# JOINT TIME-FREQUENCY ANALYSIS OF BASE ISOLATED BUILDINGS

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## SUMMARY

The study of the seismic response of base isolated buildings is of great interest to the engineering community. Their performance as indicated by measured data is generally favourable but much research is still required to further the understanding of their behaviour and dynamic characteristics under earthquake loading. This paper examines the characteristics of response data generated by a finite element model of a five story, base isolated building, subjected to earthquake ground motions. The earthquake response analysis is conducted via frequency and joint time-frequency methods of analysis.

This study illustrates the challenges encountered when applying frequency and joint time-frequency domain analysis techniques to base isolated structures. These include necessarily large time windows to capture the long period of oscillation and a general lack of coherence between the ground and roof acceleration records.

## INTRODUCTION

The objective of this study is to further the understanding of the response of a base isolated building to seismic loading. It is of interest to investigate the predominant frequencies of vibration and identify the temporal location of frequency shifts in the response. This study is the first phase of additional work being conducted by the authors to investigate the actual response of instrumented, base isolated, structures during seismic events.

The structure considered in this study is a five-story apartment building with a reinforced concrete frame supported by 16 columns. A study of the response of the building with and without base isolation is made. The building is modelled in SAP2000 (Computers and Structures, Inc., 1999). The earthquake chosen for the analysis is the Sylmar record from the 1994 Northridge Earthquake. This earthquake had a moment magnitude of 6.7 and the peak ground acceleration of the Sylmar record was 0.73 g.

With a non-linear system such as a base isolated building, a standard frequency-domain analysis may not be sufficient to describe its dynamic behaviour under transient seismic loading and hence a joint time-frequency analysis is conducted in addition to a frequency domain analysis.

The paper begins with a description of the building model. An overview of the frequency and joint time-frequency domain analysis methods used in the study is presented, followed by a discussion of the results. The potential challenges with both methods of analysis are discussed.

## DESCRIPTION OF BUILDING MODEL

The building considered in this study is a six-level apartment building, with five levels above ground and one basement level. It is a three by three bay, reinforced concrete building with the upper two stories set back. The

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building has plan dimensions of 13.4 m by 18 m. The total height above ground is 13.74 m. Figure 1 shows a schematic diagram of the building.

### Design

For the purpose of this study a preliminary design of the building was developed. The loads prescribed by the 1997 UBC were used to estimate the required member sizes. This information was used to determine the total weight of the structure and the corresponding load on the isolators. It should be noted that a final design of the members was not done and the results obtained from this study are only consistent with the members used. Superstructure response quantities such as maximum drift were not calculated.

### Superstructure and Isolation Models

The structure was modelled in SAP2000 Non-linear (Computers and Structures, Inc., 1999). The building was modelled in both fixed-base and base isolated configurations, i.e., without and with isolators. The superstructure model consists of 120 joints, 218 members, and 48 shells. The isolated structure has an additional 16 non-linear isolator elements. The floors were modelled to behave as rigid diaphragms.

The preliminary size of the isolators was selected based on the results of ISOLATE, a proprietary program developed by Dynamic Isolation Systems (DIS, 1995). SAP2000 uses a bi-linear model for the isolator elements.

### Modal Frequencies

The first six frequencies for the fixed-base and isolated structures, as calculated by SAP2000, are given in Table 1. Modes 1 and 4 correspond to the first two natural frequencies in the longitudinal direction, modes 2 and 5 the transverse direction and modes 3 and 6 are torsional modes of vibration. Due to the unsymmetrical plan of the building the longitudinal and torsional modes are coupled.

**Table 1. Computed Modal Frequencies**

| MODE | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Fixed Base Frequency (Hz) | 1.602 | 1.727 | 2.577 | 4.651 | 4.831 | 6.536 |
| Isolated Frequency (Hz) | 0.538 | 0.560 | 0.640 | 2.762 | 2.817 | 4.237 |

## ANALYSIS METHODS

### Frequency Domain Analysis

The frequency-domain analysis conducted consists of identifying the predominant frequencies of vibration for both the isolated and fixed-base building configurations. The transfer function between the ground acceleration and the resulting roof acceleration is calculated. The peaks in the transfer function, or the Frequency Response Function (FRF), are taken to be the predominant or natural frequencies of the structure. The FRF is calculated by taking the ratio of the cross spectrum of the ground/roof acceleration pair, and auto spectrum of the ground acceleration. For more information on frequency-domain analysis see Ewins (1984).

