

OPTIMAL SEMI ACTIVE AND PASSIVE CONTROL OF THE SEISMIC RESPONSE OF COUPLED FRAME-BRACING SYSTEMS

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SUMMARY

In this paper a particular *ON-OFF* type of Semi-Active control of the seismic response of braced frames is considered, in which the braces are connected to the frame by special elements, which are activated or deactivated at different times, in order to obtain the best response control. An optimal control algorithm, for n *dof* systems, is first analyzed and compared with some other ones, presented in the literature, for similar control problems. Comparisons are also systematically made with optimized Passive control strategies based on the use of dissipative connections. It is shown that Semi Active control performs better than Passive; the improvements are only minor in terms of maximum response values, but appear more relevant when the entire time history is looked at.

INTRODUCTION

The structural control methodologies have been recognized to have a high potential for protecting structures against earthquakes. It has been observed, though, that, particularly in this field, conceptual and operational simplicity, reliability and low energy consumption are essential features of any structural control system to be effectively adopted. For this reason a great attention has been given to Passive and Semi Active control strategies. In Passive Control (*PC*) the control forces develop as the effect of the presence of special devices whose parameters remain unchanged in time and are selected, at the design stage, for reducing the structural motion to certain classes of expected input actions. In Semi Active Control (*SAC*) the mechanical parameters of the control devices are actively modified, following the time evolution of the structural response. The regulation modalities of the parameters of the devices are determined on the basis of predetermined control algorithms, as a function of the excitation and/or of the structural response, which implies use of sensors, processors and actuators, as it is typical of active control; the energy required for the modification of the mechanical parameters of the devices is minimal, though, with respect to other Active or Hybrid control techniques and may be furnished, for example, by a battery. In the literature on *SAC* these systems are often referred to as *Parametric Control* [Yang *et al.* 1994], *Variable Structure Systems* [Inaudi & Hayen 1995], or *Variable Passive Control* [Lee *et al.* 1998]. Systems of this type, recently tested and put in operation, are, for example, the *Active Variable Dampers (AVD)*, in which the devices, whose parameters are actively regulated, are capable of giving supplementary energy dissipation, like friction dampers, viscous fluid dampers or controllable fluid dampers (*Electro-Rheological* or *Magneto-Rheological*), and the *Active Variable Stiffness (AVS)*, in which the structural stiffness is regulated by means of active braces which are connected or disconnected in time. In the present paper a possible application of the *SAC* is studied for controlling the seismic response of coupled frame-brace systems. In this case the control is realized by changing the parameters of different types of devices which connect the frame to the brace. The attention is given to *ON-OFF* type systems, in which the connections have only two possible states *ACTIVE* (the brace is connected) or *NON ACTIVE* (the brace is disconnected). Comparisons among different control algorithms are made. The *SAC* case results are also compared with results which are obtainable by optimal *PC* systems which make use of similar types of dissipative connections, for example elasto-plastic [Ciampi *et al.* 1995] or visco-elastic [Paolacci *et al.* 1998].

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AN ALGORITHM FOR THE OPTIMAL SEMI ACTIVE ON-OFF CONTROL

The equations of motion of an n dof system, with mass matrix \mathbf{M} , damping \mathbf{C} and stiffness \mathbf{K} , subjected to base acceleration, are the following

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{b}^u \mathbf{u}(t) - \mathbf{M} \begin{Bmatrix} 1 \\ \vdots \end{Bmatrix} \ddot{x}_g(t) \quad (1)$$

where $\mathbf{u}(t)$ represents the control force in the m controllable elements ($u_i = 0$ if the element is *OFF*). In the space of the state variables (1) reduces to a system of first order differential equations, in which $\mathbf{z}(t) = [\mathbf{x}(t), \dot{\mathbf{x}}(t)]^T$ is the vector of the state variables (of size $2n$):

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}^u \mathbf{u}(t) + \mathbf{H}\ddot{x}_g(t) . \quad (2)$$

