UNCERTAINTIES IN THE ESTIMATION OF NATURAL FREQUENCIES OF BUILDINGS IN MEXICO CITY

David MURIÀ-VILA[1], Luis FUENTES OLIVARES[2] And Ricardo GONZÁLEZ ALCORTA[3]

SUMMARY

The uncertainty in the computation of natural frequencies of building structures is studied. Experimental and analytical values of natural frequencies are compared in this research. The former values are computed taking into account the current practice criteria and more realistic hypothesis (calibrated model). The models that consider current practice criteria produce error values close to the calibrated models when soil-structure interaction effects and the contributions of masonry walls are considered.

INTRODUCTION

Important efforts in structural engineering research are focused toward the prediction of the dynamic behavior of structural systems subjected to dynamic loads. The key issue of these efforts is the characterization of a representative mathematical model able of determining the dynamical response of the system. From a theoretical viewpoint, it is possible to establish several mathematical models for a structural system. Their solutions will reproduce the dynamic response of the structure with different degrees of approximation. Thus, the main problem is to select the most representative model.

A mathematical model of a physical system can be established based on the fundamental laws of dynamics, structural analysis and constitutive laws of materials, including all aspects that participate significantly in the model. The models are sensitive to the characteristics of the system. Therefore, the uncertainty associated with them can lead to increased uncertainties on the relationship between the real and predicted response.

The current design criteria of buildings allow some damage in structural systems due to high intensity earthquakes. Therefore, it is necessary to understand the behavior of structures beyond their elastic limit. In Mexico, the possible deterioration implicit in the seismic behavior factor normally it is not considered. Methods to compute the dynamic response of structures in the professional practice are based on the use of commercially developed computer programs, in which the structural elements that participate in the stiffness and mass of the building are incorporated assuming a linear behavior. Thus, the hypothesis made for the computation of the lateral stiffness of buildings can affect significantly the structural design, because the seismic coefficients will depend of them.

In several research studies criteria to consider the stiffness deterioration in explicit way are proposed, in order to assess in a more realistic way the lateral stiffness of buildings, this phenomenon is included through stiffness reduction factors of the structural elements [Anderson *el al.*, 1991; Boroschek and Mahin, 1991; Durrani *et al.*, 1994; Foutch *et al.*, 1989; Freeman, 1980; Gamboa and Murià-Vila, 1996; Paulay and Priestley, 1992; Tena-Colunga, 1992].

The criterion of Paulay and Priestley [1992] has been included in the current New Zealand Code [NZS, 1995]. They suggested different reduction factors (RF) for moments of inertia and effective cross section area of the

[1] *Instituto de Ingeniería, UNAM. Cd Universitaria, Apto. Postal 70-472, 04510 México, DF, dmv@pumas.iingen.unam.mx*
[2] *Instituto de Ingeniería, UNAM*
[3] *Instituto de Ingeniería, UNAM*

structural elements as a function of global ductility capacity (μ). In this sense, other codes use similar criterions such as the Greek and the Japanese Codes [Paz, 1994; AIJ, 1994].

To evaluate the computation methods and the analysis considerations, some methodologies for the dynamical characteristics determination of real buildings are applied in Mexico. Among the most important are ambient vibration tests (AV) and the analyses of seismic records of instrumented buildings (EQ). Due to the stress levels, which can be present in the structures, the results obtained using these techniques can be different. Because the potential advantages presented in these methods, it is necessary to estimate the correlation between their results and the ones provided by the mathematical models that are used.

In this study, the uncertainty in the computation of the natural frequencies of vibration in six buildings is evaluated. The evaluation is done comparing the experimental values and those of calibrated models with respect to the ones computed applying the common criteria used in professional practice. The foregoing will allow assess the possible underestimation or overestimation, which must be considered for design purposes.

BUILDINGS SELECTED

Six reinforced concrete buildings of Mexico City were selected. Five of them are founded on soft soils and one in firm soil. Information on constructive data, plans and experimental dynamic characteristics of these buildings was available. A summary of their principal characteristics is presented in table 1. Buildings A, C, E and F, and B and D experienced moderate and light earthquake damage, respectively. The masonry walls of the buildings were considered in the original design as non-structural elements, except for B building.

