CHARACTERIZATION OF SEISMIC ISOLATION BEARINGS FOR BRIDGES FROM BI-DIRECTIONAL TESTING

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SUMMARY

To investigate the bi-directional behavior of lead-rubber isolation bearings, a series of bi-directional displacement-controlled tests were conducted. A coupled bilinear model with circular force interaction function is capable of representing the observed bi-directional behavior of lead-rubber bearings at a specified shear train level with a set of parameters corresponding to the maximum shear strain. Bi-directional coupling effects are observed in earthquake testing of an isolated bridge model subjected to near field ground motions. To represent the behavior of lead-rubber isolation bearing over a wide range of shear strain level, a four-parameter uni-directional model based on bounding surface theory is presented. Compared with the data from uni-directional testing of the bearings, the model with four strain-independent parameters provides a better fit than the bilinear model.

INTRODUCTION

The design of seismic isolation system for bridges is generally based on an equivalent linear model for demand analysis. Nonlinear demand analysis is becoming more prevalent with the need for better estimates of isolation bearing performance under maximum credible earthquakes (or probabilistic estimates of ground motion with long recurrence period). Since elastomeric and sliding bearings can be approximately represented by a bilinear force-deformation relationship, most nonlinear demand analysis is based on a simple hysteretic element for each bearing in a bridge. The use of a bilinear model for elastomeric bearings raises two important issues. The first is whether the coupling between transverse and longitudinal behavior of a bearing must be included, or if assuming uncoupled behavior is adequate. If uncoupled behavior is assumed, then each bearing can be modeled simply as two bilinear springs along orthogonal axes. The second issue is that, although the force-deformation relationship is approximately bilinear, the parameters are in fact a function of the strain level. Aside from the difficulty of estimating *a priori* the maximum strain, bridge bearings can undergo a wide range of strain during an earthquake, so that it may not be reasonable to use a bilinear model with a single set of parameters [Kelly, 1998].

These issues and others on seismic protection of bridges are under investigation in a research program at the University of California, Berkeley sponsored by the California Department of Transportation. In Fenves, et al. (1998), bi-directional bilinear models based on rate-dependent plasticity and the Park-Wen [Park, et al., 1986] approach were developed and calibrated with data from bi-directional experiments on high-damping rubber bearings. Even neglecting strain-induced hardening of HDR bearings, the parameters of the bilinear model were strain dependent. In this paper, the plasticity model is examined using experimental data for lead-rubber bearings, which are expected to have less sensitivity to strain than HDR bearings. To account for strain-dependency of the model parameters, this paper proposes a new model for elastomeric bearings, based on bounding surface theory [Dafalias, et al., 1976]. The objective of the paper is to demonstrate that the improved model for elastomeric bearings has a single set of parameters that can be calibrated from a uni-directional test and then used for a wide-range of strains in a bi-directional demand analysis of a bridge.

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BI-DIRECTIONAL BILINEAR MODELS

A bilinear model based on rate-independent plasticity theory [Lubliner, 1990] can be used to characterize the bi-directional behavior of elastomeric bearings. The restoring force, $\mathbf{F} = [F_x \quad F_y]^T$, in a bearing is assumed to depend only on the shear deformation, $\mathbf{U} = [U_x \quad U_y]^T$, where F_i and U_i are the restoring force and deformation in orthogonal horizontal directions, $i = x, y$. The restoring force consists of an elastic hardening component and a hysteretic force, $\mathbf{F}_p$, as follows:

$$\mathbf{F} = K_2 \cdot \mathbf{U} + \mathbf{F}_p \ , \tag{1}$$

where K_2 is the post-yield hardening stiffness and $\mathbf{F}_p = [F_{px} \quad F_{py}]^T$ is the hysteretic force. Assuming isotropic behavior, a circular interaction function $\Phi(\mathbf{F}_p)$ can be postulated for the bearing,

$$\Phi(F_p) = \left\| \mathbf{F}_p \right\| - Q_D \ , \tag{2}$$

where the zero-displacement-force intercept, Q_D, represents one-half the size of hysteretic loop for a bearing as shown in figure 1. The responses in the two directions are coupled through the circular force-interaction function. Applying the additive decomposition assumption of plasticity, the hysteretic force, $\mathbf{F}_p$, is

$$\mathbf{F}_p = (K_1 - K_2) \cdot (\mathbf{U} - \mathbf{U}^p) \ , \tag{3}$$

where K_1 is pre-yield stiffness, and $\mathbf{U}^p$ is plastic deformation. Three parameters K_1, K_2, and Q_D define the bilinear model as illustrated in figure 1.

