INDIRECT BOUNDARY ELEMENT METHOD, A TOOL TO CALCULATE SEISMIC RESPONSE OF IRREGULARLY LAYERED SEDIMENT

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SUMMARY

This paper shows the way to turn into reality the idea of the reflection and transmission coefficients in the space - frequency domain for infinitely spread irregular interface by using indirect boundary element method (i-bem) and to apply them to the wave field in irregularly stratified media. The way is a hybrid method between the strategy of reflectivity method developed in seismology and that of i-bem grown in engineering society. I consider i-bem as one of the ways to disassemble the wave field into up- and down-going waves, the role of the wave function of which is played by green's function matrices, and that of the coefficients vectors of which by the imaginary forces distributed along both faces of interfaces. The usage of the reference solution, that is the wave field in the corresponding horizontally stratified media, allows us to handle infinitely spread interfaces. This method can stack the effect of transmission and reflection on the wave field as wave goes by. Therefore, for example., The directly coming waves from the seismic source can be separated from latter phases. The test case for homogeneous basin shows the incident wave and reflected waves that bounce up and down in the basin. It is expected that the formulation shown here can make easier the consideration on the wave field in complex velocity structure and the search for good and efficient approximation.

INTRODUCTION

It may be required to calculate the seismic response of irregularly stratified media, if we have to estimate seismic ground motion in an area such complicated geological setting as alluvial basin. During the 90's decade, a substantial development took place in numerical methods for this purpose. Such domain methods as Finite Element and Finite Difference Method have reached at practical application for real size problems, whereas such boundary methods as Boundary Element Method and Boundary Integral Equation Method stay in the stage of research. Unfortunately, the capacity of computer that we can use today is not enough to calculate real size problems. The boundary methods, however, have not lost their appeal, yet and are attracting researchers, because of the possibility for substantial improvement of cost performance and accuracy, not only by computer technology, but also by the wisdom and the effort of the human beings. The topic introduced here is not for practical application, but for the pathfinding for the future development of boundary methods.

The theory for the wave field in the horizontally stratified media given in the wavenumber-frequency domain shows us the following. It is an efficient strategy to disassemble the wave field in each layer into up- and down-going waves and to stack the effect of transmission and reflection as wave propagates [Kennett (1983)]. [Takenaka and Fujuwara (1994)] have applied a similar idea to irregular structure in terms of BEM (direct formulation).

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This strategy can be useful in terms of Indirect BEM in the space-frequency domain in that up- and down-going waves are given independently by the product of Green's function matrix and the vector of the imaginary forces distributed along the interfaces [Sanchez-Sesma and Campillo (1991)]. Note that this Green's function corresponds to the wave function in the theory for horizontally stratified media, and that distributed force vector corresponds to the wave vector. This implies that the transmission and reflection coefficient matrices may be able to be defined by the relation between these distributed force vectors. In this paper, I show a way to turn this idea into reality.

APPLICATION OF INDIRECT BEM IN THE SPACE-FREQUENCY DOMAIN TO HANDLE TRANSMISSION AND REFLECTION

The main difference between the method in the wavenumber domain and that in the space domain is that the former can be applied directly to infinitely spread plane interfaces and requires the integral over wavenumber, whereas the later can handle just finite interfaces. After [Fujiwara and Takenaka (1993)] developed the idea itself, the technique in Indirect boundary element method that can handled infinitely spread interfaces is introduced by [Yokoi and Takenaka (1995)] for the two dimensional free surface. Then, by [Yokoi and Sanchez-Sesma (1998)] for tri-dimensional one and applied to irregularly stratified media by [Yokoi (1996)]. The technique takes advantage of the reference solution, i.e., the wave field in the corresponding horizontally stratified media.

Transmission and Reflection at an interface

First, consider an irregular interface between two half spaces of the material properties of which values are different each other. Hereafter, I use the notation $G^{l}_{l',l''}(x;\xi), H^{l}_{l',l''}(x;\xi)$ for the displacement and traction Green's function, respectively, in (*l*)-th layer, of which source is located at ξ on (*l*'')-th interface and of which receiver is located at x on (*l*')-th interface. The distributed forces considered along the upper and lower face of (*l*)-th interface are written as $\overline{\Phi}^{l}(\xi), \underline{\Phi}^{l}(\xi)$, respectively. Note that these forces are distributed just along the interfaces that can be handled in numerical computation. For the forces distributed away to infinitely far along interfaces, $\hat{\overline{\Phi}}^{l}(\xi), \hat{\underline{\Phi}}^{l}(\xi)$ are used. The wave field in the horizontally stratified media in that the material properties of each layer are the same as those of the considered irregularly layered media, is called the reference solution. Down-going wave in the (*l*)-th layer is noted $\left(\underline{u}^{l},\ \underline{t}^{l}\right)$ and up-going wave $\left(\overline{u}^{l},\ \overline{t}^{l}\right)$ for this reference solution. These waves also have their corresponding forces distributed along the horizontal plane interfaces. Vectors $\overline{\Psi}^{l}(\xi), \underline{\Psi}^{l}(\xi)$ are used for these along finite part of plane interfaces. The configuration is shown in Figure 1. The difference between continuous variables and operations and the corresponding discrete ones is not clearly distinguished in the description, because the theory explained below is for numerical calculation method. The difference, in contrast, between infinitely spread interfaces and limited ones is strictly distinguished. $\langle\ \rangle$ is used to denote the integral operator along infinitely spread interfaces as explained below.

Up-going incident wave to an interface

Consider the up-going incident wave to the (*l*)-th interface from beneath (Figure 1). The reflection and transmission at the interface are taken into account, but the reflection of the transmitted wave at the (*l*-1)-th interface is omitted here. The boundary condition along the (*l*)-th interface, that is the continuity of displacement and traction, can be formulated as follows. The wave field in the (*l*+1)-th layer is the sum of the contribution from the distributed force along the upper face of (*l*+1)-interface $\overline{\Phi}^{l+1}$ and that of the distributed force along the lower face of the (*l*)-th interface $\underline{\Phi}^{l}$. Whereas the wave field in the (*l*)-th layer is of the contribution from the distributed force along the upper face of the (*l*)-th interface $\overline{\Phi}^{l}$. Therefore,

$$\begin{cases} \int_{S^{l+1}} G^{l+1}_{l,l+1}(x;\xi)\overline{\Phi}^{l+1}(\xi)d\xi + \int_{S^{l}} G^{l+1}_{l,l}(x;\xi)\underline{\Phi}^{l}(\xi)d\xi = \int_{S^{l}} G^{l}_{l,l}(x;\xi)\overline{\Phi}^{l}(\xi)d\xi, \\ \int_{S^{l+1}} H^{l+1}_{l,l+1}(x;\xi)\overline{\Phi}^{l+1}(\xi)d\xi + \int_{S^{l}} H^{l+1}_{l,l}(x;\xi)\underline{\Phi}^{l}(\xi)d\xi = \int_{S^{l}} H^{l}_{l,l}(x;\xi)\overline{\Phi}^{l}(\xi)d\xi, \end{cases} \quad for\ x \in S^{l}. \tag{1}$$

