# EFFECTS OF RAYLEIGH AND LOVE WAVES ON MICROTREMOR H/V SPECTRA

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## SUMMARY

In order to simulate the horizontal-to-vertical (H/V) spectral ratios of microtremors, analytical formulas are presented for computing the H/V ratios of Rayleigh and surface waves propagating on a layered half-space. Assuming that Rayleigh to Love wave amplitude ratio in horizontal motions is 0.7, the H/V spectra of Rayleigh and surface waves are computed at four sites, where the PS log data are available. The comparison among the H/V spectra of microtremors and Rayleigh and surface waves at the sites leads to the following conclusions: (1) The higher modes of Rayleigh waves have significant effects on the H/V spectra of microtremors in the frequency range where response functions of higher modes are predominant; (2) The H/V ratios of Rayleigh waves considering higher modes are in good agreement with those of microtremors, although the H/V values of Rayleigh waves are less than those of microtremors; (3) The H/V ratios of surface waves are in fairly good agreement with those of microtremors; and (4) $V_S$ profiles of subsurface soils may be estimated from the H/V ratios of microtremors using the proposed formulas.

## INTRODUCTION

One of the convenient methods for estimating S-wave velocity ($V_S$) profile of subsurface soils is to make use of the characteristics of surface waves (Rayleigh and Love waves) in microtremors that can readily be observed on the ground surface without drilling any borehole. Recent studies, for example, have shown that (1) microtremor consists mainly of surface waves, (2) Frequency-wave number (F-k) and Spatial Auto-correlation (SAC) spectral analyses of microtremor records measured with arrays of sensors can yield dispersion characteristics of Rayleigh waves, and that (3) the horizontal-to-vertical (H/V) spectral ratio of microtremors at a site, that can be observed with a three-component sensor, is stable over a day and may reflect the soil profile of the site. Many studies indicated that the inverse analysis of the dispersion data at a site successfully results in a $V_S$ profile [e.g., Horike, 1985; Okada and Matsushima, 1986; Tokimatsu *et al*., 1992]. On the other hand, very few theoretical studies have been made to investigate a possibility using the H/V ratio of microtremors.

Nakamura and Ueno (1986) suggested that the H/V ratio of microtremors at a site corresponds roughly to the amplification factor of subsurface soils for vertically incident S-wave, assuming that microtremor consists mainly of body wave (P- and S- wave). On the other hand, Tokimatsu and Miyadera (1992), for example, indicated that the variation with frequency of the microtremor H/V ratio corresponds to that of fundamental Rayleigh mode. However, the H/V values of microtremors are not comparable with those of fundamental Rayleigh mode. This disagreement may be caused by the influence of higher Rayleigh modes and Love waves in microtremors [e.g., Tokimatsu and Tamura, 1994; Lachet and Bard, 1994]. In order to investigate a possibility of $V_S$ profiling based on conventional microtremor measurements with one station, the theoretical formulas using both Rayleigh and Love waves for simulating the H/V ratios of microtremors are required.

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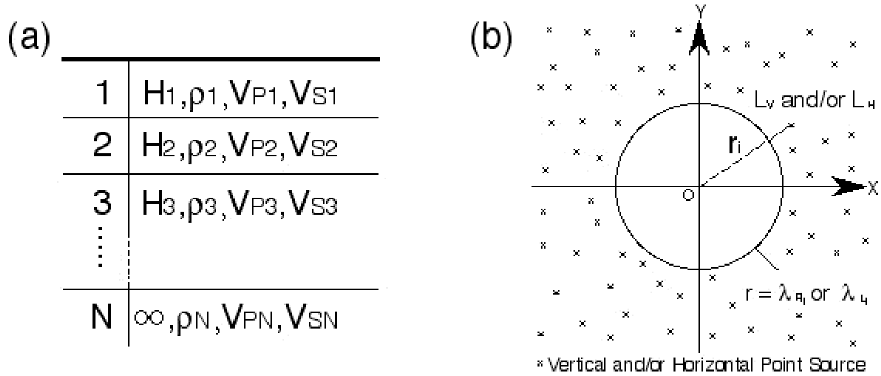


**Figure 1: Geometry of (a) soil layer and (b) source models**

The objects of this paper are to formulate H/V spectra of Rayleigh and surface waves for simulating those of microtremors and to investigate the effects of higher Rayleigh modes and Love waves on the H/V spectra of microtremors.