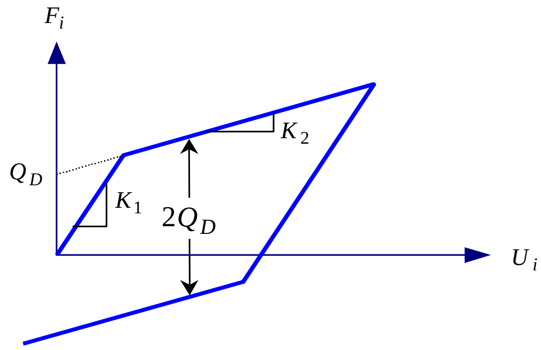

Figure 1: Bilinear Force-Displacement Relation for Isolation Bearings

The aforementioned model couples the two orthogonal directions through a circular interaction surface. The force-deformation responses in the two directions can be uncoupled by assuming a square interaction surface. The responses in the two directions are then independent and can be represented by two bilinear springs. Most dynamic analysis programs can easily include independent nonlinear springs, so the uncoupled model is most attractive if coupling effects are not important.

BI-DIRECTIONAL TESTS OF LEAD-RUBBER ISOLATION BEARINGS

The 6-DOF earthquake simulator at the Pacific Earthquake Engineering Research Center was adapted for bi-directional testing of seismic isolation bearings. For displacement-controlled testing, a rigid-frame was fixed in the horizontal space with struts attached to support points off the simulator. The simulator platform was then moved under displacement control in the horizontal plane. For earthquake testing, the struts were removed and the simulator is controlled by command signals for displacement and acceleration. Figure 2 shows the plan and elevation of the rigid-frame supported by four lead-rubber bearings and five-component load cells on the earthquake simulator. The rigid-frame supports concrete and lead blocks with a total weight of 65 kips, providing a vertical load of approximately 16 kips on each bearing.

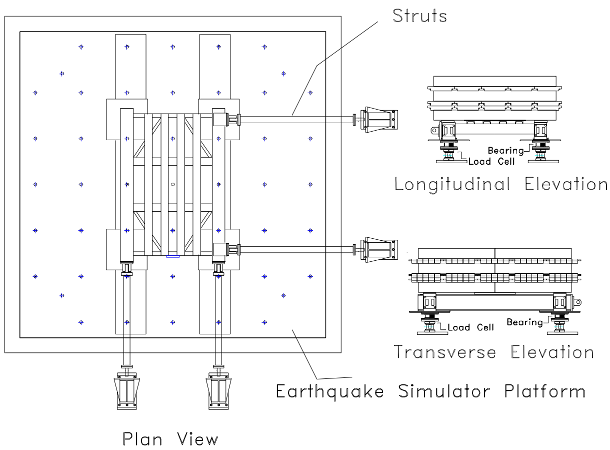

Figure 2: Configuration of Rigid-Block Test

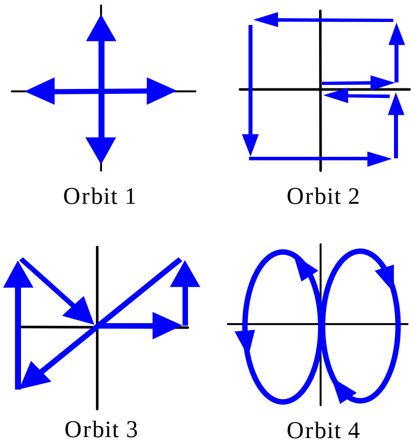

Figure 3: Displacement-Controlled Orbits

The lead-rubber bearings investigated in this study were manufactured by Dynamic Isolation Systems, Inc. The bearings consist of nineteen layers of 0.125-in. thick rubber with a bonded diameter of 6.614 in., eighteen 0.075-in. thick steel shims, a 1.18-in. diameter lead plug, and 0.50-in. thick top and bottom steel plates. The bearings have a shape factor, S_1, equal to 11.5. The nominal vertical stress on the bearings under gravity load is 470 psi.