By using the expression for the operator for integral along the infinitely spread interfaces,

$$\left\langle \begin{matrix} G_{l,l+1}^{l+1} \\ H_{l,l+1}^{l+1} \end{matrix} \right\rangle \hat{\overline{\Phi}}^{l+1} + \left\langle \begin{matrix} G_{l,l}^{l+1} \\ H_{l,l}^{l+1} \end{matrix} \right\rangle \hat{\underline{\Phi}}^{l} = \left\langle \begin{matrix} G_{l,l}^{l} \\ H_{l,l}^{l} \end{matrix} \right\rangle \hat{\overline{\Phi}}^{l}. \tag{2}$$

As it is not possible to handle infinitely spread interfaces in computer, the interface S^l is divided into S^l_i that can be made discrete and handled in computation, and the rest of the interface S^l_e

(3)

The up-going wave of the reference solution in the (l)-th layer is given as

$$\begin{cases} \overline{u}^l(x) = \int_{S_f^l} G_{l,l}^l(x;\xi)\overline{\Psi}^l(\xi)d\xi + \int_{S_e^l} G_{l,l}^l(x;\xi)\overline{\Psi}^l(\xi)d\xi, \\ \overline{t}^l(x) = \int_{S_f^l} H_{l,l}^l(x;\xi)\overline{\Psi}^l(\xi)d\xi + \int_{S_e^l} H_{l,l}^l(x;\xi)\overline{\Psi}^l(\xi)d\xi, \end{cases} \quad for\ x \in l-th\ \ layer. \tag{4}$$

Similar formulas can be given also for down-going wave in the (l)-th layer $\left(\underline{u}^{l+1},\ \ \underline{t}^{l+1}\right)$ and up-going wave in (l+1)-th layer $\left(\overline{u}^{l+1},\ \ \overline{t}^{l+1}\right)$.

The assumption $\overline{\Phi}^l(\xi) \cong \overline{\Psi}^l(\xi), \underline{\Phi}^l(\xi) \cong \underline{\Psi}^l(\xi),\ \xi \in S_e^l, \overline{\Phi}^{l+1}(\xi) \cong \overline{\Psi}^{l+1}(\xi),\ \xi \in S_e^{l+1}$, gives

$$\begin{cases} \int_{S_i^{l+1}} G_{l,l+1}^{l+1}(x;\xi)\overline{\Phi}^{l+1}(\xi)d\xi + \int_{S_i^l} G_{l,l}^{l+1}(x;\xi)\underline{\Phi}^l(\xi)d\xi + \underline{u}^{l+1}(x) + \overline{u}^{l+1}(x) - \int_{S_f^l} G_{l,l}^{l+1}(x;\xi)\underline{\Psi}^l(\xi)d\xi \\ \quad - \int_{S_f^{l+1}} G_{l,l+1}^{l+1}(x;\xi)\overline{\Psi}^{l+1}(\xi)d\xi = \int_{S_e^l} G_{l,l}^l(x;\xi)\overline{\Phi}^l(\xi)d\xi + \overline{u}^l(x) - \int_{S_f^l} G_{l,l}^l(x;\xi)\overline{\Psi}^l(\xi)d\xi, \\ \int_{S_e^{l+1}} H_{l,l+1}^{l+1}(x;\xi)\overline{\Phi}^{l+1}(\xi)d\xi + \int_{S_e^l} H_{l,l}^{l+1}(x;\xi)\underline{\Phi}^l(\xi)d\xi + \underline{t}^{l+1}(x) + \overline{t}^{l+1}(x) - \int_{S_f^l} H_{l,l}^{l+1}(x;\xi)\underline{\Psi}^l(\xi)d\xi \\ - \int_{S_f^{l+1}} H_{l,l+1}^{l+1}(x;\xi)\overline{\Psi}^{l+1}(\xi)d\xi = \int_{S_e^l} H_{l,l}^l(x;\xi)\overline{\Phi}^l(\xi)d\xi + \overline{t}^l(x) - \int_{S_f^l} H_{l,l}^l(x;\xi)\overline{\Psi}^l(\xi)d\xi, \quad for\ x \in S_i^l. \end{cases} \tag{5}$$

Discretization of these boundary integral equations gives the following simultaneous linear equations.

$$\left(\begin{bmatrix} \overline{u}_l^l \\ \overline{t}_l^l \end{bmatrix} - \begin{bmatrix} G_{l,l}^l \\ H_{l,l}^l \end{bmatrix}\overline{\Psi}^l + \begin{bmatrix} G_{l,l}^l \\ H_{l,l}^l \end{bmatrix}\overline{\Phi}^l\right) = \left(\begin{bmatrix} \underline{u}_l^{l+1} \\ \underline{t}_l^{l+1} \end{bmatrix} - \begin{bmatrix} G_{l,l}^{l+1} \\ H_{l,l}^{l+1} \end{bmatrix}\underline{\Psi}^l + \begin{bmatrix} G_{l,l}^{l+1} \\ H_{l,l}^{l+1} \end{bmatrix}\underline{\Phi}^l\right) + \left(\begin{bmatrix} \overline{u}_l^{l+1} \\ \overline{t}_l^{l+1} \end{bmatrix} - \begin{bmatrix} G_{l,l+1}^{l+1} \\ H_{l,l+1}^{l+1} \end{bmatrix}\overline{\Psi}^{l+1} + \begin{bmatrix} G_{l,l+1}^{l+1} \\ H_{l,l+1}^{l+1} \end{bmatrix}\overline{\Phi}^{l+1}\right) \tag{6}$$

The left member represents the up-going wave in the (l)-th layer, where as the first parenthesis of the right member corresponds to the down-going and the second to the up-coming wave in the (l+1)-layer.

Moving all known variables in the right member, the following simple equations are obtained.

$$\begin{bmatrix} G_{l,l}^l & -G_{l,l}^{l+1} \\ H_{l,l}^l & -H_{l,l}^{l+1} \end{bmatrix}\begin{bmatrix} \overline{\Phi}^l \\ \underline{\Phi}^l \end{bmatrix} = \left(\begin{bmatrix} \overline{u}_l^{l+1} \\ \overline{t}_l^{l+1} \end{bmatrix} - \begin{bmatrix} G_{l,l+1}^{l+1} \\ H_{l,l+1}^{l+1} \end{bmatrix}\overline{\Psi}^{l+1} + \begin{bmatrix} G_{l,l+1}^{l+1} \\ H_{l,l+1}^{l+1} \end{bmatrix}\overline{\Phi}^{l+1}\right) + \left(\begin{bmatrix} \underline{u}_l^{l+1} \\ \underline{t}_l^{l+1} \end{bmatrix} - \begin{bmatrix} G_{l,l}^{l+1} \\ H_{l,l}^{l+1} \end{bmatrix}\underline{\Psi}^l\right) - \left(\begin{bmatrix} \overline{u}_l^l \\ \overline{t}_l^l \end{bmatrix} - \begin{bmatrix} G_{l,l}^l \\ H_{l,l}^l \end{bmatrix}\overline{\Psi}^l\right) \tag{7}$$

Define the transmission and reflection operators, respectively, $\hat{\overline{\Phi}}^l = \left\langle T_{l,l+1}^U \right\rangle \hat{\overline{\Phi}}^{l+1},\ \hat{\underline{\Phi}}^l = \left\langle R_{l,l+1}^U \right\rangle \hat{\overline{\Phi}}^{l+1}$.

