALYSIS

Two cases of incremental load analysis was carried out to investigate the performance of the statically indeterminate stress at the ends of member in the ultimate state. Analytical conditions of the two cases was loading and unloading the statically indeterminate stress before incremental load analysis.

Analytical Model

Each member in frame is represented by a lineal element at the centroid of the section. Nonlinear rotational springs are inserted at the end of a member to represent inelastic deformation within the member. The stiffness of a member was varied at two loading levels corresponding to flexural cracking of concrete and yielding of tensile reinforcements and PC tendons. The stiffness after yielding was 1/1000 times the initial elastic stiffness. The effect of axial load was considered in evaluating the stiffness characteristics of a column. The equations of initial stiffness K_E of a member, relation factor α_y at yielding, flexural cracking moment M_c and ultimate moment M_y used in the analysis are as follows;

$$K_E = L / \left\{ L^2 / (3 E_c I_c) + k / (G_c A_c) \right\} \tag{1}$$

$$\alpha_y = (0.043 + 1.67 n\, p_t + 0.043 a / D + 0.33 \eta)(d / D)^2 \tag{2}$$

$$M_c = \left(1.8\sqrt{F_c} + P_e / A_c\right) Z_e + N\, D / 6 \tag{3}$$

$$M_y = 0.9 \sigma_y a_t d + \left(f_{py} a_p\right)(1 - 0.5 q) d \tag{4}$$

where, L: length of face to the inflection point of a member, E_c and G_c: elastic and shear moduli of concrete, I_c and G_c: moment inertia and modulus of a member considering the effect of reinforcement, A_c: area of concrete section, k: shape factor for shear (1.2 for columns and 1.0 for beams), n: modular ratio of steel to concrete, p_t: tensile reinforcement ratio including the effect of PC tendon, a: shear span, d and D: effective and overall

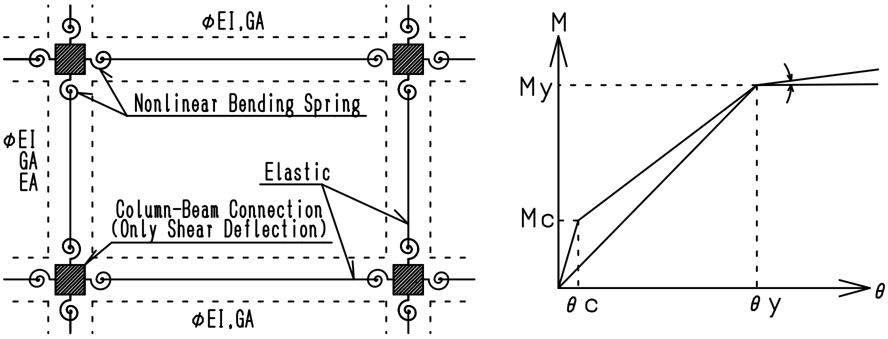

Fig.4 Analytical Model and Nonlinear Characteristics of member

member depth, $\eta=(N+P_e)/(b \cdot D \cdot F_c)$, b: member width, at and a_p: area of tensile reinforcement and PC tendon, F_c: compressive strength of concrete, σ_y: 1.1 times the nominal yield strength of reinforcement, f_{py}: yield strength of PC tendon, and $q=(f_{py} \cdot a_p)/(b \cdot D \cdot F_c)$. Analytical frame model and nonlinear characteristics of member are shown in Fig.4.

Relation of analytical Model and Undergoing the Stress

In the analytical model by inserting the nonlinear rotational springs at the ends of a member, undergoing of the statically indeterminate stress was carried out to vary the initial starting point in the initial elastic stiffness line before incremental load analysis. Therefore, the undergoing statically indeterminate stress at the ends of a member should be less than the flexural cracking of the ends of a member.

ANALYTICAL RESULTS

Two cases of incremental load analysis was carried out as a push over method. A case 1 was to undergoing the statically indeterminate stress at the ends of a member, and another Case 2 was analyzed unloading the statically indeterminate stress before incremental load analysis. The relation of lateral load and story drift angle are shown in Fig.5, and member stress of each case in two loading stages of 0.2 and 0.3 in terms of a base shear coefficient C_B in Fig.6. Member ductility factors at the two maximum story drift angles of 0.01 and 0.02 times the radian are shown in Fig.7.