### H/V SPECTRA OF RAYLEIGH AND SURFACE WAVES

The soil model is assumed to be a semi-infinite elastic medium made up of N parallel, solid, homogeneous, iso-tropic layers (Figure 1(a)). Each layer is characterized by thickness, H, mass density, ρ, P-wave velocity, $V_P$, and S-wave velocity, $V_S$. The origin of an orthogonal coordinate system is placed on the free surface (Figure 1(b)). To model sources of microtremors, it is assumed that the Fourier time transformed vertical and hori-zontal point forces, $L_V(f)$ and $L_H(f)$, are randomly distributed on the free surface [e.g., Lachet and Bard, 1994]. At the origin, vertical and two orthogonal horizontal displacements induced by the sources are observed. In this study, the amplitude of horizontal motion at the origin is defined as square root of sum of squares of the two orthogonal horizontal displacements.

Surface and body waves are generated from each source and propagate on the medium. Based on the studies by Harvey (1981) and Tokimatsu and Tamura (1995), the displacements at the origin may be expressed by only surface waves under the following conditions: (1) each distance between the origin and source is longer than a wavelength of Rayleigh or Love wave ($\lambda_{Rj}$ or $\lambda_{Lj}$; Figure 1(b)), and (2) the effective periods are less than natural site period. Under these conditions, the vertical and horizontal power of the *j* th Rayleigh mode from the *i* th vertical point source may be expressed by Harkrider (1964):

$$P^V_{VRij} = L_V^2 A_{Rj}^2 |H_0^{(2)}(k_{Rj} r_i)|^2 exp(-2hk_{Rj} r_i) \tag{1}$$

$$P^V_{HRij} = L_V^2 A_{Rj}^2 (u/w)_j^2 |H_1^{(2)}(k_{Rj} r_i)|^2 exp(-2hk_{Rj} r_i) \tag{2}$$

in which *A* is medium response factor [Harkrider, 1964], *k* is wave number, *u/w* is H/V ratio of Rayleigh mode at the free surface, subscript *R* presents Rayleigh wave, *r* is distance between the origin and the source, $H_n^{(2)}()$ is Hankel function of the second kind of the order *n*, and *h* is damping ratio of soil. Similarly, the vertical and horizontal power of the *j* th Rayleigh mode from the *i* th horizontal point source may be expressed by

$$P^H_{VRij} = (L_H^2/2)\, A_{Rj}^2 (u/w)_j^2 |H_1^{(2)}(k_{Rj} r_i)|^2 exp(-2hk_{Rj} r_i) \tag{3}$$

$$P^H_{HRij} = (L_H^2/2)\, A_{Rj}^2 (u/w)_j^4 |H_0^{(2)}(k_{Rj} r_i)|^2 exp(-2hk_{Rj} r_i) \tag{4}$$

The horizontal power of the *j* th Love mode from the *i* th horizontal point source may be

$$P^H_{HLij} = (L_H^2/2) A_{Lj}^2 |H_0^{(2)}(k_{Lj} r_i)|^2 exp(-2hk_{Lj} r_i) \tag{5}$$

in which subscript *L* presents Love wave. Assuming the statistical independence among the loading phases of all sources, the vertical and horizontal relative powers of all waves at the origin are given by integrating Eqs. (1)-(5) for all sources and modes. The results of the integration are

$$P_{VS} = P_{VR} = \Sigma (A_{Rj}/k_{Rj})^2 \{1 + (\alpha^2/2)(u/w)_j^2\} \tag{6}$$

$j$

$$P_{HS} = P_{HR} + P_{HL}, \tag{7}$$

$$P_{HR} = \sum_j (A_{Rj}/k_{Rj})^2 (u/w)_j^2 \{1+(\alpha^2/2)(u/w)_j^2\}, \quad P_{HL} = \sum_j (\alpha^2/2)(A_{Lj}/k_{Lj})^2$$

where $\alpha$ is the H/V ratio of loading force, $L_H/L_V$. Using Eqs. (6) and (7), the H/V ratio of surface waves, $(H/V)_S$, that of Rayleigh waves, $(H/V)_R$, and Rayleigh to Love wave amplitude ratio in horizontal motions, $R/L$, are presented as follows:

$$(H/V)_S = (P_{HS} / P_{VS})^{1/2} \tag{8}$$

$$(H/V)_R = (P_{HR} / P_{VR})^{1/2} \tag{9}$$

$$R/L = (P_{HR} / P_{HL})^{1/2} \tag{10}$$

Eqs. (8)-(10) are formulated in three-dimensional wave propagating field. In two-dimensional wave field, $(H/V)_S$, $(H/V)_R$, and $R/L$ can also be formulated by Eqs. (8)-(10), because the relative amplitudes of $j$ th Rayleigh and Love modes in Eqs. (1)-(5) are only replaced by $A_{Rj}/k_{Rj}$ and $A_{Lj}/k_{Lj}$, respectively [Regan and Harkrider, 1980; Hisada *et al.*, 1991].