By using this solution, the transmitted and reflected wave field can be numerically estimated as follows.

$$\left\langle \begin{matrix} G_{l,l}^l \\ H_{l,l}^l \end{matrix} \right\rangle \hat{\overline{\Phi}}^l = \left\langle \begin{matrix} G_{l,l}^l \\ H_{l,l}^l \end{matrix} \right\rangle \left\langle \hat{T}_{l,l+1}^U \right\rangle \hat{\overline{\Phi}}^{l+1} \approx \begin{bmatrix} \overline{u}_l^l \\ \overline{t}_l^l \end{bmatrix} - \begin{bmatrix} G_{l,l}^l \\ H_{l,l}^l \end{bmatrix}\overline{\Psi}^l + \begin{bmatrix} G_{l,l}^l \\ H_{l,l}^l \end{bmatrix}\overline{\Phi}^l,\ \left\langle \begin{matrix} G_{l,l}^{l+1} \\ H_{l,l}^{l+1} \end{matrix} \right\rangle \hat{\underline{\Phi}}^l = \left\langle \begin{matrix} G_{l,l}^{l+1} \\ H_{l,l}^{l+1} \end{matrix} \right\rangle \left\langle \hat{R}_{l,l+1}^U \right\rangle \hat{\overline{\Phi}}^{l+1} \approx \begin{bmatrix} \underline{u}_l^{l+1} \\ \underline{t}_l^{l+1} \end{bmatrix} - \begin{bmatrix} G_{l,l}^{l+1} \\ H_{l,l}^{l+1} \end{bmatrix}\underline{\Psi}^l + \begin{bmatrix} G_{l,l}^{l+1} \\ H_{l,l}^{l+1} \end{bmatrix}\underline{\Phi}^l. \tag{8}$$

It is not necessary to obtain the value of $\hat{\overline{\Phi}}^l = \left\langle T_{l,l+1}^U \right\rangle \hat{\overline{\Phi}}^{l+1}$, or $\hat{\underline{\Phi}}^l = \left\langle R_{l,l+1}^U \right\rangle \hat{\overline{\Phi}}^{l+1}$. The quantities that are used in the next step of computation are the displacement-traction vector $\left\langle \begin{matrix} G_{l,l}^l \\ H_{l,l}^l \end{matrix} \right\rangle \hat{\overline{\Phi}}^l$ and $\left\langle \begin{matrix} G_{l,l}^{l+1} \\ H_{l,l}^{l+1} \end{matrix} \right\rangle \hat{\underline{\Phi}}^l$ as shown above.

Down-going incident wave to an interface

Consider down-going incident wave to the (l-1)-th interface from upside (Figure 1). The reflection and transmission at this interface are taken into account, but the reflection of the transmitted wave at the (l+1)-th interface is omitted. The boundary condition along the (l)-th interface, that is the continuity of displacement and traction, can be formulated as follows. The wave field in the (l)-th layer is the sum of the contribution from the distributed force along the lower face of (l-1)-interface $\underline{\Phi}^{l-1}$ and that of the distributed force along the upper face of the (l)-th interface $\overline{\Phi}^l$. Whereas the wave field in the (l)-th layer is of the contribution from the distributed force along the upper face of the (l)-th interface $\underline{\Phi}^l$. Therefore, the boundary integral equation expressed by integral operators as Eq. (2) is

$$\left\langle \begin{matrix} G_{l,l-1}^l \\ H_{l,l-1}^l \end{matrix} \right\rangle \hat{\underline{\Phi}}^{l-1} + \left\langle \begin{matrix} G_{l,l}^l \\ H_{l,l}^l \end{matrix} \right\rangle \hat{\overline{\Phi}}^l = \left\langle \begin{matrix} G_{l,l}^l \\ H_{l,l}^l \end{matrix} \right\rangle \hat{\underline{\Phi}}^l. \tag{9}$$

The assumption $\overline{\Phi}^{l}(\xi)\cong\overline{\Psi}^{l}(\xi),\underline{\Phi}^{l}(\xi)\cong\underline{\Psi}^{l}(\xi),\ \xi\in S_e^l,\overline{\Phi}^{l+1}(\xi)\cong\overline{\Psi}^{l+1}(\xi),\ \xi\in S_e^{l+1}$, gives the following integral equation that corresponds to Eq. (5).

$$\begin{cases}\int_{S_i^{l-1}}G_{l,l-1}^{l}(x;\xi)\underline{\Phi}^{l-1}(\xi)d\xi+\int_{S_i^{l}}G_{l,l}^{l}(x;\xi)\overline{\Phi}^{l}(\xi)d\xi+\bar{u}^{l}(x)+\underline{u}^{l}(x)-\int_{S_f^{l}}G_{l,l}^{l}(x;\xi)\overline{\Psi}^{l}(\xi)d\xi \\ -\int_{S_f^{l-1}}G_{l,l-1}^{l}(x;\xi)\underline{\Psi}^{l-1}(\xi)d\xi=\int_{S_i^{l}}G_{l,l}^{l}(x;\xi)\underline{\Phi}^{l}(\xi)d\xi+\underline{u}^{l+1}(x)-\int_{S_f^{l}}G_{l,l}^{l}(x;\xi)\underline{\Psi}^{l}(\xi)d\xi, \\ \int_{S_i^{l-1}}H_{l,l-1}^{l}(x;\xi)\underline{\Phi}^{l-1}(\xi)d\xi+\int_{S_i^{l}}H_{l,l}^{l}(x;\xi)\overline{\Phi}^{l}(\xi)d\xi+\bar{t}^{l}(x)+\underline{t}^{l}(x)-\int_{S_f^{l}}H_{l,l}^{l}(x;\xi)\overline{\Psi}^{l}(\xi)d\xi \\ -\int_{S_f^{l-1}}H_{l,l-1}^{l}(x;\xi)\underline{\Psi}^{l-1}(\xi)d\xi=\int_{S_i^{l}}H_{l,l}^{l}(x;\xi)\underline{\Phi}^{l}(\xi)d\xi+\underline{t}^{l+1}(x)-\int_{S_f^{l}}H_{l,l}^{l}(x;\xi)\underline{\Psi}^{l}(\xi)d\xi,\end{cases}\quad for\ x\in S_i^l. \qquad (10)$$

Discretization of these boundary integral equations gives the following simultaneous linear equations.

$$\left(\begin{bmatrix}\underline{u}_l^l\\ \underline{t}_l^l\end{bmatrix}-\begin{bmatrix}G_{l,l-1}^l\\ H_{l,l-1}^l\end{bmatrix}\underline{\Psi}^{l-1}+\begin{bmatrix}G_{l,l-1}^l\\ H_{l,l-1}^l\end{bmatrix}\underline{\Phi}^{l-1}\right)+\left(\begin{bmatrix}\bar{u}_l^l\\ \bar{t}_l^l\end{bmatrix}-\begin{bmatrix}G_{l,l}^l\\ H_{l,l}^l\end{bmatrix}\overline{\Psi}^{l}+\begin{bmatrix}G_{l,l}^l\\ H_{l,l}^l\end{bmatrix}\overline{\Phi}^{l}\right)=\left(\begin{bmatrix}\underline{u}_l^{l+1}\\ \underline{t}_l^{l+1}\end{bmatrix}-\begin{bmatrix}G_{l,l}^{l+1}\\ H_{l,l}^{l+1}\end{bmatrix}\underline{\Psi}^{l}+\begin{bmatrix}G_{l,l}^{l+1}\\ H_{l,l}^{l+1}\end{bmatrix}\underline{\Phi}^{l}\right) \qquad (11)$$