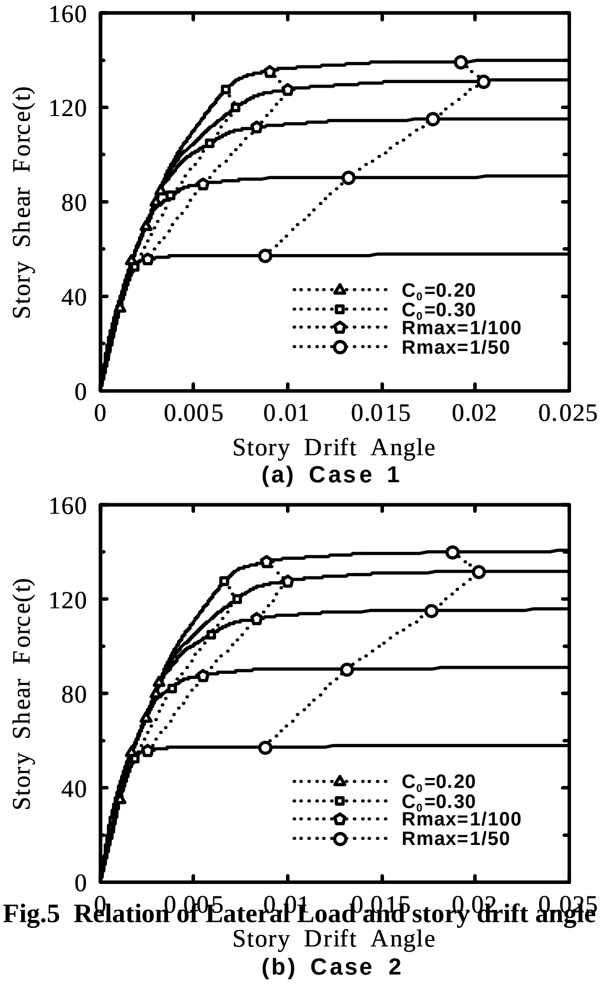

Fig.5 Relation of Lateral Load and story drift angle

Maximum Story Drift Angles

Maximum story drift angles at two loading stages of 0.2 and 0.3 in terms of a base shear coefficient was not varied in the two analytical cases.