## ESTIMATION OF RAYLEIGH TO LOVE WAVE AMPLITUDE RATIO

In computing $(H/V)_S$ and $(H/V)_R$ using Eqs. (8)-(10), the values of $\alpha$ or $R/L$ are required in addition to the soil model. In this study, the value of $R/L$ in horizontal motions is estimated from microtremor array data.

### Dispersion Characteristics of Microtremor Horizontal Motions

F-k spectral analyses of vertical, radial, and transverse motions [e.g., Tokimatsu, 1995] and three-dimensional SAC analyses [Matsushima and Okada, 1990] are conducted for the microtremor array records at four sites in Japan, hereby called Sites A, B, C, and D. Site A is Yumenoshima in Tokyo, and Site B is Rokko Island in Kobe city, and Sites C and D are Asahi and Kotobuki elementary schools in Kushiro city. The details of the array measurements can be found elsewhere [Tokimatsu, 1995; Tokimatsu *et al.*, 1997].

Figures 2(a) and (b) show the variation with period of the propagating azimuths of vertical, radial, and transverse motions determined by F-k spectral analyses of the array records at sites A and B. In each site, the azimuths of the three motions at a frequency coincide with each other. Figures 3 and 4 summarize the dispersion characteristics of radial and transverse motions determined by F-k and SAC analyses at sites A and B. Also shown in the figures in lines are the dispersion curves of horizontal Rayleigh and Love waves computed theoretically from the soil profile at the sites. The effects of higher modes are taken into account in the analyses [Tokimatsu *et al.*, 1992]. In each site, the dispersion curves of observed radial and transverse motions show good agreement with those of theoretical Rayleigh and Love waves, respectively. These trends are also confirmed at sites C and D. Tokimatsu (1995) indicated that the characteristics of Rayleigh and Love waves could be derived from radial and

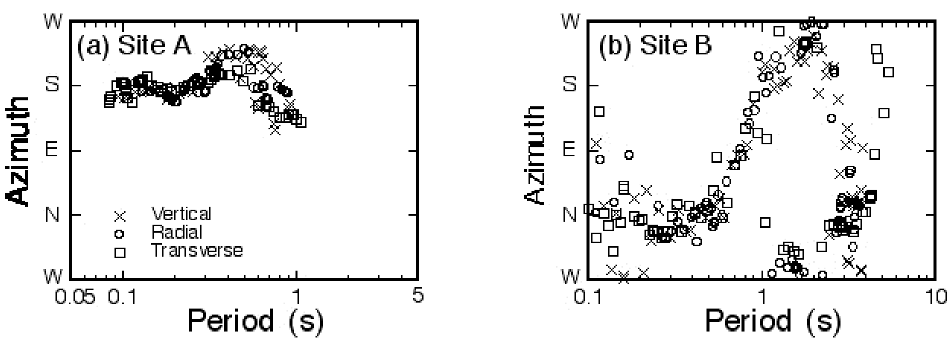


**Figure 2: Variation with period of propagating azimuths of microtremor vertical, radial, and transverse motions at sites A and B**

transverse horizontal motions at sites A-D.