Bi-Directional Displacement Controlled Tests

To investigate the bi-directional behavior of the lead-rubber bearings, displacement-controlled testing was conducted with the four orbits in figure 3. The orbits are run at slow rate (approximately 100 sec per cycle) for three cycles to various shear strain levels. To evaluate rate-effects, orbit 4 is also run at 2 sec per cycle.

The model parameters are calibrated using the experimental data from the displacement-controlled tests. The calibration involves the minimization of a residual function corresponding the force difference between the measured data and the model for a given displacement history. Since the displacement increment in the orbits may not be equally spaced, a residual function is normalized by the length of the displacement path:

$$normalized \quad residual = \frac{\int \left| \mathbf{F}_{exp} - \mathbf{F}_{model} \right|^T \cdot |\, d\mathbf{U}\,|}{\int \sqrt{d\mathbf{U}^T \cdot d\mathbf{U}}} \tag{4}$$

The residual in equation 4 can be used to compare fits between different displacement histories because of the normalization by length. The downhill simplex method [Nelder and Mead, 1965] is used for the minization because it is efficient and does not required gradients of the residual with respect to the model parameters.

Figure 4 shows the experimental behavior of one bearing for displacement orbit 2 at the 100% maximum shear strain in the X and Y directions. The coupled bi-directional behavior of the bearing is observed in the force locus plot: with fixed 100% shear strain in the X direction, loading along the Y direction reduces the force in the X. The force reduction almost equals one-half the size of the hysteretic loop as shown in the Force-Y:Deformation-Y plot in figure 4. This behavior is consistent with the postulated circular interaction surface, and it cannot be modeled correctly with an uncoupled bilinear model. The dotted lines in figure 4 give the coupled bilinear model response with the best-fit parameters obtained from the calibration process. The model shows very good agreement with experimental behavior at single shear strain level and it accurately represents the coupled behavior of the lead-rubber bearings under bi-directional loading.

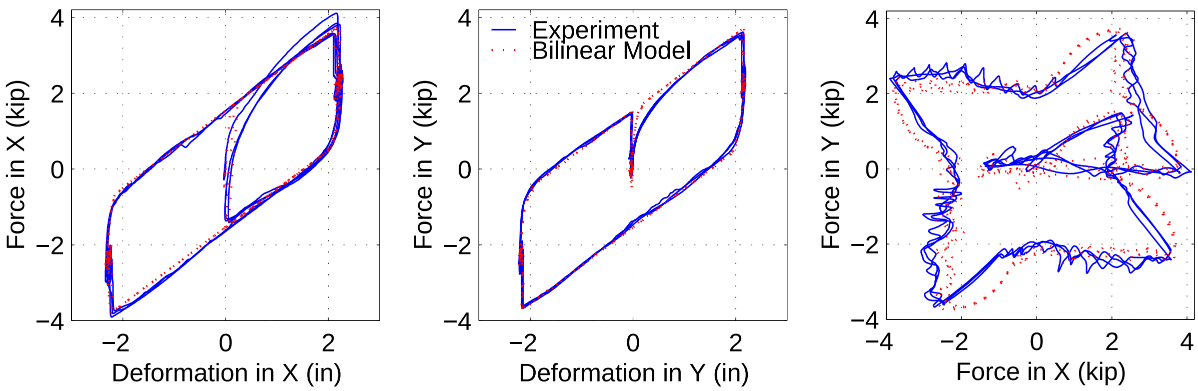

Figure 4: Experiment Compared with Coupled Bilinear Model – Orbit 2 at 100 % Shear Strain Level

In figure 5, the best-fit parameters for the coupled bilinear model are plotted as a function of maximum shear strain using the data from one bearing for all the displacement-controlled tests. The calibrated pre-yield stiffness, K_1, from orbit 1 is significantly greater than the values from the other orbits. The reason for the large K_1 values for orbit 1 is unclear, but the subsequent tests show more uniform K_1 with a lower stiffness. Although K_1 is small for orbit 2, the normalized residual is not very sensitive to K_1 for that case. The calibrated post-yield stiffness K_2 and Q_D are very consistent for the different orbits. It is important to note that for the bilinear model, the calibrated post-yield stiffness, K_2, decreases monotonically with maximum shear strain. The yield force, Q_D, for the hysteretic component increases with maximum shear strain up to 175% and decreases thereafter. The decrease in Q_D might be caused by degradation of the lead core under large shear strain. These observations demonstrate that the coupled bilinear model can represent the bi-directional behavior of lead-rubber bearings well at a specified shear level. The parameters for bilinear model, however, are dependent on the maximum shear strain. This presents practical problems in using bilinear models for demand analysis because the maximum shear strain is not known *a priori*. One approach to address the problem is to perform the analysis iteratively until the maximum shear strain is consistent with the bilinear model parameters. Another approach is to seek a model that has a constant parameterization for a wide range of shear strain, as discussed in the next section.