In order to determine the optimal control process different performance indexes might be used. In this case an *instantaneous* optimal control scalar function, [Yang *et al.* 1987], is used, quadratic in the vector $\mathbf{z}(t)$; this is expressed in the following form, where the square matrix \mathbf{Q} , of order $2n$, represents a 'weighting matrix':

$$J(t) = \frac{1}{2} [\mathbf{z}^T(t) \mathbf{Q} \mathbf{z}(t)] . \quad (3)$$

At a given time t , the entire system response, (vector $\mathbf{z}(t)$), is considered known. Since it is not possible to modify the actual value of the performance function $J[\mathbf{z}(t)]$, the control strategy is based on the selection of a control force which guarantees the *future* value of such a function to be as little as possible, (*Optimal Incremental Control*). This is obtained by minimizing the contribution that the control force \mathbf{u} gives to the time derivative of $J(t)$, which may be expressed as follows:

$$\dot{j}^u(t) = [\mathbf{z}^T \mathbf{Q} \mathbf{B}^u] \mathbf{u} = \sum_{i=1}^m \dot{j}_i^u = \sum_{i=1}^m [\mathbf{z}^T \mathbf{Q} \mathbf{B}^u]_i u_i . \quad (4)$$

Eq (4) has a value which is as negative as possible, if negatives are the individual contributions to the sum; in other words the i -th control element shall be active if, and only if, it contributes with a non-positive term to the sum (4). The control algorithm, finally, selects the optimal state of the i -th control element ($i=1, \dots, m$), independently of those assumed by the other $m-1$:

$$[\mathbf{z}^T(t) \mathbf{Q} \mathbf{B}^u]_i u_i(t) \leq 0 \Rightarrow \quad i\text{-th element ACTIVE,} \quad \text{NON ACTIVE otherwise} \quad (5)$$

If the \mathbf{Q} matrix is assumed to have the following form, the performance index has the meaning of *Total Recoverable Energy*:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{K} & \mathbf{O} \\ \mathbf{O} & \mathbf{M} \end{bmatrix} \Rightarrow \quad J(t) = E_r(t) = 1/2 [\mathbf{x}^T(t) \mathbf{K} \mathbf{x}(t) + \dot{\mathbf{x}}^T(t) \mathbf{M} \dot{\mathbf{x}}(t)] \quad (6,7)$$

In such a case eq (5) may be explicitated as:

$$[\dot{\mathbf{x}}^T(t) \mathbf{b}^u]_i u_i(t) \leq 0 \Rightarrow \quad i\text{-th element ACTIVE,} \quad \text{NON ACTIVE otherwise} \quad (8)$$

Since matrix \mathbf{Q} , (6), is positive definite, the performance index (7) is a possible Lyapounov function for the controlled system. The activation conditions (8) guarantee that the control forces give only negative contributions to the time derivative of $J(t)$. As a consequence, if the uncontrolled system is per-se stable, such derivative has to remain at least semi-definite negative, because it was already so in the uncontrolled case. The stability of the algorithm (8) is then demonstrated. In the following the control algorithms derived by the above said procedure shall be defined 'optimal' in relative displacements and velocities, because based on a performance index which is quadratic in these two quantities.

THE STRUCTURAL MODEL

The structural model adopted in this work (Fig. 1) is made of two structural parts, assumed to remain indefinitely elastic, connected through a single, controllable and massless, interaction element. The first structure represents the frame and the other, whose mass is considered negligible, the brace. The problem has only one dynamic degree of freedom and is governed by equations (9):

$$\begin{cases} m\ddot{y}(t) + c\dot{y}(t) + ky(t) = u(t) - m\ddot{y}_G(t) \\ k_2 y_2(t) = -u(t) \end{cases} \quad (9)$$