### Joint Time-Frequency Analysis

If the modal characteristics of a structure are thought to be time variant a joint time-frequency analysis can be useful in improving the understanding of the structure’s behaviour. For this study, the time-frequency analysis is conducted using the Short Time Fourier Transform (STFT). The ground and roof acceleration signals are multiplied by a window function. The resulting signal is transformed into the frequency domain via the Fourier Transform. The window function is propagated through the signal giving spectral values for each time step. The resulting expression is a function of both time and frequency and is thus termed the time-frequency spectrogram.

The cross time-frequency spectrogram is then divided by the auto time-frequency spectrogram to yield a function analogous to the FRF (Black, 1998). More information on analysis in the time-frequency domain can be found in Black (1998) or Cohen (1995).

## RESULTS OF ANALYSES

The fixed-base and base isolated structures were subjected to the fault normal component of the Sylmar record of the 1994 Northridge Earthquake. The motion was applied in the transverse direction of the building. Non-linear analysis was conducted using 30 Ritz vectors. The modal damping for the fixed-base structure was assumed to be five percent of critical damping, while for the isolated structure it was taken as three percent.

### Results of Frequency Domain Analysis

Figure 2 presents the Frequency Response Function calculated for various input-output signal combinations in the transverse direction. Figure 2.a presents the FRF for the base and roof accelerations for the fixed-base structure. The first two natural frequencies are clearly seen to be 1.72 Hz and 4.83 Hz, which correspond to the second and fifth modes in Table 1.

Figure 2.b shows the FRF for the calculated motions above the isolator and the roof. The peaks correspond to the first two transverse modes of the superstructure, as shown in Figure 2.a. It is of interest to note that the frequencies are smaller than the fixed-base frequencies, which implies a lengthening of the period. This is consistent with a structure resting on a flexible base, such as the isolation system. It is not possible to comment on the magnitude of the response in the superstructure modes from the FRF. This is due to the fact that the denominator of the FRF is the motion above the isolator, which may have very little power at those frequencies, thus amplifying the peaks. The fact that the fixed-base structure was modelled with a higher value of viscous modal damping is also evident, as the peaks in Figure 2.a are wider than those in Figure 2.b.

Figure 2.c and Figure 2.d present the Frequency Response Functions for the calculated motions below and above the isolators, and between the base and roof signals, respectively. In Figure 2.d the peaks at 0.56 Hz and 2.8 Hz correspond to the first two isolated modes in the transverse direction. The peaks at 1.3 Hz and 2.7 Hz, however, are not easily identified. The peak at 2.7 Hz could be the second longitudinal mode but the structure was excited only in the transverse direction, which is symmetric, and thus no response should occur in this mode. An exhaustive attempt was made to explain these peaks but no conclusion could be reached. To investigate this phenomenon, the auto-power spectrum (PSD) for various signals were calculated and are presented in Figure 3.

The PSDs did not explain the origin of the above-mentioned peaks, and in fact, it identified other anomalous, but possibly related results. The PSD of the fixed-base structure, presented in Figure 3.a, clearly shows the fundamental transverse mode. It would be expected that similar peaks corresponding to the fundamental periods of the isolated structure would also be seen in Figures 3.b and Figures 3.c. These peaks are present, but two other peaks, one at about 1 Hz and one at 3 Hz, eclipse the fundamental frequencies. The PSD calculated from the earthquake does not show significant peaks at either 1 or 3 Hz and thus the peaks observed in the PSD of the isolated structure cannot be explained by the presence of input into the system at these frequencies.

To ascertain whether this motion was due to higher modes of vibration being aliased at the sampling rate of 50 Hz, the model was analysed limiting the response to the first 5 modes. The peaks at 1 and 3 Hz were present for this case as well and thus are not a result of aliasing. To determine if these peaks were specific to this earthquake, the model was subjected to two other ground motions, the El Centro record from the 1940 Imperial Valley Earthquake and the Newhall Record from the 1994 Northridge Earthquake. Peaks near 1 and 3 Hz were present for these cases as well. A closer look at the fixed-base structure PSD was made to determine if the peaks were present for this structure, but they were not. It is likely that the unexplained peaks in the FRFs and the PSDs are in fact the same frequencies and that the observed differences are numerical in nature. At the time of writing however, this had not been substantiated.