The calibrated parameters for displacement orbit 4 at the slow rate are very close to those for orbit 4 at the fast rate (2 sec/cycle). The exception to this observation is that Q_D at 210% maximum shear strain is smaller for the rapid loading than for the slow loading, possibly due to further degradation of the lead core. It can be concluded that the rate of loading is not important for modeling the behavior of lead-rubber bearings.

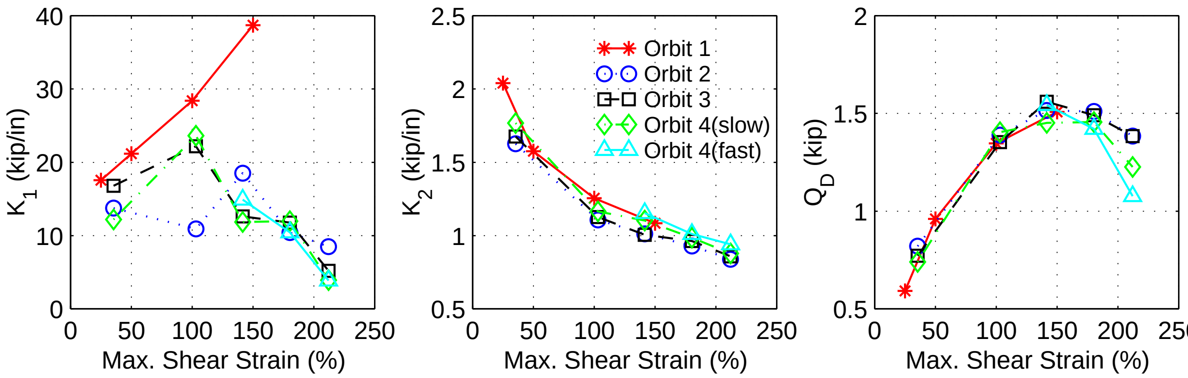

Figure 5: Best-Fit Parameters for Coupled Bilinear Model as a Function of Maximum Shear Strain

Bi-Directional Earthquake Excitation Tests

To evaluate the effects of different types of ground motions for isolated bridges, and the efficacy of bi-directional models for isolation bearings, a series of bi-directional earthquake excitation tests were conducted.

Five pairs of bi-directional ground motion representing different earthquake sources, intensities, and durations, and different soil types were used in the experimental tests with specified length and amplitude scale factors. Because of space restrictions, selected data for one earthquake test are presented, the 1978 Tabas record (M=6.7, R=6.7 km, length scale=5, amplitude scale=1).

Figure 6 shows the experimental displacement orbit and force locus. The best-fit parameters for the bilinear model, calibrated for displacement orbit 4 with 150% shear strain (2 sec/cycle), are used to simulate the bearing response under earthquake excitations. The Newmark time integration method with Newton-Raphson iteration is used to solve the nonlinear equations of motion for the isolated rigid frame modeled as a 3-DOF system with no viscous damping.