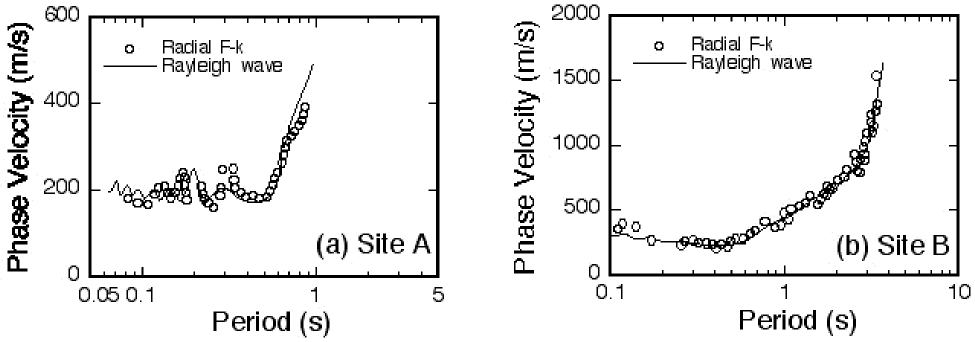


**Figure 3: Dispersion curves of microtremor radial motions compared with those of Rayleigh waves at sites A and B**

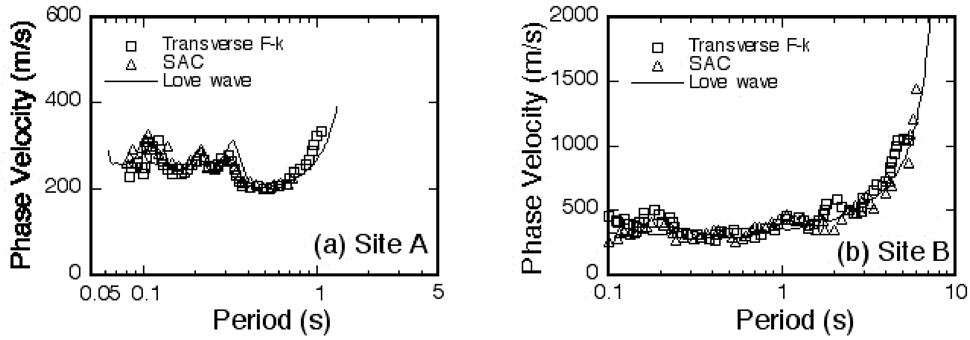


**Figure 4: Dispersion curves of microtremor transverse motions compared with those of Love waves at sites A and B**

**Rayleigh to Love Wave Amplitude Ratio in Microtremor Horizotal Motions**

Based on the propagating azimuths and phase velocities of microtremors shown in Figures 2-4, beam-formed radial and transverse horizontal motions at each period are made using the observed array data, and the spectral amplitude ratio between the two motions, $(R/L)_{FK}$, are determined. The spectral ratio between the two motions, $(R/L)_{SAC}$, are also evaluated from three-dimensional SAC analyses [Matsushima and Okada, 1990].

Figures 5(a) and (b) show the variation with frequency of $(R/L)_{FK}$ and $(R/L)_{SAC}$ estimated every 3 hours over a 24-hour period at site A. The amplitudes of both $(R/L)_{FK}$ and $(R/L)_{SAC}$ are stable over a day, and are very similar. Figure 6 shows the $(R/L)_{FK}$ estimated at sites A-D. The amplitudes of $(R/L)_{FK}$ at all sites are 0.4-1 in a period range of 0.1-5 s. The estimated values are comparable with those of $(R/L)_{SAC}$ estimated at Hachinohe city, Japan, by Matsushima and Okada (1990). Thus, in this study, it is assumed that the $R/L$ value for computing $(H/V)_S$ and $(H/V)_R$ is 0.7 at any frequency.