### Results of Joint Time-Frequency Analysis

Figure 4 shows the time-frequency spectrum of the fixed-base structure excited in the transverse. Stable ridges are seen at 1.7 Hz and 4.8 Hz, which correspond to the first two modes in that direction. Figure 5 shows a plot of the time-frequency response function for the isolated building. It is seen that the isolated frequencies, 0.56 Hz and 2.8 Hz, dominate the response during the heavy shaking portion of the record, up to about 10 seconds. During the free vibration response however, the structure vibrates at frequencies near 1 and 3 Hz as seen in the frequency-domain analysis. Although the time-frequency response does show that the response at 1 and 3 Hz occurs during free vibration, it does not give further insight into the origin of these frequencies.

Through the course of this study, several limitations with the application of joint time-frequency methods became apparent. Firstly, isolated structures generally have long periods, about 2 seconds or longer. To fully capture this response in the frequency domain, a relatively large time window length is needed. A window length of at least 3 to 4 seconds is required, but preferably it should be about 5 to 6 seconds. A window length of 6 seconds was used for the calculation of the time-frequency plots in Figures 4 and 5. A window of 6 seconds should capture the fundamental mode of the isolated building but the resulting time resolution is poor. This characteristic of all joint time-frequency methods of analysis is described by the uncertainty principle (for more information, see Black, 1998 or Cohen, 1995). As the window length gets larger and larger, the joint time-frequency analysis approaches a standard frequency domain analysis.

Another limitation with frequency and joint time-frequency analysis of base-isolated buildings is the lack of coherence between the base and roof motion. Coherence is a measure of the linear relation between the input and the output of a system. This lack of coherence, although expected, was more substantial than anticipated. Figures 6 and 7 show the individual components of the FRF, as well as the coherence, for the fixed and isolated cases respectively. It is seen that the coherence is almost exact (a value of 1) and the FRF is “clean”, for the fixed-base case. This is as it should be because the structure is linear, in this case, and the data was generated analytically. For the isolated structure however, there is a dramatic decrease in the coherence between the input and output signals. In general, the FRF is much “less clean” than the fixed base case. This lack of coherence may significantly affect the results and possibly lead to the identification of spurious predominant frequencies.

The limitations mentioned above are not as prevalent in the analysis of non-isolated structures, as their fundamental periods are shorter and the building response is more linearly related to the ground motion.

## CONCLUSIONS

A frequency domain analysis of the base isolated structure showed unidentifiable peaks in the FRF near 1.3 and 2.7 Hz. To further investigate the possible origin of these peaks, auto-power spectral densities were calculated for various signals. Dominant peaks near 1 and 3 Hz were seen in all records considered. These peaks are present only in the isolated structure during free vibration and they are not limited to the response of the structure to a particular earthquake. Similar to the peaks seen in the FRF, these predominant frequencies could not be explained. Analysis in the joint time-frequency domain showed that the isolated structure responds at the isolated frequency during heavy shaking and near 1 and 3 Hz during free vibration.

It is likely that the peaks seen at 1.3 and 2.7 Hz in the frequency domain analysis are the same peaks seen near 1 and 3 Hz in the joint time-frequency domain analysis. The discrepancy between these values is possibly numerical in nature. This however, had not been substantiated at the time of writing. The implication of a lack of coherence between the input and output signals needs to be investigated as it may be responsible for the unexplained peaks.

Although Black (1998) showed that joint time-frequency analysis of instrumented structures subjected to earthquake shaking can be a powerful technique to improve the understanding of seismic response, this study illustrates potential limitations with the utility of joint time-frequency analysis for base-isolated structures. These weaknesses stem from the fact that a large window length is required to capture the predominant frequency of a base-isolated building and thus the time resolution is limited. As well, the highly non-linear behaviour of base isolated structures results in a lack of coherence between the input and output quantities and hence the quality of the results should be carefully considered.

## ACKNOWLEDGEMENTS

The authors wish to acknowledge Professor Nicos Makris (UC Berkeley) for his assistance in the development of the analytical model and Professor Anil Chopra (UC Berkeley) for his feedback on the preliminary findings.

The assistance of Professor Carlos Ventura (The University of British Columbia) with the development of the joint time-frequency analysis method used in this study is appreciated.