Table 1. Characteristics of buildings

Blg.	Nº Stories	Structure	Construction Year	Dimensions (m)					Frequencies (Hz)			
				H	ΔH	D	T	L	Soil	T	L	R
A	20	F_{BP}, S_{PC}, FW, W_M	1979	58	2.90	2.30	17.8	37.5	0.5	0.40	0.58	0.72
B	17	F_B, S_R, FW, W_M	1952	44	2.40	5.44	13.5	40.5	> 3.0	0.80	1.20	1.36
C	9	F_{BP}, S_{PC}, FW, W_M	1977	27	2.65	3.30	9.4	23.2	0.7	0.64	1.12	1.85
D	8	F_{BP}, S_R, FW, W_M	1970	24	2.65	2.65	13.8	36.4	~1.0	0.88	1.28	1.59
E	14	F_{BP}, S_R, FW, W_M	1981	42	2.60	3.30	19.6	32.0	0.5	0.44	0.73	0.80
F	8	F_B, S_F, FW, W_M	1969	30	3.45	8.95	22.2	29.5	0.5	2.27	1.89	3.57

F_{BP} - Box and piles foundation
F_B - Box foundation
S_R - Reticular flat slab
S_{PC} - Precast slab
S_F - Flat slab
FW - Reinforced concrete frame and shear walls
W_M - Masonry walls

L - longitudinal
T - transversal
R - torsion
H - total height
ΔH - story height
D - deep box foundation

UNCERTAINTIES OF THE NATURAL VIBRATION FREQUENCIES

The statistical trend of the errors found comparing the vibration frequencies obtained from the mathematical models of the buildings designed by professional practice (models engineering) with the experimental and computed with the calibrated models was analyzed in order to establish the uncertainty level in the computation of the vibration frequency of buildings using the criteria commonly used in structural models. Three-dimensional linear models were elaborated using the ETABS program [Habibullah, 1995].

The study was divided in two parts: in the first one, the natural frequencies of vibration of the buildings are estimated using the AV models (low stress levels) for analysis considerations related to a linear behavior. In the second part, frequencies of vibration are estimated with the EQ models (high stress levels) which in addition to the previous considerations, the deterioration of the stiffness of the elements of the system is taken into account.

In order to get the response of the calibrated models of the buildings closest to the experimental information obtained from ambient vibration or earthquake loads, those models take into account the characteristics of the system and are called AV or EQ calibrated models, respectively.

The considerations for the analyses of the AV calibrated models are:

- Young's modulus for low stress levels
- "Real mass" and its distribution according to its plan location
- Rigid zones (RZ) in the beam-column connections
- Participation of the slab in accordance to Mexico's Federal District Code [RCDF, 1996]
- Masonry and concrete walls only when connected to the structure
- Parapets only if interaction with the structural elements is observed
- Soil-structure interaction (SSI) effects for buildings on soft soils. The stiffness for instrumented buildings were determined from experimental information using the procedure proposed by Luco [1980]. For non-instrumented structures, the RCDF [1993] proposal was used
- Staircases and parking ramps.

In addition to these items, for the EQ calibrated models, non-linear aspects such as Young's modulus for high stress, and deterioration RF of the gross section of the structural elements were also included.

In the engineering models, the principal hypotheses used in the professional practice were used. These hypotheses were obtained from an inquiry to 10 structural design firms in Mexico. The inquiry showed that the SSI effects, masonry walls and RZ, are considered only for less than a half of the companies. Only one of them takes into account the deterioration effect assumed for the bending stiffness of columns, beams and walls as 100, 60 and 80% of the gross section, respectively. Only two firms considered the "real" distribution of mass.

AV and EQ engineering models were developed to compare the results with the AV and EQ calibrated models and the experimental data.

The general considerations of the EQ engineering models were:

- Design Young's modulus
- Design mass uniformly distributed
- Gross sections of structural elements
- Participation of the slab in accordance with the RCDF [1996]
- No staircases and parking ramps
- RZ of 0 and 100 % in the beam-column connections.

AV engineering models were elaborated to compare with the AV calibrated models and the ambient vibration data. The differences of these models from previous models were the Young's modulus and the mass assumed the same considerations to the linear calibrated models; and RZ became little meaningful thus, the value of the calibrated models was used.