The first parenthesis of the left member represents the down-going incident wave, whereas the second corresponds to the up-going wave in the (l)-th layer. The right member represents the down-going wave in the (l+1)-th layer. Moving all known variables in the right member, the following simple formula is obtained.

$$\begin{bmatrix}G_{l,l}^l & -G_{l,l}^{l+1}\\ H_{l,l}^l & -H_{l,l}^{l+1}\end{bmatrix}\begin{bmatrix}\overline{\Phi}^l\\ \underline{\Phi}^l\end{bmatrix}=-\left(\begin{bmatrix}\underline{u}_l^l\\ \underline{t}_l^l\end{bmatrix}-\begin{bmatrix}G_{l,l-1}^l\\ H_{l,l-1}^l\end{bmatrix}\underline{\Psi}^{l-1}+\begin{bmatrix}G_{l,l+1}^{l+1}\\ H_{l,l+1}^{l+1}\end{bmatrix}\underline{\Phi}^{l-1}\right)-\left(\begin{bmatrix}\bar{u}_l^l\\ \bar{t}_l^l\end{bmatrix}-\begin{bmatrix}G_{l,l}^l\\ H_{l,l}^l\end{bmatrix}\overline{\Psi}^{l}\right)+\left(\begin{bmatrix}\underline{u}_l^{l+1}\\ \underline{t}_l^{l+1}\end{bmatrix}-\begin{bmatrix}G_{l,l}^{l+1}\\ H_{l,l}^{l+1}\end{bmatrix}\underline{\Psi}^{l}\right) \qquad (12)$$

Define the transmission and reflection operators as follows, respectively. $\hat{\underline{\Phi}}^l=\langle\hat{T}_{l,l-1}^D\rangle\hat{\underline{\Phi}}^{l-1}$, $\hat{\overline{\Phi}}^l=\langle\hat{R}_{l,l-1}^D\rangle\hat{\underline{\Phi}}^{l-1}$.

By using this solution of Eq.(12), the transmitted and reflected wave field can be calculated as follows.

$$\left\langle\begin{matrix}G_{l,l}^{l+1}\\ H_{l,l}^{l+1}\end{matrix}\right\rangle\hat{\underline{\Phi}}^l=\left\langle\begin{matrix}G_{l,l}^{l+1}\\ H_{l,l}^{l+1}\end{matrix}\right\rangle\langle\hat{T}_{l,l-1}^D\rangle\hat{\underline{\Phi}}^{l-1}\approx\begin{bmatrix}\underline{u}_l^{l+1}\\ \underline{t}_l^{l+1}\end{bmatrix}-\begin{bmatrix}G_{l,l}^{l+1}\\ H_{l,l}^{l+1}\end{bmatrix}\underline{\Psi}^l+\begin{bmatrix}G_{l,l}^{l+1}\\ H_{l,l}^{l+1}\end{bmatrix}\underline{\Phi}^l,\ \left\langle\begin{matrix}G_{l,l}^{l}\\ H_{l,l}^{l}\end{matrix}\right\rangle\hat{\overline{\Phi}}^l=\left\langle\begin{matrix}G_{l,l}^{l}\\ H_{l,l}^{l}\end{matrix}\right\rangle\langle\hat{R}_{l,l-1}^D\rangle\hat{\underline{\Phi}}^{l}\approx\begin{bmatrix}\bar{u}_l^{l}\\ \bar{t}_l^{l}\end{bmatrix}-\begin{bmatrix}G_{l,l}^{l}\\ H_{l,l}^{l}\end{bmatrix}\overline{\Psi}^l+\begin{bmatrix}G_{l,l}^{l}\\ H_{l,l}^{l}\end{bmatrix}\overline{\Phi}^l. \qquad (13)$$

The quantities that are used in the next step of computation are the displacement - traction vector $\left\langle\begin{matrix}G_{l,l}^{l+1}\\ H_{l,l}^{l+1}\end{matrix}\right\rangle\hat{\underline{\Phi}}^l$ and $\left\langle\begin{matrix}G_{l,l}^{l}\\ H_{l,l}^{l}\end{matrix}\right\rangle\hat{\overline{\Phi}}^l$ as shown above.

Transmission and reflection operators for a layer

Consider a layer caught between two half spaces with irregular interfaces. Name it (*l*)-th layer, the half space hanging over (*l*-1)-th layer, another half space lays under it (*l*+1)-th layer, and the interface between (*l*-1)-th and (*l*)-th layers is (*l*)-th interface (Figure 2).

Up-going incident wave

Define the transmission operator for the upcoming incident wave in (*l*+1)-th layer and the up-going wave in (*l*-1)-th layer, and the reflection operator for the same incident wave and the reflected wave in (*l*+1)-layer (Figure 2).

$$\hat{\overline{\Phi}}^{l-1}=\langle\hat{T}_{l-1,l+1}^U\rangle\hat{\overline{\Phi}}^{l+1},\quad \hat{\underline{\Phi}}^{l}=\langle\hat{R}_{l-1,l+1}^U\rangle\hat{\overline{\Phi}}^{l+1}.$$

Consideration on the multiple reflection in (*l*)-th layer gives the following relation.

$$\begin{cases}\hat{\overline{\Phi}}^{l-1}=\langle T_{l-1,l}^U\rangle\left\{I+\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle+\left(\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\right)^2+\left(\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\right)^3+\cdots\right\}\langle T_{l,l+1}^U\rangle\hat{\overline{\Phi}}^{l+1}\\ \quad=\langle T_{l-1,l}^U\rangle\left(I-\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\right)^{-1}\langle T_{l,l+1}^U\rangle\hat{\overline{\Phi}}^{l+1},\\ \hat{\underline{\Phi}}^{l}=\langle R_{l,l+1}^U\rangle\hat{\overline{\Phi}}^{l+1}+\langle T_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\left\{I+\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle+\left(\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\right)^2+\left(\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\right)^3+\cdots\right\}T_{l,l+1}^U\hat{\overline{\Phi}}^{l+1}\\ \quad=\left\{\langle R_{l,l+1}^U\rangle+\langle T_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\left(I-\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\right)^{-1}\langle T_{l,l+1}^U\rangle\right\}\hat{\overline{\Phi}}^{l+1}.\end{cases} \qquad (14)$$

A conventional expression is used for the ascending polynomial series in the last member. This gives the transmission and reflection operator as follows.

$$\langle T_{l-1,l+1}^U\rangle=\langle T_{l-1,l}^U\rangle\left(I-\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\right)^{-1}\langle T_{l,l+1}^U\rangle,\ \langle R_{l-1,l+1}^U\rangle=\langle R_{l,l+1}^U\rangle+\langle T_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\left(I-\langle R_{l,l+1}^D\rangle\langle R_{l-1,l}^U\rangle\right)^{-1}\langle T_{l,l+1}^U\rangle. \qquad (15)$$