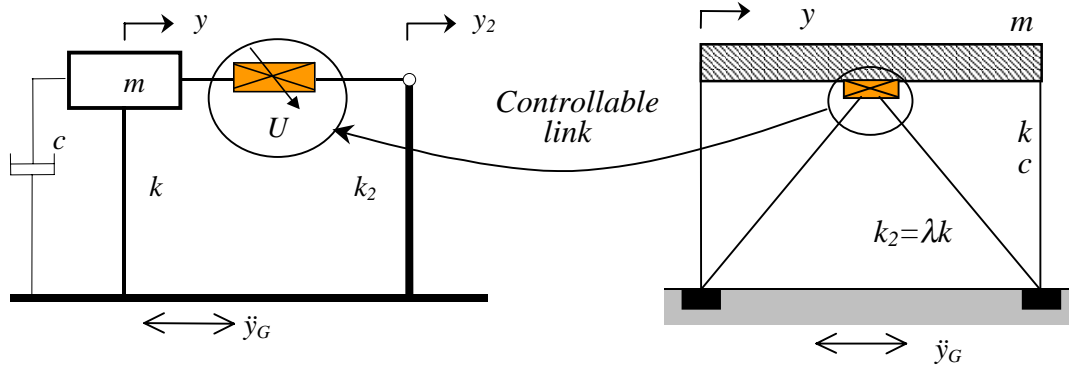


Figure 1. The structural model for the frame-bracing system

The model may be considered as a particular, and simpler, case of the two dof model used for the study of the Semi Active and passive control of adjacent structures [Ciampi *et al.* 1999], where both structures have comparable masses. The first of eq. (9) is the equation of motion of the frame, with mass m , stiffness $k = m \omega_0^2$ and damping coefficient $c = 2 m \omega_0 \xi_0$; the second is the equilibrium equation of the brace, having stiffness $k_2 = \lambda k$. The control force $u(t)$ depends on the connection type; in particular three types of connection elements have been here considered:

1. Perfectly Rigid Element (*RE*), which is able to assure, when active, perfect solidarity between brace and frame
2. Linear Viscous Element (*VE*), with damping constant $c_u = 2 m \omega_0 \xi_u = \xi_u c_{cr}$;
3. Rigid Plastic Element (*RPE*), with yielding force $f_y = \eta_u m \dot{y}_{G,max}$.

It is important to observe that, at the time in which the control element is deactivated, the control force passes from $u(t) = u[z(t)] \neq 0$ to $u(t^+) = 0$. As a consequence of the second of eq. (9), the brace, which is massless, should immediately release all the elastic energy stored up to that moment. In reality this unloading process of the brace occurs in a short, but finite, time, which is related to period and damping properties of the brace.

In this work the response of the structural model has been studied for the following input cases: free vibrations, sinusoidal base motion, earthquake-like base excitation. For this last case the base accelerations are a set of 5 artificially generated accelerograms, of 20 sec. duration, compatible with the EC8 elastic response spectrum, for soil type C, [EC8, 1989]; for the same case all the reported maximum response results have been computed as average values over the above said set, with the exception of the time histories which necessarily relate to a single accelerogram. From the point of view of a multi-objective protection, the maximum values over the time histories of both frame displacement, ($y(t)$), and absolute acceleration, ($\dot{y}_a(t)$), are significant response quantities; the first in fact is related to the stress state in the frame, the second to the total base shear, apart from being a comfort indicator. In particular Y and A are defined, respectively, as such maximum values, normalized with respect to the corresponding quantities for the unbraced frame. Finally the quantity G , average value between Y and A , is introduced as a global comparison index, so that the smaller are the G values the better the performance of the control system in a global sense.

OPTIMAL SELECTION OF THE CONNECTION PARAMETERS

For both cases of Passive and Semi Active *ON-OFF* control, it is necessary to select the parameters of the connection element, which optimize the performance of the control system. This has been done, in this work, by minimizing an Energy Index, which has been proposed and validated with reference to passive control, but which can be computed for every type of oscillatory response of any type of controlled system; in particular it

has been already used with good results for the *SAC* case, [Ciampi *et al.* 1999]). This index, denominated *EDI* (Energy Dissipation Index), represents a measure of the part of the Energy input to the structure which is dissipated by the control system; a precise definition is found in [Paolacci *et al.* 1998]. For the connection elements which are not dissipative, as the Rigid Element (*RE*), it is still possible to define *EDI*, by assuming, as dissipated, the elastic energy of the brace which is instantaneously released when the brace is disconnected. For comparison, other possible criteria for the optimal selection of the parameters of the connection have been also considered, in particular the minimization of the maximum value, over the time history, of the total recoverable Energy of the frame, that is the sum of the elastic and the kinetic Energies, ($E_r = E_s + E_k$).