AV MODELS

The natural frequencies of vibration for the translational components of movement (in the large dimension direction, L, and in the minor dimension, T) and of torsion (R) obtained from experimental data from ambient vibration tests, and from analytical AV calibrated and engineering models are presented in table 2.

The relative error values (e_{ce}) of the vibration natural frequencies of the AV calibrated and engineering models (F_{cav} or F_{iav}) of each building, were computed with respect to the frequencies obtained from the ambient vibration tests (F_e) using $e_{ce} = (F_{cav} - F_e)100/F_e$.

In the analysis of the ambient vibration records of the building F, a non-linear behavior was evident. This non-linearity was primarily associated to the coupling between its foundation box and the subway station through the construction joint [Murià-Vila *et al.*, 1997]. The computed vibration frequencies of the building are within the expected range of variation.

Table 2. Frequencies (Hz) of AV calibrated and engineering models

Blg.	Calibrated Models			Engineering Models											
				Masonry						Without Masonry					
				SSI			Without SSI			SSI			Without SSI		
	T	L	R	T	L	R	T	L	R	T	L	R	T	L	R
A	0.38	0.57	0.70	0.38	0.56	0.68	0.82	1.08	1.60	0.35	0.54	0.67	0.80	0.94	1.56
B	0.75	0.99	1.18	---	---	---	0.76	0.98	1.20	---	---	---	0.70	0.77	1.07
C	0.62	1.08	1.52	0.61	1.04	1.37	1.20	2.05	1.55	0.49	0.57	0.83	0.64	0.70	0.87
D	0.82	1.22	1.54	0.83	1.23	1.52	1.59	1.77	1.65	0.82	1.21	1.45	1.58	1.67	1.50
E	0.49	0.76	0.91	0.47	0.75	0.90	0.56	0.96	0.93	0.46	0.70	0.83	0.53	0.86	0.84
F	2.31	1.94	3.75	2.29	1.94	3.72	2.55	2.11	3.85	2.21	1.91	3.63	2.45	2.08	3.75
$\bar{e}_{ce}$	5.76		8.89	4.35		10.04	48.95		29.05	11.71		14.87	34.56		34.12
σ	4.53		5.68	1.92		8.77	36.47		45.84	14.59		22.66	30.94		44.60

T – transversal component L – longitudinal component R – torsional component
$\bar{e}_{ce}$ - average of absolute relative error σ - standard deviation

Calibrated models

The averages of the absolute relative error values of the frequencies of the buildings were 6% for translation and 9% for torsion. The magnitude of the relative error values shows that it was possible to achieve representative models for the buildings.

Engineering models

In the assessment of the relative error values of these models, four cases were considered: with SSI effects and masonry walls, with SSI effects and without masonry walls, without SSI effects and with masonry walls, and without SSI effects and nor masonry walls.

From the averages of the absolute relative error values with respect to the experimental (table 2) it can be observed that:

- For models with SSI effects and masonry walls the error values turn out to be practically equal to the ones obtained with the calibrated models.
- For models without masonry walls, the average error values increases to 12% in the translational components and 15% in torsion.
- For the cases in which the SSI effects are ignored, the error values are increased significantly with values between 29 and 49%.

EQ MODELS

Calibrated models of two instrumented buildings

The buildings E and F are seismically instrumented. The experience obtained from the analysis of seismic records of these buildings shows that some of them presented non-linear behavior during seismic movements [Murià-Vila *et al.*, 1997; Murià-Vila and Toro, 1997].

Building E shows variations of up to 67 percent between the identified vibration frequencies due to deterioration by damage accumulation for several earthquakes effects occurred between 1993 and 1995 [Meli *et al.*, 1998; Murià-Vila and Toro, 1997]. For building F, the results show large variations (up to 75 %) of the vibration frequencies. In this case, variations are not associated with damage, rather with non-linear interaction between adjacent structures, and they show a high sensitivity with the movement amplitude.

For building E, a calibrated and refined model (EQR model) was developed. In the EQR model it was taken into account that the structure has suffered damage associated to the action of several earthquake. It was necessary to consider cracked sections in structural elements. Additionally, special analyses were done in order to determine particular characteristics of equivalent diagonals ends to represent the concrete walls, and to define broad of equivalent beams of the reticular slabs. These and other aspects are discussed in detail in Gamboa and Murià-Vila [1996].