Down-going incident wave

Define, the transmission operator for the down-going incident wave in (l-1)-th layer and the down-going wave in (l+1)-th layer, and the reflection operator for the incident wave and the reflected wave in (l-1)-layer (Figure 2).

$$\underline{\hat{\Phi}}^{l+1}=\left\langle \hat{T}^{D}_{l-1,l+1}\right\rangle \underline{\hat{\Phi}}^{l-1},\ \overline{\hat{\Phi}}^{l-1}=\left\langle \hat{R}^{D}_{l-1,l+1}\right\rangle \underline{\hat{\Phi}}^{l-1}.$$

Consideration on the multiple reflection in (l)-th layer gives the following relation.

$$\begin{cases}\overline{\hat{\Phi}}^{l-1}=\left\langle R^{D}_{l-1,l}\right\rangle \underline{\hat{\Phi}}^{l-1}+\left\langle T^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\left\{I+\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle+\left(\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\right)^{2}+\left(\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\right)^{3}+\cdots\right\}\left\langle T^{D}_{l-1,l}\right\rangle \underline{\hat{\Phi}}^{l-1}\\ \quad=\left\{\left\langle R^{D}_{l-1,l}\right\rangle+\left\langle T^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\left(I-\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\right)^{-1}\left\langle T^{D}_{l-1,l}\right\rangle\right\}\underline{\hat{\Phi}}^{l-1},\\ \underline{\hat{\Phi}}^{l+1}=\left\langle T^{D}_{l,l+1}\right\rangle\left\{I+\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle+\left(\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\right)^{2}+\left(\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\right)^{3}+\cdots\right\}\left\langle T^{D}_{l-1,l}\right\rangle \underline{\hat{\Phi}}^{l-1}\\ \quad=\left\langle T^{D}_{l,l+1}\right\rangle\left(I-\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\right)^{-1}\left\langle T^{D}_{l-1,l}\right\rangle \underline{\hat{\Phi}}^{l-1}.\end{cases} \tag{16}$$

A conventional expression is used for the ascending polynomial series in the last member. Therefore,

$$\begin{cases}\left\langle R^{D}_{l-1,l+1}\right\rangle=\left\{\left\langle R^{D}_{l-1,l}\right\rangle+\left\langle T^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\left(I-\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\right)^{-1}\left\langle T^{D}_{l-1,l}\right\rangle\right\}\\ \left\langle T^{D}_{l-1,l+1}\right\rangle=\left\langle T^{D}_{l,l+1}\right\rangle\left(I-\left\langle R^{U}_{l-1,l}\right\rangle\left\langle R^{D}_{l,l+1}\right\rangle\right)^{-1}\left\langle T^{D}_{l-1,l}\right\rangle,\end{cases} \tag{17}$$

Note that Eqs. (15) and (17) have similar structure as those of Kennett (1983).

Transmission and reflection operator for stratified media

Repetition in similar way as described above lets us obtain the transmission and reflection coefficients for irregularly stratified media between the shallowest interface and the deepest (L)-th interface. For upcoming incident wave, $\overline{\hat{\Phi}}^{1}=\left\langle T^{U}_{1,L}\right\rangle \overline{\hat{\Phi}}^{L+1}$, $\underline{\hat{\Phi}}^{L}=\left\langle R^{U}_{1,L}\right\rangle \overline{\hat{\Phi}}^{L+1}$, and for down-going incident wave, $\overline{\hat{\Phi}}^{1}=\left\langle R^{D}_{1,L}\right\rangle \underline{\hat{\Phi}}^{0}$, $\underline{\hat{\Phi}}^{L}=\left\langle T^{D}_{1,L}\right\rangle \underline{\hat{\Phi}}^{0}$, where the distributed force along the (L+1)-th interface that does not exist $\overline{\hat{\Phi}}^{L+1}$ denotes the force equivalent to the real source located in (L+1)-layer. This means that the displacement and traction along the lower face of the deepest interface given by $\overline{\hat{\Phi}}^{L+1}$ is exactly same as those given by the real source. If this condition is fulfilled, the distributed force along the upper face of the deepest interface $\overline{\hat{\Phi}}^{L}$ caused by $\overline{\hat{\Phi}}^{L+1}$ is exactly same as that given by the real source. This distributed force is obtained by solving the following integral equations expressed by the integral operators shown below.

$$\begin{bmatrix}{}^{0}u^{L+1}_{L}\\ {}^{0}t^{L+1}_{L}\end{bmatrix}+\left\langle\begin{matrix}G^{L+1}_{L,L}\\ H^{L+1}_{L,L}\end{matrix}\right\rangle\underline{\hat{\Phi}}^{L}=\left\langle\begin{matrix}G^{L}_{L,L}\\ H^{L}_{L,L}\end{matrix}\right\rangle\overline{\hat{\Phi}}^{L}. \tag{18}$$

The similar procedure as above based on the approximation $\overline{\Phi}^{L}(\xi)\cong\overline{\Psi}^{L}(\xi),\underline{\Phi}^{L}(\xi)\cong\underline{\Psi}^{L}(\xi),\ \xi\in S^{L}_{e}$, gives

$$\begin{cases}{}^{0}u^{L+1}_{L}(x;x_s)+\underline{u}^{L+1}_{L}(x;x_s)-\int_{S^{L}_{L}}G^{L+1}_{L,L}(x;\xi)\underline{\Psi}^{L}(\xi)d\xi+\int_{S^{L}_{i}}G^{L+1}_{L,L}(x;\xi)\underline{\Phi}^{L}(\xi)d\xi\\ =\int_{S^{L}_{L}}G^{L}_{L,L}(x;\xi)\overline{\Phi}^{L}(\xi)d\xi+\overline{u}^{L}_{L}(x;x_s)-\int_{S^{L}_{L}}G^{L}_{L,L}(x;\xi)\overline{\Psi}^{L}(\xi)d\xi,\\ {}^{0}t^{L+1}_{L}(x;x_s)+\underline{t}^{L+1}_{L}(x;x_s)-\int_{S^{L}_{L}}H^{L+1}_{L,L}(x;\xi)\underline{\Psi}^{L}(\xi)d\xi+\int_{S^{L}_{i}}H^{L+1}_{L,L}(x;\xi)\underline{\Phi}^{L}(\xi)d\xi\\ =\int_{S^{L}_{L}}H^{L}_{L,L}(x;\xi)\overline{\Phi}^{L}(\xi)d\xi+\overline{t}^{L}_{L}(x;x_s)-\int_{S^{L}_{L}}H^{L}_{L,L}(x;\xi)\overline{\Psi}^{L}(\xi)d\xi,\end{cases}\quad for\ x\in S_{L}. \tag{19}$$

In discrete form,

$$\begin{bmatrix}{}^{0}u^{L+1}_{L}\\ {}^{0}t^{L+1}_{L}\end{bmatrix}+\left(\begin{bmatrix}\underline{u}^{L+1}_{L}\\ \underline{t}^{L+1}_{L}\end{bmatrix}-\begin{bmatrix}G^{L+1}_{L,L}\\ H^{L+1}_{L,L}\end{bmatrix}\underline{\Psi}^{L}+\begin{bmatrix}G^{L+1}_{L,L}\\ H^{L+1}_{L,L}\end{bmatrix}\underline{\Phi}^{L}\right)=\left(\begin{bmatrix}\overline{u}^{L}_{L}\\ \overline{t}^{L}_{L}\end{bmatrix}+\begin{bmatrix}G^{L}_{L,L}\\ H^{L}_{L,L}\end{bmatrix}\overline{\Phi}^{L}-\begin{bmatrix}G^{L}_{L,L}\\ H^{L}_{L,L}\end{bmatrix}\overline{\Psi}^{L}\right) \tag{20}$$