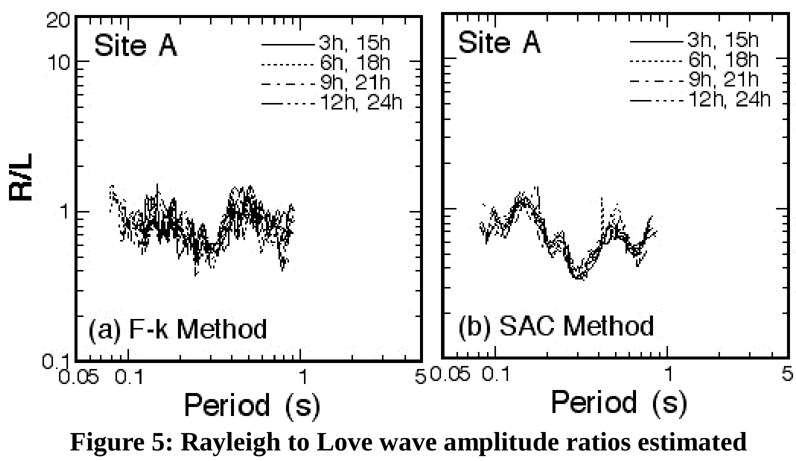


**Figure 5: Rayleigh to Love wave amplitude ratios estimated by (a) F-k and (b) SAC method at site A**

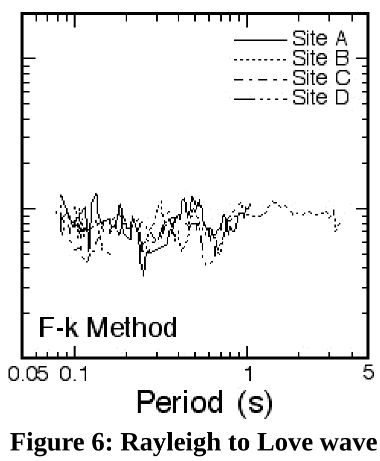


**Figure 6: Rayleigh to Love wave amplitude ratios at sites A-D**

## COMPARISON OF THEORETICAL AND MICROTREMOR H/V RATIOS

To examine the applicability and limitation of the proposed Eqs. (8)-(10) to the H/V ratios of microtremors, three-component microtremor data observed at four sites nearby Tokyo, hereby called Sites A, E, F, and G, are used in this study. The details of the observations can be found elsewhere [e.g., Tokimatsu, 1995]. The shallow $V_S$ profiles from down-hole method at the sites are shown in Figure 7 [Ishihara *et al.*, 1989; Tokimatsu, 1995]. Tables 1 and 2 show the deep soil profiles at the sites modeled by the results of other geophysical inves-tigations [Shima *et al.*, 1976; Higashi and Kudo, 1992].

### Definition of H/V Spectrum

In the past studies by authors, the microtremor H/V spectral ratio, $(H/V)_{mR}$, was defined as

$$(H/V)_{mR} = (S_{NS}\,S_{EW})^{1/2}/\,S_{UD} \qquad (11)$$

where $S_{UD}$ is the Fourier amplitude of vertical motion, and $S_{NS}$ and $S_{EW}$ are those of the two orthogonal horizontal motions. In this definition, $(S_{NS}S_{EW})^{1/2}$ was assumed to correspond to the amplitude of Rayleigh waves. However, this definition can not be equivalent to the theoretical $(H/V)_R$ in Eqs. (9). Thus, the microtremor H/V spectral ratio, $(H/V)_{mS}$, which may be equivalent to $(H/V)_S$ in Eqs. (8), is defined as

$$(H/V)_{mS} = (S_{NS}^{\,2} + S_{EW}^{\,2})^{1/2}/\,S_{UD} \qquad (12)$$

### Effects of Rayleigh Waves on Mocrtremor H/V Spectrum

The microtremor H/V spectra $(H/V)_{mR}$ determined by Eq. (11) at sites A and G are shown in Figure 8 in crosses. The broken and solid lines in the figure are the H/V spectra of fundamental and superposed Rayleigh modes, $(H/V)_{R0}$ and $(H/V)_R$, respectively, computed for the soil profiles at the sites.

At site A, the variation with period of the computed H/V spectrum $(H/V)_{R0}$ is in good agreement with the observed one, $(H/V)_{mR}$. At site G, however, the computed spectrum $(H/V)_{R0}$ is inconsistent with the observed $(H/V)_{mR}$ in a period range of 0.5-3 s. Besides, the values of $(H/V)_{R0}$ are not comparable with those of the observed $(H/V)_{mR}$. On the other hand, the amplitude of $(H/V)_R$, considering the effects of higher modes, is in good agreement with the observed $(H/V)_{mR}$ at each site.