COMPARISON OF TWO ALGORITHMS FOR THE SAC OF BRACED FRAMES

Two significant control strategies, which are found in the literature, with reference to the same braced frame control problem, are compared with the previously discussed optimal control algorithm and between them:

1. “*AVS*”, Active Variable Stiffness: [Kobori & Kamagata 1994];
2. “*IH*”, “Inaudi & Hayen”, [Inaudi & Hayen 1995], [Hayen & Iwan 1994].

The “*AVS*” algorithm was proposed by Kobori and Kamagata (1994), and implemented in the first semi actively controlled building in the world, constructed by the Kajima Corp. in Tokyo. In this application the braces are connected to the frame when the elastic energy of the frame is increasing, disconnected otherwise, so that the corresponding algorithm may be written as:

$$\dot{y} \geq 0 \Rightarrow \quad \textit{Element ACTIVE}, \quad \textit{NON ACTIVE otherwise} \quad (10)$$

It can be observed that this algorithm does not belong to the optimal control strategies previously introduced, because there is no direct minimization of the contribution of the control force to a performance function.

The “*IH*” control strategy was first proposed with reference to a two dof model in which two structures (a *main structure* and a *secondary structure*) are connected through a controllable device [Hayen & Iwan 1994]; the performance index, $J(t)$, to be minimized, is the relative recoverable energy of the *main structure*. The following algorithm, which may be considered a special case of (8), is obtained:

$$u \dot{y} \leq 0 \Rightarrow \quad \textit{Element ACTIVE}, \quad \textit{NON ACTIVE otherwise} \quad (11)$$

If the secondary structure is massless, as in the present case where it represents a brace, by using the second of eq.(9) the algorithm specializes as follows:

$$\dot{y}_2 \geq 0 \Rightarrow \quad \textit{Element ACTIVE}, \quad \textit{NON ACTIVE otherwise} \quad (12)$$

where y is the frame displacement and y_2 the brace displacement. The algorithm implies that the brace is always active, with the exception of a discrete number of time instants at which y attains a maximum value; at these times the brace is instantaneously deactivated, to be immediately reactivated again, [Inaudi & Hayen 1995]. This algorithm is optimal in relative displacements and velocities, because it is a specialization of algorithm (8).

For a first comparison of the performances of the two algorithms, their behavior in the free vibration case has been analyzed, for a particular structural configuration in which the connection is perfectly rigid, (*RE*), and there is no damping in the frame, ($\xi_0 = 0$). In this case there are no dissipation sources other than the ones due to control. The first two columns of Table 1 present the ratio between two consecutive maximum values of the displacements and the vibration period of the free vibrations of the system (normalized with respect to the period of the frame alone, $T_0 = 2\pi / \omega_0$). These quantities are independent of the vibration amplitude, because the considered algorithms make the system nonlinear but *homogeneous*, [Inaudi *et al.* 1994]. The last column contains the value, λ , of the relative stiffness of the frame for which the critical damping is attained. It is evident the difference between the (non-optimal) ‘*AVS*’ algorithm and the (optimal) ‘*IH*’. The damping of the motion obtained with the second is for every λ (less than 1), superior. In particular with ‘*IH*’ the critical damping is attained for $\lambda = 1$, while with ‘*AVS*’ it occurs only in the limit for $\lambda \rightarrow \infty$. For the ‘*IH*’ algorithm $\lambda = 1$ separates the different behaviors of the system, pre-critical and post-critical, as it is visible in the representation in the phase plane, (Fig.2).

Table 1. Comparison of the performance of the two algorithms: Free Vibrations

Strategy	Y_{n+1}/Y_n	T/T_0	λ_{cr}
'AVS'	$1/(1+\lambda)$	$(1+\sqrt{1+\lambda})/(2\sqrt{1+\lambda})$	∞
'Inaudi & Hayen' (IH)	$(1-\lambda)^2/(1+\lambda)^2$	$1/\sqrt{1+\lambda}$	1