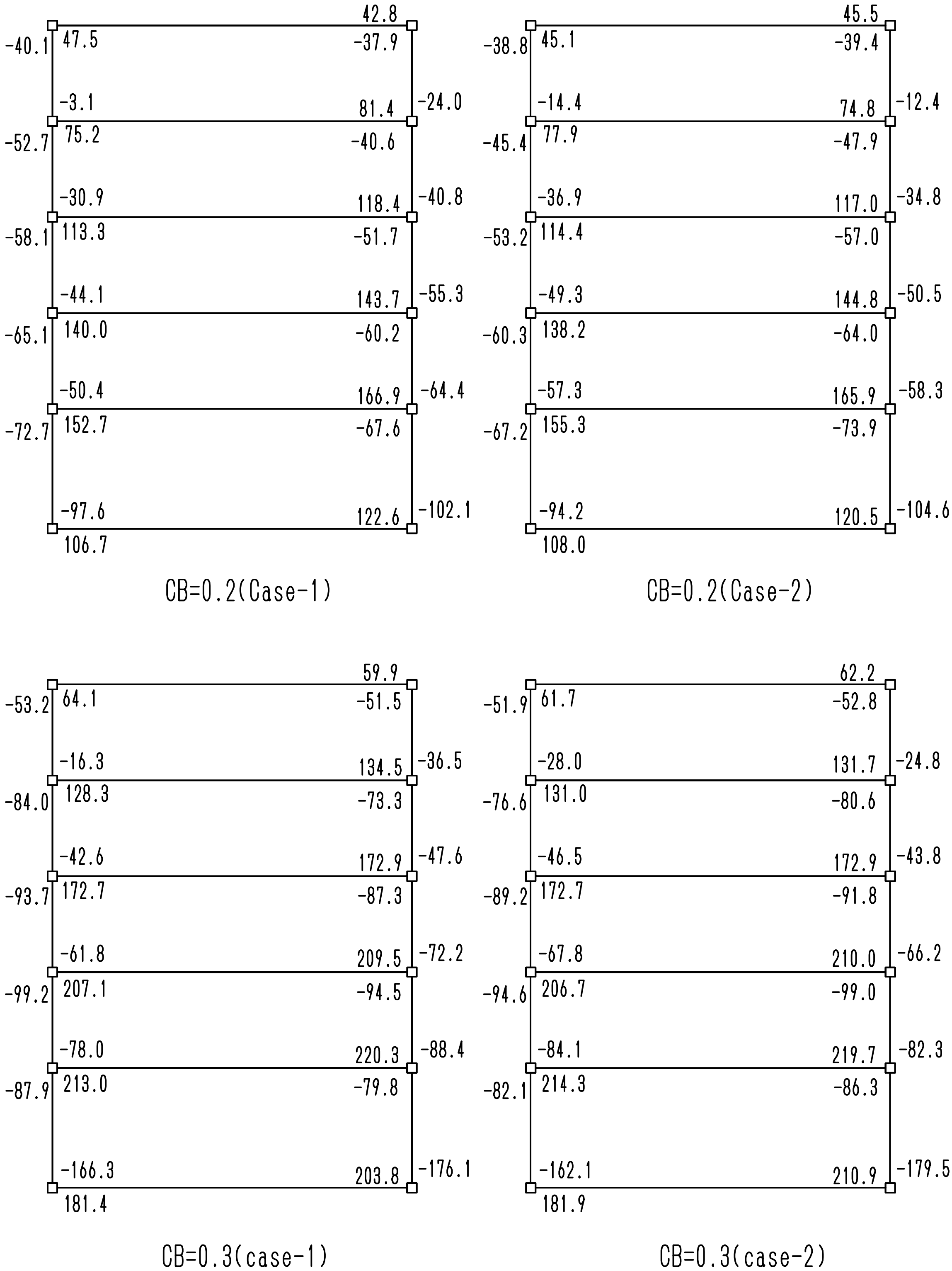

Fig.6 Member stress of each case

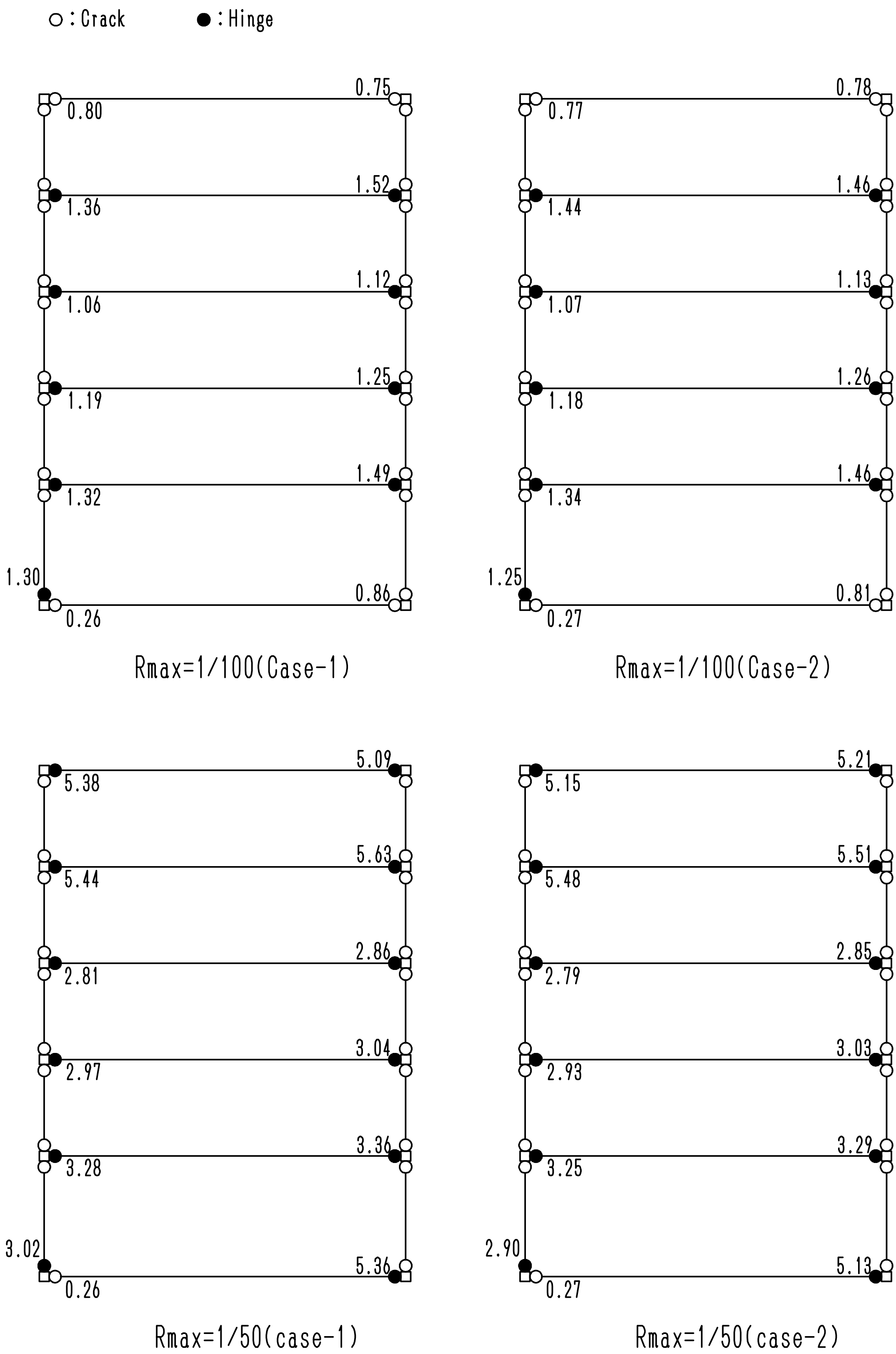

Fig.7 Member Ductility Factors

Member Stresses before Yielding

The statically indeterminate stress of member in the upper story (forth and fifth stories) was summed up to the stress by seismic loading at loading stage of 0.2 in CB favorably. Also, that of member in the lower story (first and second stories) developing of story drift was summed up slightly to the stresses by seismic loading. However, the statically indeterminate stress was reduced significantly in the loading stage of 0.3 in terms of a base shear coefficient.

Member Ductility Factors

The ductility factors of beams at developing to 0.01 and 0.02 times the radian in the story drift angle are not varied favorably in the two cases of analysis, because the statically indeterminate stress at the ends of beams was too slightly. Beside, that of columns are varied slightly, the deviation was about 0.1 times the ductility factor.

CONCLUSIONS

To examine the performance of the statically indeterminate stress in the ultimate stage, a 5-story cast-in-situ partially prestressed concrete model frames to be one structural frame was planed and designed accordance with Japanese PRESSS Guidelines. And, two cases of incremental load analysis was carried out under loading and unloading the statically indeterminate stress before incremental load analysis. From the comparison of the results in the two cases of incremental load analysis, following conclusions may be drawn:

1) The statically indeterminate stress was summed up involving stress by seismic lateral loading completely in the initial elastic stage, and the statically indeterminate stress is not varied.

2) The statically indeterminate stress of member in the upper story was summed up to the stress by seismic load at loading stage of 0.2 in CB to suppose the moderate earthquake favorably. Also, that of member in the lower story developing of story drift was summed up slightly to the stresses by seismic load.

3) The statically indeterminate stress of member in the upper story was reduced significantly in the loading stage of 0.3 in CB to suppose strong ground motion.

4) The ductility factors of beams at developing equal of more than 0.01 times the radian in the maximum story drift angle are not varied favorably in the two analyses under loading and unloading the statically indeterminate stress at the ends of beams, and that of columns are varied slightly.

5) The statically indeterminate stress corresponding to equal of less than 0.3 times the yielding flexural moment could be taken no account favorably at the ultimate state.

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