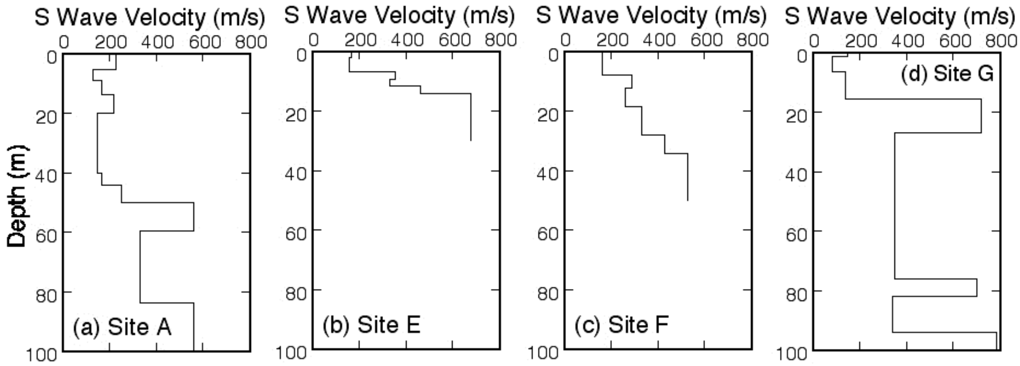


**Figure 7: Shallow $V_S$ profiles at sites A and E-G [(a)Ishihara *et al.*, 1989; (b)-(d)Tokimatsu, 1995]**

**Table 1: Deep soil profiles at sites A, E, and F [Shima *et al.*, 1976]**

| Depth(km) | ρ (t/m³) | Vp(km/s) | Vs(km/s) |
|---|---|---|---|
| 0.0 - 1.5 | 1.9 | 1.8 | 0.7 |
| 1.5 - 2.3 | 2.2 | 2.8 | 1.5 |
| 2.3 - | 2.5 | 5.6 | 3.0 |

**Table 2: Deep soil profile at site G [Higashi and Kudo, 1992]**

| Depth(km) | ρ (t/m³) | Vp(km/s) | Vs(km/s) |
|---|---|---|---|
| 0.0 - 0.3 | 2.0 | 2.3 | 0.8 |
| 0.3 - 2.0 | 2.3 | 3.0 | 1.5 |
| 2.0 - 3.2 | 2.5 | 4.2 | 2.4 |
| 3.2 - | 2.8 | 5.5 | 2.8 |

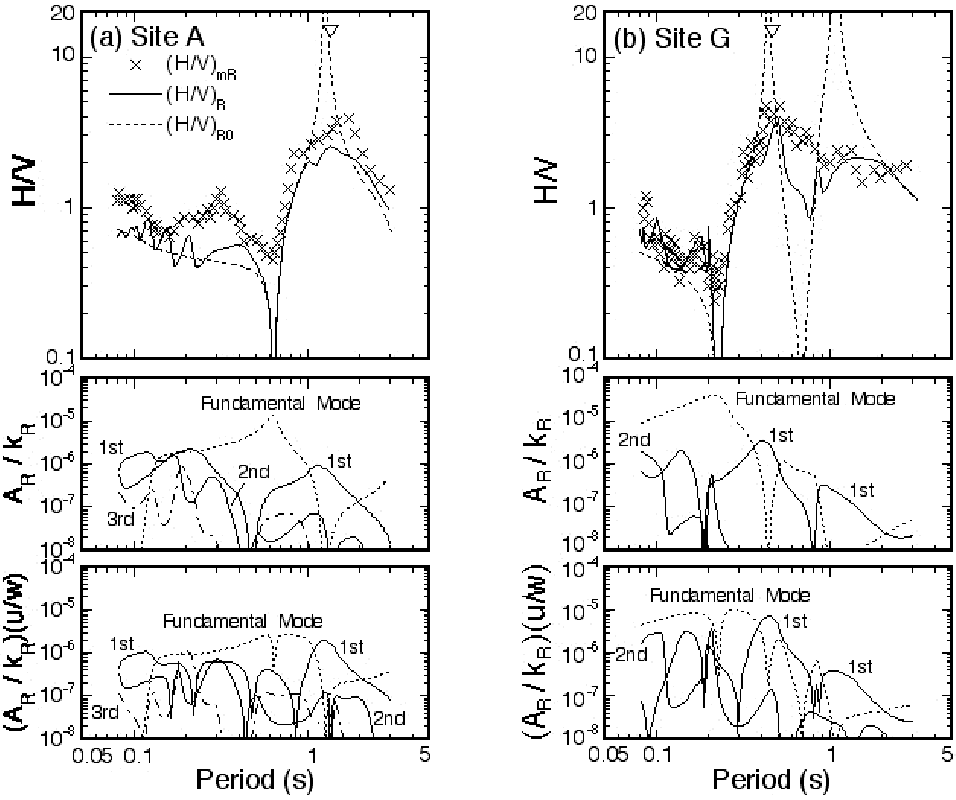


**Figure 8: The H/V ratios of microtremors compared with those of Rayleigh waves at sites A and G**

To investigate why the good agreement exists between the $(H/V)_R$ and $(H/V)_{mR}$, the vertical and horizontal response functions, $A_R/k_R$ and $(A_R/k_R)(u/w)$, up to 3rd higher Rayleigh-mode, are also shown in Figure 8 in lines. In the figure, the good agreements between the $(H/V)_R$ and $(H/V)_{mR}$ are found mainly in a period range of 1.2 s and over 0.5 s at sites A and G, respectively. In these period ranges, the values of both vertical and horizontal response functions of higher Rayleigh-modes are larger than those of fundamental mode.