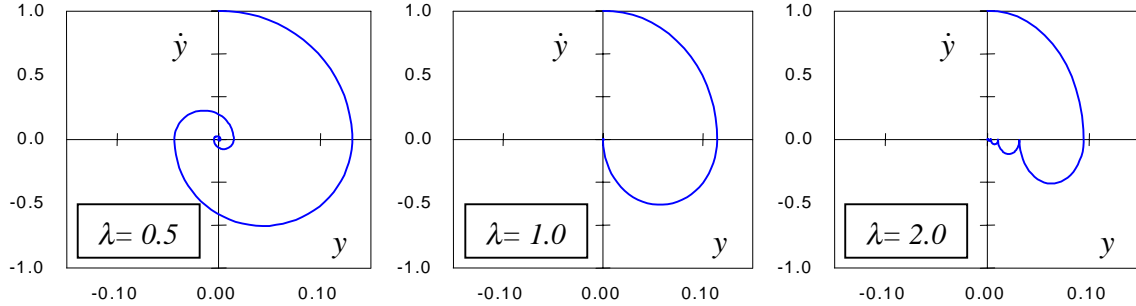


Figure 2. Controlled free vibrations in the phase plane: 'IH' algorithm

Further evaluations of the effectiveness of the control algorithms can be made by studying the system response to sinusoidal base input. The stationary response of the system has the same period as the input, and it is possible to compute in closed form the expression of the energy dissipated in one cycle, and consequently the equivalent viscous damping, which does not depend on the amplitude of the motion. In fact if ΔE is the energy dissipated in one cycle and E_0 the maximum elastic energy stored in the frame ($E_0 = \frac{1}{2} k y_0^2$), it is possible to obtain the following values of the ratio $\Delta E / E_0$: 2λ for the 'AVS' and 8λ for the 'IH' control strategy. Again the algorithm 'IH' appears superior with respect to 'AVS', because its dissipation is 4 times the one of the other, as also evident in Fig.3, which shows the energy dissipation cycles corresponding to the two control algorithms.

In order to evaluate the influence of parameter λ on the response of the system, for the 'IH' strategy, the maximum response, for different values of such parameter, is reported in Fig. 4, as a function of β , which is the ratio of the frequency of the excitation to the one of the frame. A resonance peak which decreases with increasing λ and moves right, towards values of β greater than one, is observed; the greater reductions of the resonance peak are already obtained for $\lambda = 1- 2$, while, for $\lambda > 2$, no further significant reductions are noted, so that it may be concluded that it is not convenient to use λ values greater than 2.

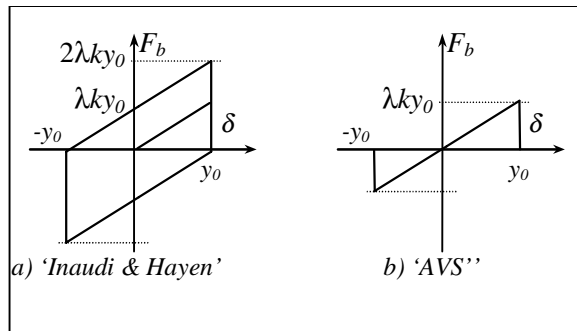


Figure 3. Energy dissipation cycles

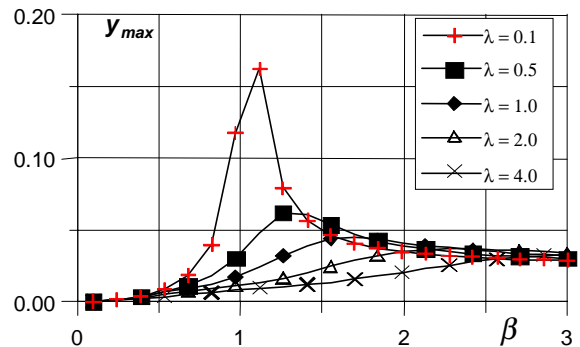


Figure 4

The simple structural model of Fig.1 has been finally subjected to the set of 5 generated accelerograms. Results shown here refer to the case $T_0 = 0.5$ sec, which falls within the region of the maximum spectral content for the excitation, and $\xi_0 = 0$. In Fig.5 are reported, as a function of λ , the values of the global response quantity G , and those of EDI . The 'IH' algorithm is also for this case superior, both in terms of lower values of G and in terms of larger values of EDI . (Such differences would be even more significant for $\xi_0 > 0$!). It may be observed that, as for the previous case, the important improvements occur already for $\lambda=1$, while for increasing λ they tend to become negligible.