Due to the time required to elaborate refined models and the need of considering, in the professional practice, some deterioration that may be permitted for structures, it was decide to include them in a simple way. In order to decide which of them is appropriate for the selected buildings, the relationship between effective properties of cracked and gross sections of representative structural elements was evaluated. The average values of this relationship for different structural elements were compared with the ones proposed by Paulay and Priestley [1992] to estimate the effective lateral stiffness of buildings. Consequently, with the proposed reduction factors (RF) of Paulay and Priestley assuming that deterioration of the structural systems due to seismic effects corresponds to ductility capacities of 3 and 6, EQ calibrated models of the E and F buildings were constructed. Values of RF for the reticular flat slabs and the masonry walls were established [Gamboa and Murià-Vila, 1996]. For the slabs, the RCDF [1996] proposal was used, and reducing the moments of inertia with RF values of 0.30 and 0.15, corresponding to μ values equal to 3 and 6, equivalent beams were computed. For the confined masonry, RF of the shear area according to interstory drifts obtained from experimental tests [Flores and Alcocer, 1996] was established.

Frequencies obtained with the EQR and EQ calibrated models for the two buildings are presented in table 3. They are compared with the ones obtained from the analysis of the records of two seismic events: October 24, 1993 (93 event) and September 14, 1995 (95 event), with small and moderate intensity, respectively.

Table 3. Frequencies (Hz) of EQR and EQ calibrated models of E and F buildings compared with those identified with earthquake records

	E Building			F Building		
	T	L	R	T	L	R
Event 93	0.35	0.57	0.50	1.80-2.08	1.54-1.70	3.10-3.39
Event 95	0.28	0.45	0.43	1.38-1.69	1.53-1.58	---
EQR	0.33	0.50	0.57	---	---	---
EQ (μ = 3)	0.34	0.52	0.58	1.83	1.60	3.02
EQ (μ = 6)	0.28	0.45	0.49	1.60	1.44	2.70

For building E, it is observed that vibration frequencies of the models with μ=3 are similar and that they agree with the obtained from 93 event. On the other hand, frequencies of the EQ model with μ=6 are very similar to the 95 event. For building F with EQ models and μ equal to 3 and 6, frequency values are within the interval of computed experimental frequencies.

Calibrated models

Based on the foregoing, EQ calibrated models of other buildings were developed. The following cases were studied: RF associated with μ=3 and RZ=50%, FRC associated with μ=6 and RZ=50%, and RF associated with μ=6 and RZ=0%.

The rigid zones (RZ) of the beam-column connections in reinforced concrete structures are assumed normally 50% of their dimensions [Horvilleur and Cheema, 1995]. Nevertheless, when the ductility capacities that they can be developed in the structural elements are large Paulay and Priestley [1992] recommend not considering RZ.

The natural vibration frequencies obtained with the EQ models are presented in table 4. For the EQ models with μ=3 and RZ=50% with respect to those which consider μ=6 and RZ equal to 0 and 50%, differences were found in the natural frequencies associated with average error values of about a 16 and 18%, respectively (table 4). Additionally, the results obtained indicate that, in this case, it is not meaningful the difference between the frequencies of the models with RZ equal to 0 and 50%.

Table 4. Frequencies (Hz) of EQ calibrated models

Blg.	$\mu = 3$ RZ = 50 %			$\mu = 6$ RZ = 50 %		
	T	L	R	T	L	R
A	0.30	0.43	0.57	0.26	0.37	0.48
B	0.56	0.74	0.87	---	---	---
C	0.56	0.98	1.14	0.48	0.76	0.84
D	0.75	1.09	1.24	0.68	0.79	1.07
E	0.34	0.52	0.58	0.28	0.45	0.49
F	1.83	1.60	3.02	1.60	1.44	2.70

Table 5. Frequencies of EQ engineering models (RZ = 100%)