The first term denotes the contribution of real source along the deepest interface. Moving all known variables in the right member, the following simple formula is obtained.

$$\begin{bmatrix}G^{L}_{L,L} & -G^{L+1}_{L,L}\\ H^{L}_{L,L} & -H^{L+1}_{L,L}\end{bmatrix}\begin{bmatrix}\overline{\Phi}^{L}\\ \underline{\Phi}^{L}\end{bmatrix}=\begin{bmatrix}{}^{0}u^{L+1}_{L}\\ {}^{0}t^{L+1}_{L}\end{bmatrix}+\left(\begin{bmatrix}\underline{u}^{L+1}_{L}\\ \underline{t}^{L+1}_{L}\end{bmatrix}-\begin{bmatrix}G^{L+1}_{L,L}\\ H^{L+1}_{L,L}\end{bmatrix}\underline{\Psi}^{L}\right)-\left(\begin{bmatrix}\overline{u}^{L}_{L}\\ \overline{t}^{L}_{L}\end{bmatrix}-\begin{bmatrix}G^{L}_{L,L}\\ H^{L}_{L,L}\end{bmatrix}\overline{\Psi}^{L}\right) \tag{21}$$

By using this solution, the transmitted wave field can be numerically estimated as follows.

$$\left\langle \begin{matrix} G_{L,L}^{L} \\ H_{L,L}^{L} \end{matrix} \right\rangle \hat{\bar{\Phi}}^{L} = \left\langle \begin{matrix} G_{L,L}^{L} \\ H_{L,L}^{L} \end{matrix} \right\rangle \left\langle T_{L,L+1}^{U} \right\rangle \hat{\bar{\Phi}}^{L+1} \approx \begin{bmatrix} \bar{u}_{L}^{L} \\ \bar{t}_{l}^{L} \end{bmatrix} - \begin{bmatrix} G_{L,L}^{L} \\ H_{L,L}^{L} \end{bmatrix} \overline{\Psi}^{L} + \begin{bmatrix} G_{L,L}^{L} \\ H_{L,L}^{L} \end{bmatrix} \overline{\Phi}^{L}, \quad (22)$$

Free surface

For the shallowest layer, traction free condition along the surface must be fulfilled. The integral equation expressed by the integral operator is

$$\left\langle H_{0,0}^{1} \right\rangle \underline{\hat{\Phi}}^{0} + \left\langle H_{0,1}^{1} \right\rangle \hat{\bar{\Phi}}^{1} = 0 \quad for\ x \in S_{0}. \quad (23)$$

Define the reflection operator at the surface as $\underline{\hat{\Phi}}^{0} = \left\langle \hat{R}_{0,1}^{U} \right\rangle \hat{\bar{\Phi}}^{1}$.

The similar procedure as above based on the approximation $\overline{\Phi}^{1}(\xi) \cong \overline{\Psi}^{1}(\xi), \xi \in S_{e}^{1}, \quad \underline{\Phi}^{0}(\xi) \cong \underline{\Psi}^{0}(\xi), \quad \xi \in S_{e}^{0}$, gives

$$\begin{aligned} &\underline{t}_{0}^{1} + \int_{S_{i}^{0}} H_{0,0}^{1}(x;\xi) \underline{\Phi}^{0}(\xi) d\xi - \int_{S_{i}^{0}} H_{0,0}^{1}(x;\xi) \underline{\Psi}^{0}(\xi) d\xi + \\ &\qquad \bar{t}_{0}^{1} + \int_{S_{i}^{1}} H_{0,1}^{1}(x;\xi) \overline{\Phi}^{1}(\xi) d\xi - \int_{S_{i}^{1}} H_{0,1}^{1}(x;\xi) \overline{\Psi}^{1}(\xi) d\xi = 0 \quad for\ x \in S_{0}. \end{aligned} \quad (24)$$

Discretization of this integral equation gives

$$\left(\underline{t}_{0}^{1} + H_{0,0}^{1} \underline{\Phi}^{0} - H_{0,0}^{1} \underline{\Psi}^{0} \right) + \left(\bar{t}_{0}^{1} + H_{0,1}^{1} \overline{\Phi}^{1} - H_{0,1}^{1} \overline{\Psi}^{1} \right) = 0. \quad (25)$$

Moving all known variables to the right member, the following simple formula is obtained.

$$H_{0,0}^{1} \underline{\Phi}^{0} = -\left(\bar{t}_{0}^{1} + H_{0,1}^{1} \overline{\Phi}^{1} - H_{0,1}^{1} \overline{\Psi}^{1} \right) - \left(\underline{t}_{0}^{1} - H_{0,0}^{1} \underline{\Psi}^{0} \right). \quad (26)$$

By using this solution, the wave field reflected at the surface can be numerically estimated as follows.

$$\left\langle \begin{matrix} G_{0,0}^{1} \\ H_{0,0}^{1} \end{matrix} \right\rangle \underline{\hat{\Phi}}^{0} = \left\langle \begin{matrix} G_{0,0}^{1} \\ H_{0,0}^{1} \end{matrix} \right\rangle \left\langle R_{0,1}^{U} \right\rangle \hat{\bar{\Phi}}^{1} \approx \begin{bmatrix} \underline{u}_{0}^{1} \\ \underline{t}_{0}^{1} \end{bmatrix} - \begin{bmatrix} G_{0,0}^{1} \\ H_{0,0}^{1} \end{bmatrix} \underline{\Psi}^{0} + \begin{bmatrix} G_{0,0}^{1} \\ H_{0,0}^{0+1} \end{bmatrix} \underline{\Phi}^{0}. \quad (27)$$

Displacement at the surface

The followings are derived for the irregularly stratified media having free surface for up-going incident wave.

$$\begin{aligned} \hat{\bar{\Phi}}^{1} &= \left\langle T_{1,L+1}^{U} \right\rangle \hat{\bar{\Phi}}^{L+1} + \left\langle R_{1,L+1}^{D} \right\rangle \left\langle R_{0,1}^{U} \right\rangle \left\langle T_{1,L+1}^{U} \right\rangle \hat{\bar{\Phi}}^{L+1} + \left\langle R_{1,L+1}^{D} \right\rangle \left\langle R_{0,1}^{U} \right\rangle \left\langle R_{1,L+1}^{D} \right\rangle \left\langle R_{0,1}^{U} \right\rangle \left\langle T_{1,L+1}^{U} \right\rangle \hat{\bar{\Phi}}^{L+1} + \cdots \\ &= \left(I - \left\langle R_{1,L+1}^{D} \right\rangle \left\langle R_{0,1}^{U} \right\rangle \right)^{-1} \left\langle T_{1,L+1}^{U} \right\rangle \hat{\bar{\Phi}}^{L+1}. \end{aligned} \quad (28)$$