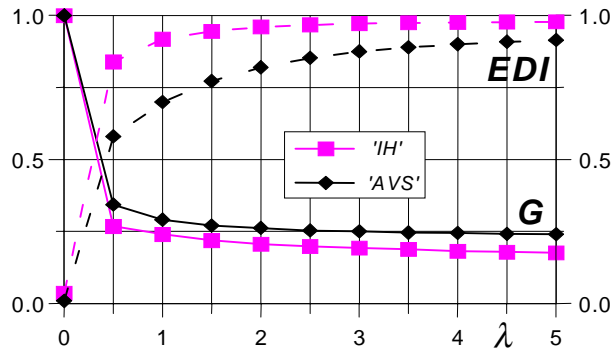


Figure 5.

The conclusion of the previous investigation is that the ‘IH’ algorithm, which coincides with a particular case of the optimal algorithm previously introduced, is superior to ‘AVS’, and will be therefore used in the following parts of the paper. It has been also demonstrated that it is not convenient to use values greater than 2 for the relative stiffness λ . Similar conclusions about λ had been also reached for the optimized Passive Control case, by using elasto-plastic, (Ciampi *et al.* 1995), or viscoelastic, (Paolacci *et al.* 1998), connections.

PASSIVE VERSUS SEMI ACTIVE CONTROL FOR SEISMIC EXCITATION

In this last part of the paper optimized Passive Control and Semi Active Control of the seismic response of the structural braced frame model will be compared, as a result of an extensive numerical investigation. Only a small part of the results are reported here; they refer to the following values of the parameter of the system: $T = T_0 = 0.1 - 3.0$ sec., $\xi_0 = 0.05$, $\lambda = 2$; with the following restrains on the range of the connection parameters: $0 \leq \xi_u \leq 1$, $0 \leq \eta_u \leq 1$.

Passive Control

In Table 2 are synthetically reported the results obtained by using the Rigid Plastic (*RPE*) and the viscous (*VE*) connections, for three values of the period of the frame, $T = 0.3$ sec., 1.0 sec. e 2.0 sec. For both connections the optimal parameter selected by the *EDI* and the E_r criterion are given. According to the chosen global point of view, the best choice should be associated to the minimum value for *G*.

The following observation can be done in terms of global response reduction (*G*): the viscous connection is more effective than the rigid plastic one, specially in terms of displacements, and this may be attributed to its capability of dissipating also at low deformation levels. Both criteria, (max *EDI* and min E_r), work well in selecting the optimal connection parameters, with comparable global response reductions, specially for the longer periods.

As a conclusion, with reference to passive control, in Figure 6 are reported the design spectra both for the *VE* and for the *RPE* parameters, obtained by using the maximum *EDI* criterion. In order to verify the design performances, in Figures 7, 8 and 9, the corresponding spectra of the significant response quantities are reported. It may be noted that the methodology leads to a parameter selection design spectrum which is very regular, (the same does not occur when using the other criterion, see Table 2): for the *VE* the optimal value of ξ_u is substantially independent of the period, at least for $T > 0.25$ sec., and has a value $\xi_u \approx 0.6$; while for the *RPE*, η_u follows the variation law of the spectrum of the total force, obtained in the case of perfectly rigid connection (thin line in Fig.6), with a proportionality coefficient, which is substantially constant with the period, and has a value of about 0.1.

Table 3. Passive Control

		T = 0.3 s				T = 1.0 s				T = 2.0 s						
	Crit.	η_u	ξ_u	Y	A	G	η_u	ξ_u	Y	A	G	η_u	ξ_u	Y	A	G
<i>RPE</i>	<i>EDI</i>	0.21	0.56	0.66	0.61	0.21	0.51	0.64	0.57	0.14	0.51	0.69	0.60			
	E_r	0.50	0.37	0.62	0.49	0.36	0.43	0.65	0.54	0.14	0.51	0.69	0.60			
<i>VE</i>	<i>EDI</i>	0.61	0.42	0.61	0.52	0.65	0.33	0.62	0.48	0.57	0.40	0.72	0.56			
	E_r	1.00	0.37	0.61	0.49	0.57	0.34	0.61	0.48	0.65	0.40	0.74	0.57			