Blg.	Masonry						Without Masonry					
	SSI			Without SSI			SSI			Without SSI		
	T	L	R	T	L	R	T	L	R	T	L	R
A	0.36	0.53	0.65	0.75	0.99	1.46	0.30	0.48	0.64	0.74	0.85	1.29
B	---	---	---	0.74	0.97	1.19	---	---	---	0.70	0.91	1.09
C	0.58	1.02	1.42	1.09	1.92	1.50	0.45	0.51	0.75	0.62	0.68	0.88
D	0.78	1.17	1.46	1.51	1.75	1.68	0.78	1.13	1.29	1.51	1.67	1.40
E	0.43	0.69	0.81	0.50	0.84	0.83	0.42	0.63	0.73	0.48	0.74	0.75
F	1.98	1.74	3.23	2.15	1.87	3.32	1.90	1.72	3.13	2.04	1.85	3.21
$\bar{e}_{ie}$ ($\mu = 3$)	13.84		20.59	69.91		52.17	14.29		16.01	49.90		37.15
σ	10.74		12.41	44.21		52.18	14.45		13.59	43.00		44.49
$\bar{e}_{ie}$ ($\mu = 6$)	35.10		45.17	110.88		86.42	27.02		25.90	77.93		55.26
σ	14.43		21.21	53.25		69.12	14.30		15.38	57.02		65.87

T – transversal component L – longitudinal component R – torsional component
$\bar{e}_{ie}$ - average of absolute relative error σ - standard deviation

Engineering models

For the assessment of the relative error values of the vibration natural frequency of the engineering models of the buildings with respect to the EQ calibrated models, four cases for the AV models without rigid zone (RZ = 0%) and four with RZ =100%, were considered.

From the natural frequencies obtained for the engineering models, it was observed that the differences increase significantly when models that consider masonry walls and SSI effects are compared with those that do not take into account these aspects (table 5). Again, small differences in the frequencies obtained for the models with RZ equal to 0 and 100% were found. In table 5 only the values for RZ=100% are shown.

Comparative analysis

The average of absolute relative error values (e_{ic}), of the EQ engineering models frequencies (F_{ieq}) with respect to the EQ calibrated models (F_{ceq}) computed as $e_{ic} = (F_{ieq} - F_{ceq})100/F_{ceq}$ is shown in table 5.

Averages of the absolute relative error values were computed. The engineering models that consider SSI effects, are those which have similar results that the EQ calibrated model with μ=3. The average error values vary between 9 and 21%. The average error values of the engineering models with respect to the EQ calibrated models with μ=6 are greater than 20%. When SSI effects are ignored, error values increased from 37 to 110%. All these average error values are associated with high dispersions.

CONCLUSIONS

The inquiry revealed two aspects that are important to emphasize: SSI effects are assumed occasionally in the professional practice in spite of their relevant participation on the dynamic properties of buildings in soft soils. SSI effects alone have large uncertainties that require detailed analyses, subject that is out of the scope of this study. Other relevant aspect is that, in the professional practice, the deterioration of the stiffness of structural elements is not considered in an initial structural design of buildings in spite of the fact that it is accepted that structures may be damaged for a design earthquake when assuming a seismic response modification factor. Nevertheless, the deterioration is considered for a structural revision of existing buildings, although design firms did not specify with clarity how to proceed in such cases, it would depend on each particular structure.

When the deterioration of the structure is considered in the analyses, the frequencies of vibration of the calibrated models using stiffness reduction factors corresponding to ductility capacity of 3 of the instrumented buildings, showed an acceptable approximation with respect to the ones obtained from the analyses of the records of the seismic event with small intensity (93 event). On the other hand, with the model calibrated with reduction factors corresponding to ductility of 6 for building E, its fundamental frequencies were similar to the ones obtained with the moderate intensity earthquake (95 event). This is explained by the accumulated damage in structural elements of the building because of action of various previous earthquakes that affected this building.

Frequencies computed for Building E refined calibrated model were similar to the calibrated model with constant stiffness reduction factors. This suggests that simpler models with a sufficient degree of approximation for the assessment of the dynamic properties of existing buildings can be done using these factors.

By comparing the natural frequencies of vibration of calibrated models, with stiffness reduction factors, and the engineering models, it was found that the ones obtained from the engineering models were approximated to those that consider reduction factors for ductility capacity of 3, with average error values between 9 and 21%. It is observed again the importance of SSI effects and of the masonry walls; as not including them in the modeling, increases the error values significantly, over 35%.

All the engineering models overestimate the frequencies of vibration of the calibrated models. Exception are the models that ignore the masonry walls of two of the reinforced concrete buildings, where occurs the contrary because the masonry walls contribute significantly to the stiffness (buildings C and D).