The last change is a conventional expression just to make the formula shorter. There is not any real boundary at the depth of the real source, then, $\left\langle R_{L,L+1}^{D} \right\rangle \equiv \langle 0 \rangle$. This means that the application of this operator to any distributed force vector gives a vector of zero. Therefore,

$$\begin{aligned} \left\langle T_{1,L+1}^{U} \right\rangle &= \left\langle T_{1,L}^{U} \right\rangle \left(I - \left\langle R_{L,L+1}^{D} \right\rangle \left\langle R_{0,1}^{U} \right\rangle \right)^{-1} \left\langle T_{L,L+1}^{U} \right\rangle = \left\langle T_{1,L}^{U} \right\rangle \left\langle T_{L,L+1}^{U} \right\rangle, \\ \left\langle T_{1,L+1}^{D} \right\rangle &= \left\langle T_{L,L+1}^{D} \right\rangle \left(I - \left\langle R_{1,L}^{U} \right\rangle \left\langle R_{L,L+1}^{D} \right\rangle \right)^{-1} \left\langle T_{1,L}^{D} \right\rangle = \left\langle T_{L,L+1}^{D} \right\rangle \left\langle T_{1,L}^{D} \right\rangle, \\ \left\langle R_{1,L+1}^{D} \right\rangle &= \left\langle R_{1,L}^{D} \right\rangle + \left\langle T_{1,L}^{U} \right\rangle \left\langle R_{L,L+1}^{D} \right\rangle \left(I - \left\langle R_{1,L}^{U} \right\rangle \left\langle R_{L,L+1}^{D} \right\rangle \right)^{-1} \left\langle T_{1,L}^{D} \right\rangle = \left\langle R_{1,L}^{D} \right\rangle. \end{aligned} \quad (29)$$

and

$$\hat{\bar{\Phi}}^{1} = \left(I - \left\langle R_{1,L}^{D} \right\rangle \left\langle R_{0,1}^{U} \right\rangle \right)^{-1} \left\langle T_{1,L}^{U} \right\rangle \left(\left\langle T_{L,L+1}^{U} \right\rangle \hat{\bar{\Phi}}^{L+1} \right) = \left(I - \left\langle R_{1,L}^{D} \right\rangle \left\langle R_{0,1}^{U} \right\rangle \right)^{-1} \left\langle T_{1,L}^{U} \right\rangle \hat{\bar{\Phi}}^{L}. \quad (30)$$

The reflection at the surface is summed by using the reflection operator $\left\langle R_{0,1}^{U} \right\rangle$,

$$\begin{aligned} u_{0}^{1} &= \left\langle G_{0,0}^{1} \right\rangle \underline{\hat{\Phi}}^{0} + \left\langle G_{0,1}^{1} \right\rangle \hat{\bar{\Phi}}^{1} = \left(\left\langle G_{0,0}^{1} \right\rangle \left\langle R_{0,1}^{U} \right\rangle + \left\langle G_{0,1}^{1} \right\rangle \right) \hat{\bar{\Phi}}^{1} \\ &= \left(\left\langle G_{0,0}^{1} \right\rangle \left\langle R_{0,1}^{U} \right\rangle + \left\langle G_{0,1}^{1} \right\rangle \right) \left(I - \left\langle R_{1,L}^{D} \right\rangle \left\langle R_{0,1}^{U} \right\rangle \right)^{-1} \left\langle T_{1,L}^{U} \right\rangle \hat{\bar{\Phi}}^{L}. \end{aligned} \quad (31)$$

Approximation

Eq. (31) shows the way to obtain the full wave field in irregularly stratified media. This, however, includes infinite sums of ascending polynomials and such complicated calculation as shown by Eqs. (15) and (17). One of approximations that have to be taken is the truncation of the ascending polynomials. More approximations are used in order to make calculation easier depending on the target for each problem.

Waves directly coming from the source

If only the waves directly coming from the real source to the surface is required, Eq. (30) can be simplified more. The reflection operators for every interface are forced to be zero. $\left\langle R^U_{l,l+1}\right\rangle \equiv \langle 0\rangle \cdot \left\langle R^D_{l,l+1}\right\rangle \equiv \langle 0\rangle$. Then,

$\left\langle T^D_{l-1,l+1}\right\rangle = \left\langle T^D_{l,l+1}\right\rangle\left\langle T^D_{l-1,l}\right\rangle,\ \left\langle T^U_{l-1,l+1}\right\rangle = \left\langle T^U_{l-1,l}\right\rangle\left\langle T^U_{l,l+1}\right\rangle,$

therefore,

$$u_0^1 = \left(\left\langle G^1_{0,0}\right\rangle\left\langle \hat{R}^U_{0,1}\right\rangle + \left\langle G^1_{0,1}\right\rangle\right)\left\langle T^U_{1,L}\right\rangle\hat{\hat{\Phi}}^L = \left(\left\langle G^1_{0,0}\right\rangle\left\langle \hat{R}^U_{0,1}\right\rangle + \left\langle G^1_{0,1}\right\rangle\right)\left(\prod_{l=1}^{L-1}\left\langle T^U_{l,l+1}\right\rangle\right)\hat{\hat{\Phi}}^L. \tag{32}$$

We can start the computation from Eq. (21) and Eq, (22). The iterative application of the transmission operator can be performed by solving Eq. (7) and calculate the wave field by Eq. (8).

Multiple reflection in homogeneous basin

Consider a homogeneous basin staying on a homogeneous basement of different material property. Eq.(31) can be reduced as follows.

$$\begin{aligned} u_0^1 &= \left(\left\langle G^1_{0,0}\right\rangle\left\langle R^U_{0,1}\right\rangle + \left\langle G^1_{0,1}\right\rangle\right)\left(I - \left\langle R^D_{1,2}\right\rangle\left\langle R^U_{0,1}\right\rangle\right)^{-1}\hat{\hat{\Phi}}^1 \\ &= \left(\left\langle G^1_{0,0}\right\rangle\left\langle R^U_{0,1}\right\rangle + \left\langle G^1_{0,1}\right\rangle\right)\left\{I + \left\langle R^D_{1,2}\right\rangle\left\langle R^U_{0,1}\right\rangle + \left(\left\langle R^D_{1,2}\right\rangle\left\langle R^U_{0,1}\right\rangle\right)^2 + \left(\left\langle R^D_{1,2}\right\rangle\left\langle R^U_{0,1}\right\rangle\right)^3 + \cdots\right\}\hat{\hat{\Phi}}^1. \end{aligned} \tag{33}$$

The operation of $\left\langle R^D_{1,2}\right\rangle$ is performed by Eqs. (7) and (8), that of $\left\langle R^U_{0,1}\right\rangle$ by Eqs. (26) and (27). A numerical example is shown in Figure 3 for the problem solved by Dravinski and Mossessian (1987).

FURTHER APPLICATION AND DEVELOPMENT

As shown above, the formulation introduced here makes easier the search for a good and efficient approximation of wave field with a clear theoretical back ground of wave propagation. The computation process is composed of products of Green's function matrix to force vector, sums among displacement and traction vectors and simultaneous linear equations. The last one can be solved by using two formers in the iterative methods. Therefore, there is not any operation among matrices in computation based on the method presented here. This feature makes computation faster and required main memory smaller. Moreover, it is expected that this allow us to apply Fast Multipole Method into the calculation. Then, it is expected that the calculation can be accelerated much more than the present performance.