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**Figure 1. Schematic of the Structure**

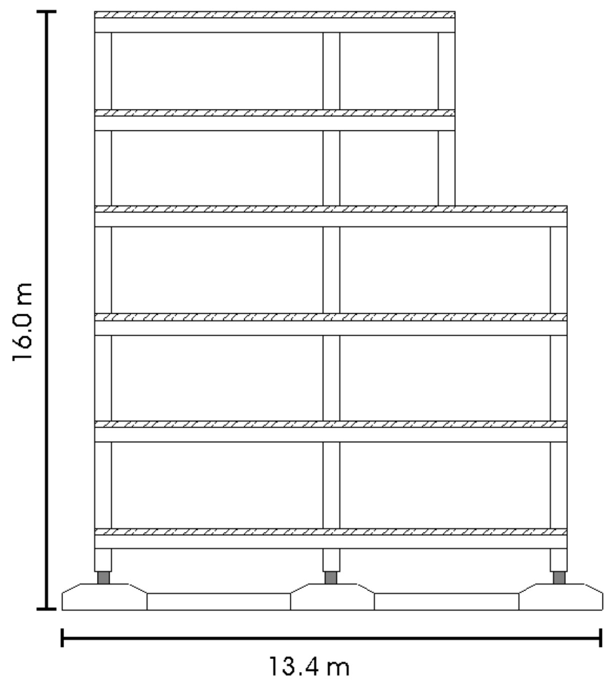

**Figure 2. Frequency Response Function Calculated from Various Input-Output Combinations**

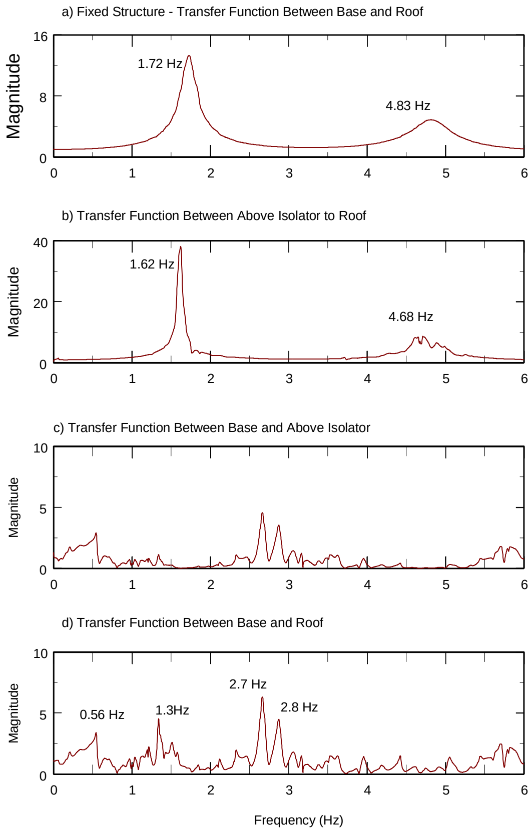


**Figure 3. Power Spectral Densities of Various Signals**

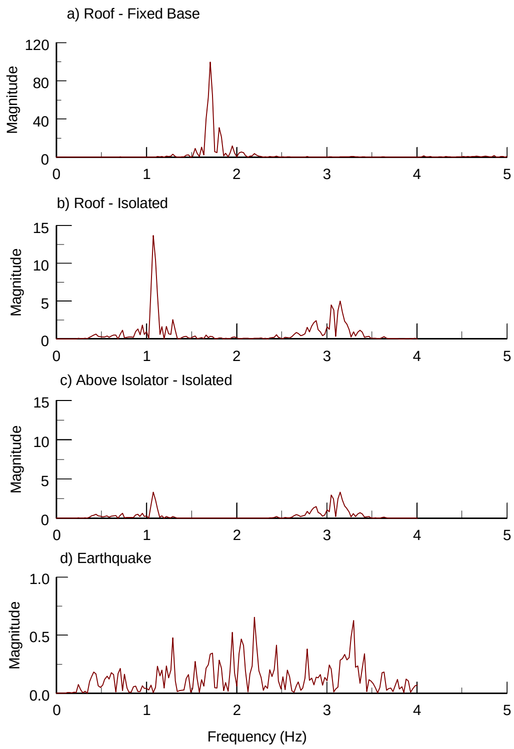