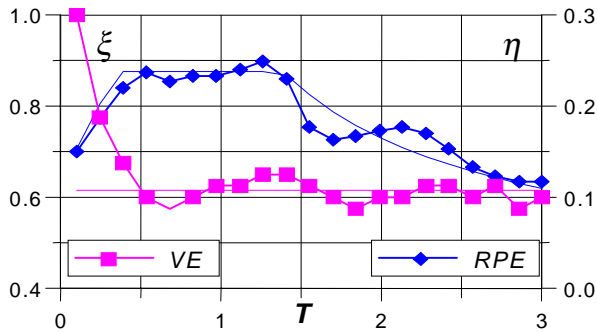


Figure 6. Optimal design parameters

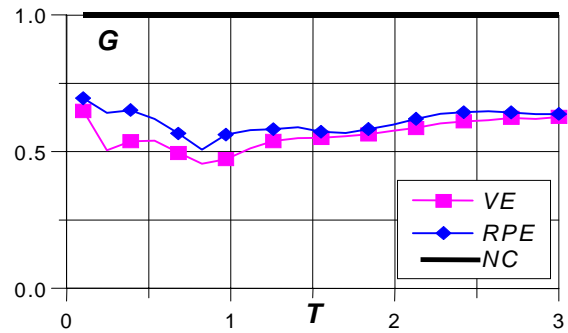


Figure 7. Global performance index, G

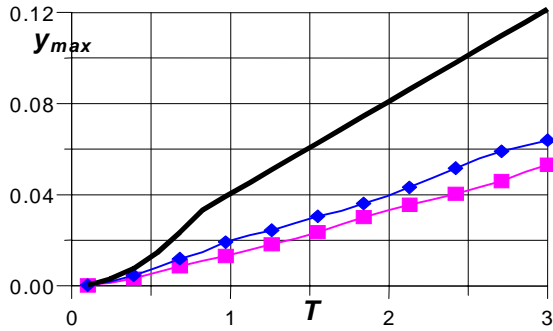


Figure 8. Maximum displacements

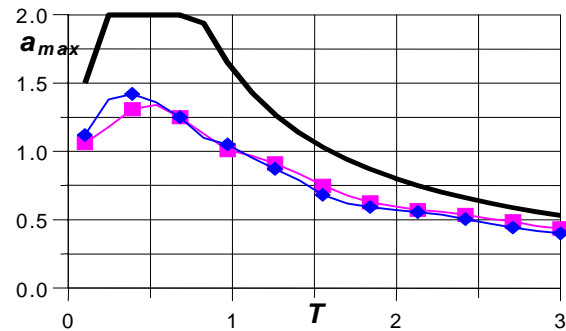


Figure 9. Maximum absolute accelerations

Semi Active Control

In Table 3 are reported the results obtained by using the optimal SAC strategy, for the different connections, including the perfectly rigid one. In this case the best connection seems to be the RPE, at least in the field of parameters which has been investigated. For this type of connection EDI selects the best parameter also for low periods, while substantial improvements are found with respect to the corresponding optimized passive control case. For the other connection type, VE, the maximum value of EDI is not attained within the considered range of variation of the parameter, so that always the maximum allowed value is selected.

Table 4. Semi Active Control

		T = 0.3 s				T = 1.0 s				T = 2.0 s						
	Crit.	η_u	ξ_u	Y	A	G	η_u	ξ_u	Y	A	G	η_u	ξ_u	Y	A	G
RPE	EDI	0.64		0.29	0.59	0.44	0.42		0.31	0.58	0.44	0.36		0.33	0.75	0.54
(η_u)	Er	0.64		0.29	0.59	0.44	0.71		0.27	0.72	0.49	0.36		0.33	0.75	0.54
VE	EDI	1.00		0.37	0.62	0.49	1.00		0.30	0.64	0.47	1.00		0.35	0.76	0.56
(ξ_u)	Er	1.00		0.37	0.62	0.49	1.00		0.30	0.64	0.47	1.00		0.35	0.76	0.56
RE		---		0.31	0.67	0.49	---		0.26	0.82	0.54	---		0.32	0.94	0.63

Finally Fig. 10 presents a time history of the system response for $T = 1.0$ sec., obtained with the rigid plastic element, in which are compared PC, SAC and the uncontrolled case. It appears, Fig. 10a and b, that using SAC not only reduces the maximum values of the important response quantities but extends the improvements to the complete time history (see Fig. 10.a for the displacements). For the SAC case it may be also observed that the hysteresis cycles are more dissipative, with smaller values of displacements and absolute accelerations (Fig.10.b).