In spite of the fact that the number of cases studied is reduced and that it is necessary to consider other typical structural systems of building used in Mexico City, the results obtained show that the analytical computation of the natural frequencies of vibration of buildings have large dispersions. This scatter can be reduced upon considering SSI effects and the probable deterioration of the structure during its lifetime.

ACKNOWLEDGMENTS

The valuable comments of José Alberto Escobar Sánchez and Arturo Tena-Colunga are appreciated. Jaime Torres participated in the structural analyses. This research was funded by the Federal District Government.

REFERENCES

AIJ [1994], “Structural Design Guidelines for Reinforced Concrete Buildings”, *Architectural Institute of Japan*, March 1994

Boroschek R. y Mahin S.A. (1991). "Seismic response of a lightly-damped torsionally-coupled building", Report No. UCB/EERC-91/18, University of California at Berkeley

Durrani N., Mau S. T., Abouhashish A. y Y. Li (1994), “Earthquake Response of Flat-Slab Buildings”, *Journal of Structural Engineering,* ASCE, **120** (3), March 1994, pp. 947-964

Flores L.E. and Alcocer S.M., [1996], “Calculated response of confined masonry structures”, *Eleventh World Conference on Earthquake Engineering*, Acapulco, paper No. 1830

Foutch D. A., Hjelmstad K. D., Del Valle E. and Figueroa E. (1989), “The Mexico Earthaquake of September 19, 1985- Case Studies of Seismic Strengthening for Two Buildings in Mexico City”, *Earthquake Spectra*, **5** (1), February 1989, pp. 437-457

Freeman S. A., Czanercki R. M. y Honda K. K. (1980), “Significance of Stiffness Assumptions of Lateral Force Criteria”, Special Publication ACI SP 63-18, pp. 437-457

Gamboa V. and Murià-Vila D., [1996], “Cálculo de la respuesta estructural del edificio Jal ante el sismo de Copala”, *Memorias del X Congreso Nacional de Ingeniería Estructural*, Mérida, Yucatán, November 1996, pp. 277-286

Habibullah A., [1996], *Three Dimensional Analysis of Building Systems Users Manual*, March, 1996

Horvilleur J. F. and Cheema M.A., [1995], “Análisis lateral de marcos de concreto reforzado sometidos a carga laterales sísmicas”, *Memorias del IX Congreso Nacional de Ingeniería Estructural*, SMIE, Zacatecas, Zac., October 29 - November 1, 1995

Luco, J. E., [1980], “Soil-structure interaction and identification of structural models”, *Proc. 2nd ASCE Speciality Conference in Civil Engineering and Nuclear Power*, Knoxville, Tennesse, **II**, paper 10-1, pp. 10.1.1-10.1.31.

Meli R., Faccioli E., Murià-Vila D., Quaas R. and Paolucci R., [1998], “ A study of site effects and seismic response of an instrumented building in Mexico City”, *Journal of Earthquake Engineering*, **2** (1), pp. 89-111

Murià-Vila D. and Toro Jaramillo A., [1997], “Effects of several seismic events recorded at a building founded on soft soil”, *11th European Conference on Earthquake Engineering*, Balkema, Rotterdam, 1998

Murià-Vila D., Torres J., Fuentes L. and González R., [1997], "Incertidumbre en la estimación de las frecuencias naturales de vibración en edificios", Instituto de Ingeniería, UNAM, Proy. 7517, sponsored by DDF, September

NZS, [1995], “Concrete Structures Standard”, Superseding NZS 3101, *Standard New Zealand*

Paulay T. and Priestley N., [1992], *Seismic Design of Reinforced Concrete and Masonry Buildings*, John Wiley & Sons Inc.

Paz M., [1994], *International Handbook of Earthquake Engineering*, Chapman Hall. pp. 239-248

RCDF [1993], “Normas Técnicas Complementarias para el Diseño por Sismo del Reglamento de Construcciones del Distrito Federal”, *Gaceta Oficial del Distrito Federal*, August 2

RCDF [1996], “Normas Técnicas Complementarias para Diseño y Construcción de Estructuras de Concreto”, *Gaceta Oficial del Distrito Federal*, March 25

Tena Colunga, A., [1992], “Seismic evaluation of unreinforced masonry structures with flexible diafragms”, *Earthquake Spectra*, **8** (2), May 1992, pp. 305-318.