**Effects of Love Waves on Mocrtremor H/V Spectrum**

In Figure 8, the values of $(H/V)_R$ are less than those of the observed $(H/V)_{mR}$. This misfit is considered to be due to the definition of the H/V ratio of microtremors, $(H/V)_{mR}$, in Eq. (11) as stated above. In Figure 9, therefore, the H/V spectra of microtremors, $(H/V)_{mS}$, determined by Eq. (12) at sites A and E-G are shown in open circles. The solid lines in the figure are the H/V spectra of surface waves, $(H/V)_S$, computed for the soil profiles at the sites, assuming that the value of horizontal Rayleigh to Love wave amplitude ratio is 0.7 at given frequencies. At each site, both the values and variation with frequency of theoretical $(H/V)_S$ show fairly good agreement with those of observed $(H/V)_{mS}$, and the fitness between theoretical and observed H/V ratios is better than that shown in Figure 8. This indicates that the proposed H/V spectrum of surface waves, $(H/V)_S$, simulate well that of microtremors, $(H/V)_{mS}$, defined by Eq. (12). Besides, a comparison of Figure 8 with Figure 9 can point out that the $(H/V)_S$ is almost parallel to the $(H/V)_R$ in log-log scale at each site, indicating that the variation with frequency of H/V ratio of microtremors is mainly controlled by that of Rayleigh waves and the value of H/V ratio of microtremors is controlled by Love waves.

**Possibility of $V_S$ Profiling Using Mocrtremor H/V Spectrum**

To investigate a possibility of soil profiling using microtremor H/V spectra based on the proposed theoretical formulas, sensitivity analysis of the H/V ratio of surface waves for soil model is performed. The sensitivity $D$ of the H/V ratio of surface waves $(H/V)_S$ for the parameter $p$ of soil model can be expressed as

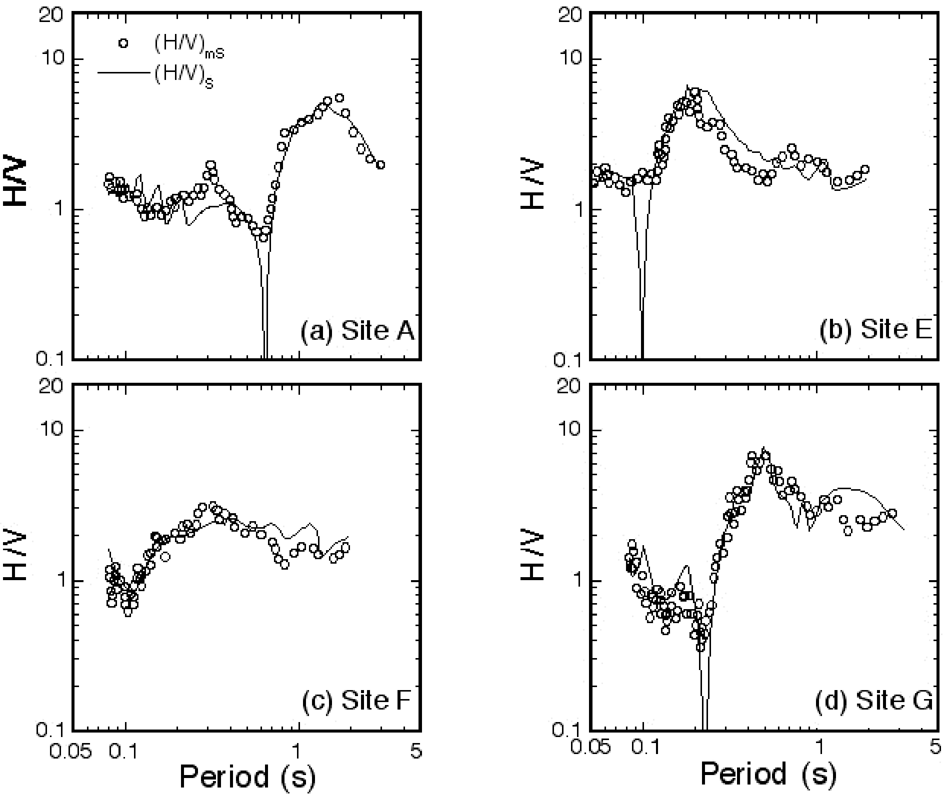


**Figure 9: The H/V ratios of microtremors compared with those of surface waves at sites A and E-G**

$$D = \left| \left( p / (H/V)_S \right) \left( \delta (H/V)_S / \delta p \right) \right| \tag{13}$$