CONCLUSIONS

The present study compares Passive and Semi Active Control of braced frames, which make use of special connection elements as control devices. The Semi Active case is limited to ON-OFF type strategies; future developments should include the possibility of continuously changing the mechanical parameters of the connections. Within the present limitations the most interesting conclusions are the following. For the case of passive control the viscous connection performs better than the rigid-plastic one; this superior behavior may be attributed to the capability of dissipating energy more continuously and even for small amplitude motions. In the Semi Active control case, unlike the Passive one, the rigid-plastic connection shows a better performance. It is

shown, also, that the optimal value of the stiffness of the bracing, for both control cases, is of the same order of magnitude of the one of the frame. Semi Active control performs better than Passive; the improvements are only minor in terms of maximum response values, but they appear more relevant when the entire time history is taken into consideration.

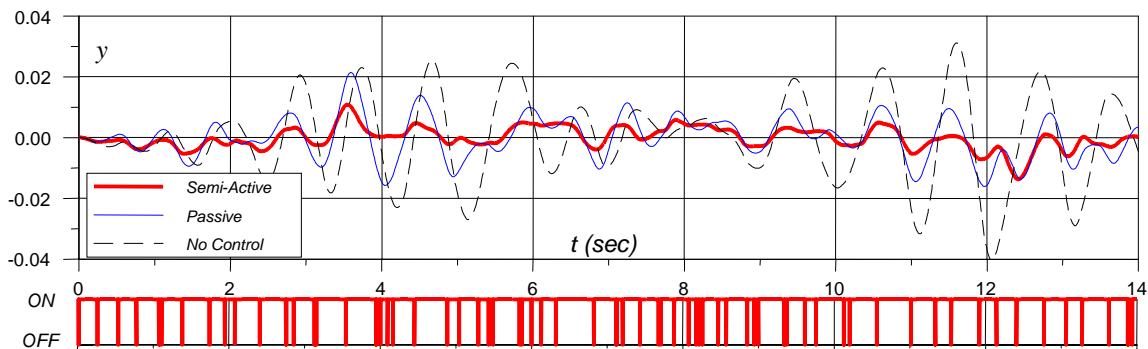


Figure 10a. Time history of the displacement response and of the control states

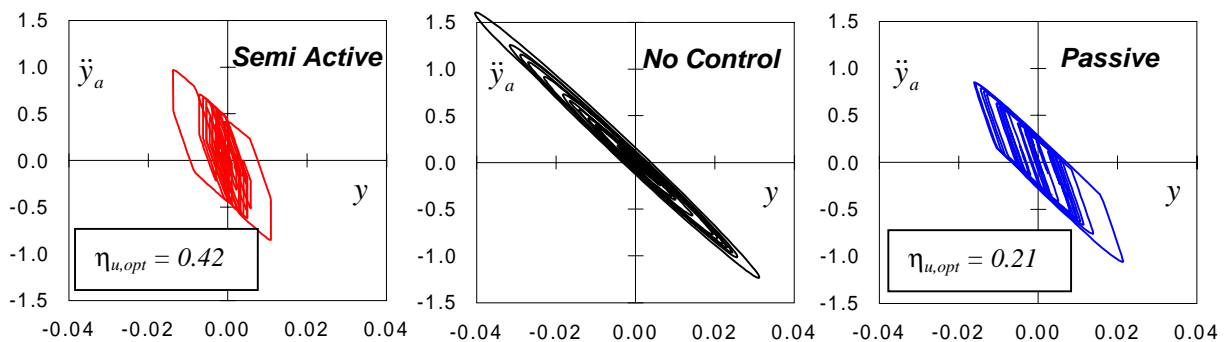


Figure 10b. Force (acceleration) – displacement dissipation cycles

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