REFERENCES

Dravinski, M. & T. K. Mossessian, 1987, "Scattering of plane harmonic P, SV, and Rayleigh waves by dipping layers of arbitrary shape.", *Bull. Seism. Soc. Am.*, **77**, 212-235.

Fujiwara, H & H. Takenaka, 1993, ”Calculation of elastic wave field by the BEM with the full-space Green’s function”, *Proceedings of the 88th SEGJ Conference*, 157-160.

Kennett, B. L. N., 1983, *Seismic Wave Propagation in Stratified Media.*, Cambridge Monographs on Mechanics and Applied Mathematics, Cambridge University Press.

Sanchez-Sesma, F. J. & M.Campillo, 1991, ”Diffraction of P, SV and Rayleigh waves by topographic features: a boundary integral formulation”, *Bull. Seism. Soc. Am.*, **81**, 2234-2253.

Takenaka, H. & H. Fujiwara, 1994, "Reflectivity method in the space domain based on the boundary element method -I. Theory of R/T-matrix approach", *Abstracts for Japan Earth and Planetary Science Joint Meeting*, 289.

Yokoi, T. & H. Takenaka, 1995, ”Treatment of an infinitely extended free surface for indirect formulation of the Boundary Element Method”, *J. Phys. Earth*, **43**, 79-103.

Yokoi, T., 1996, ”An Indirect Boundary Element Method based on recursive matrix operation to compute waves in irregularly stratified media with infinitely extended interfaces”, *J. Phys. Earth*, **44**, 39-60.

Yokoi, T. & F. J. Sanchez-Sesma, 1998, ”A hybrid calculation technique of the Indirect Boundary Element Method and the analytical solution for three-dimensional problems of topography”, *Geophys. J. Int.*, **133**, 121-139.

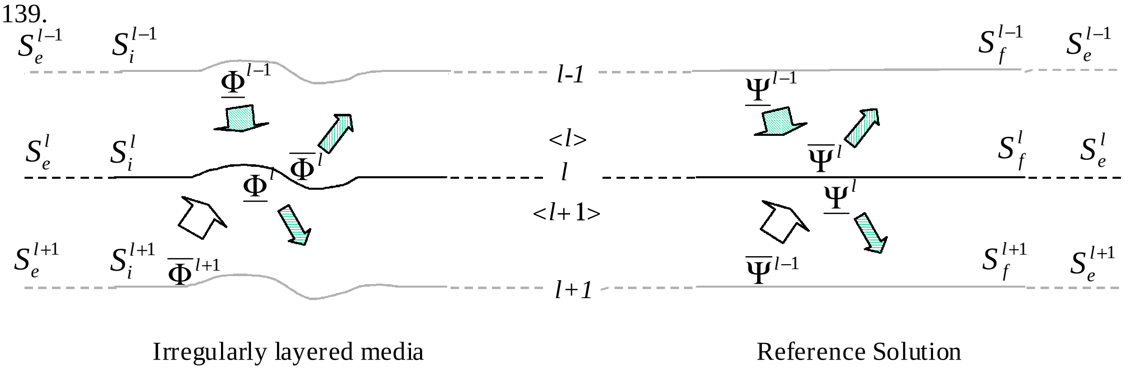

Figure 1: Configuration for the wave field in irregularly stratified media (*left*) and its reference solution, i.e., the wave field in the corresponding horizontally stratified media (*right*). The reference solution is calculated beforehand by the analytical solution in the wavenumber domain. The reflection and transmission at *l*-th interface are discussed by this configuration.

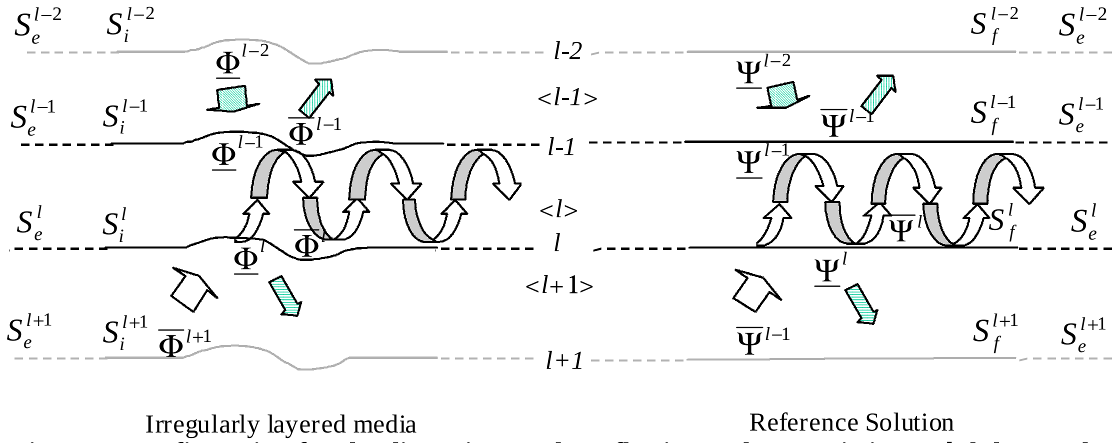

Figure 2: Configuration for the discussion on the reflection and transmission at *l*-th layer. The multiple reflection in *l*-th layer drawn by curved arrows are taken into account for the derivation of Eqs. (14) and (16).

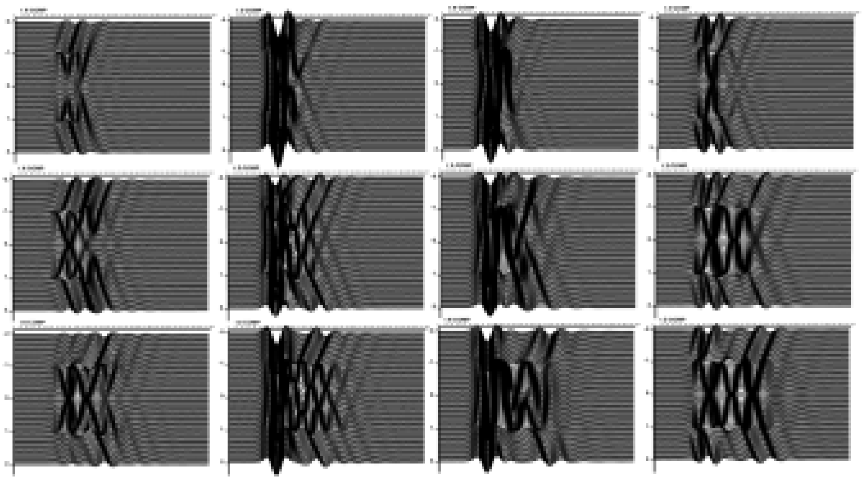

Figure 3: Examples of wave field calculated the method presented in this paper. These are the paste up of wave forms for the problem of Dravinski & Mossessian (1987). The columns show the horizontal and vertical components due to vertical P-wave incidence, and those due to SV-wave, respectively. (*Top raw*) shows the directly coming wave plus the contribution of the surface, (*middle raw*) these plus waves once bounced, (*bottom raw*) these plus waves twice bounced.