In Eq. (13), the larger the value of $D$, the higher becomes the sensitivity of $(H/V)_S$ for the parameter $p$. Figure 10(a) shows the soil model used in the sensitivity analyses. In the analyses, the deep soil layers are assumed to be those shown in Table 1. Figures 10(b)-(e) show the results of the analyses for thickness, mass density, P-wave velocity, and S-wave velocity at each layer of the test model, respectively. The values of sensitivities of $(H/V)_S$ for thickness and S-wave velocity (Figures 10(b) and (e)) are larger than those for mass density and P-wave velocity (Figures 10(c) and (d)). The same trends are confirmed for several soil models. This indicates that either thickness or S-wave velocity at each layer of the soil model or both of them, i.e., $V_S$ profile, may be estimated from the H/V ratio of microtremors at a site using the proposed theoretical formulas.

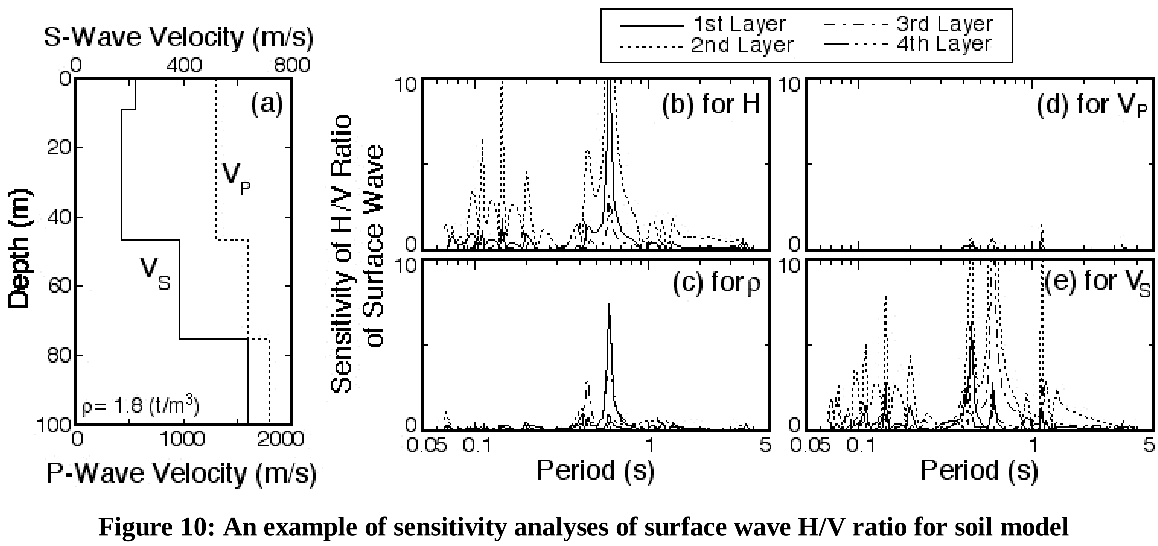


**Figure 10: An example of sensitivity analyses of surface wave H/V ratio for soil model**

## CONCLUSIONS

Analytical formulas are presented for computing the H/V ratios of Rayleigh and surface waves which are generated from point sources randomly distributed on a layered half-space. Assuming that the value of Rayleigh to Love wave amplitude ratio in horizontal motions is 0.7, the H/V spectra of Rayleigh and surface waves are computed at four sites, where the PS log data are available. The comparison among the H/V spectra of microtremors and Rayleigh and surface waves at the sites leads to the following conclusions: (1) The higher modes of Rayleigh waves have significant effects on the H/V spectra of microtremors in the frequency range where response functions of higher modes are predominant; (2) The H/V spectra of Rayleigh waves considering higher modes are in good agreement with those of microtremors, although the H/V values of Rayleigh waves are less than those of microtremors; (3) The H/V spectra of surface waves simulate well those of microtremors; and (4) either thickness or S-wave velocity at each layer of soil model or both, i.e., $V_S$ profiles, may be estimated from the H/V spectra of microtremors based on the inverse analysis using the proposed formulas.

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