**Figure 4. Time-Frequency Spectrum for Fixed Base Structure**

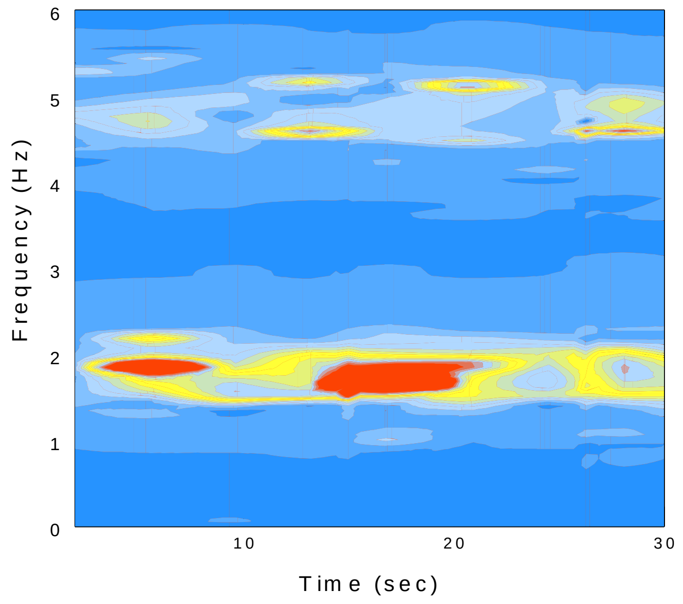


**Figure 5. Time-Frequency Spectrum of Base Isolated Structure**

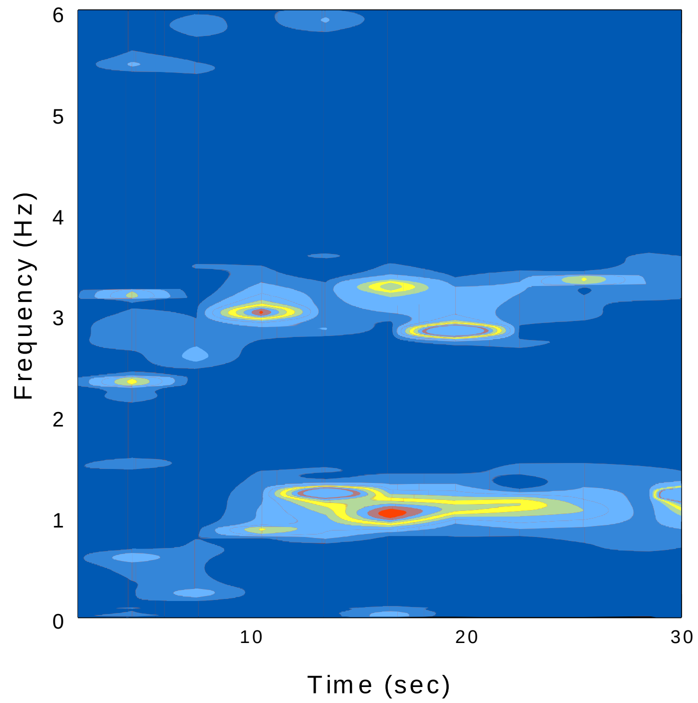

**Figure 6. Frequency Response Function and Coherence for Fixed-Base Structure**

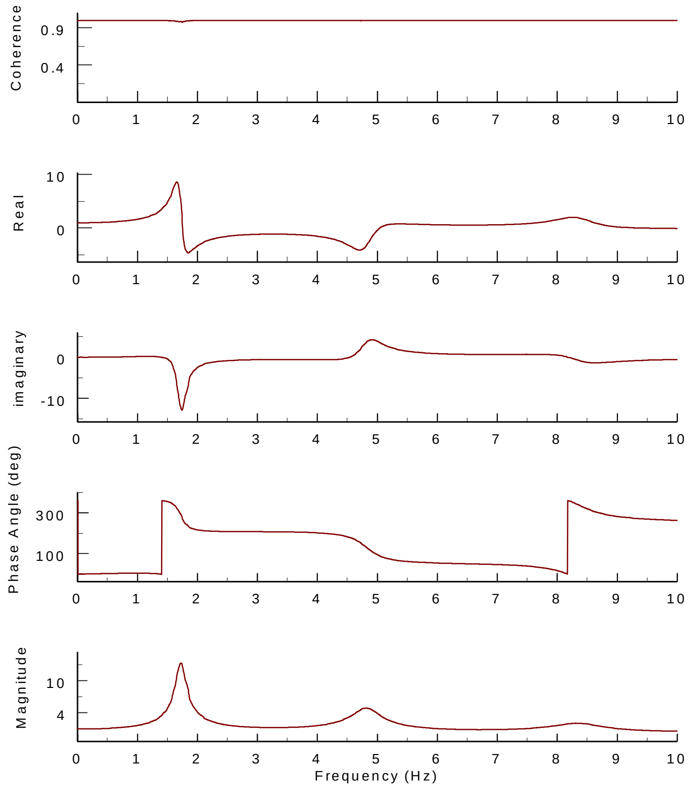


**Figure 7. Frequency Response Function and Coherence for Base Isolated Structure**