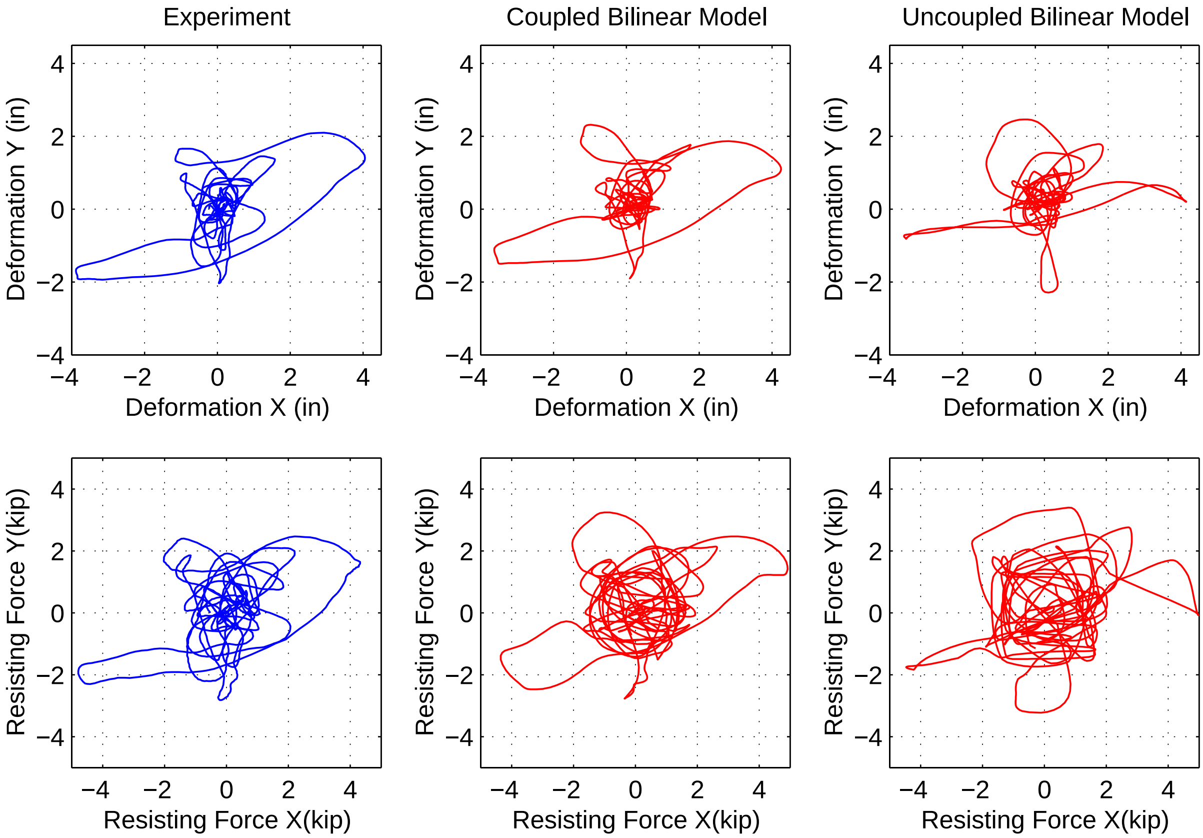

Figure 6: Experimental and Model Responses of a Bearing for 1978 Tabas Ground Motion

The experimental data show strong bi-directional coupling in the earthquake response for this near-fault record. With parameters correctly chosen for the maximum shear strain, the coupled bilinear model is able to simulate the effects of bi-directional coupling on the earthquake response. In contrast, the uncoupled bilinear model provides only a reasonable estimate of the maximum displacement but not the correct phase.

IMPROVED MODEL FOR ISOLATION BEARINGS

As shown in figure 5, the parameters for the bilinear model for lead-rubber bearings are dependent on the maximum shear strain. Therefore, an improved model with strain-independent parameters is needed to simulate the behavior of isolation bearings over a wide shear strain range. An uni-directional model applying the bounding surface theory [Dafalias and Popov, 1978] is presented in this section.

The restoring force $\boldsymbol{F}$ in an isolator consists of an elastic restoring force and hysteretic force and is expressed as in equation 1 and 3 but with a one-dimensional form. In equation 1 and 3, K_2 is the post-yield asymptotic stiffness and K_1 is the elastic stiffness within the yield surface defined in equation 5 and shown in figure 7. In one-dimensional space, the yield surface is defined as

$$\Phi(F_p,\alpha)=\left|F_p-\alpha\right|-F_y\cdot\left(1-\frac{K_2}{K_1}\right)\ , \tag{5}$$

in which α is the "back force", F_y is the yield force of an isolator as shown in figure 7. The associated flow rule is defined as

$$\dot{U}^p=\gamma\cdot\frac{\partial\Phi}{\partial F_p}=\gamma\cdot\operatorname{sgn}(F_p-\alpha)\ , \tag{6}$$

where γ is the plastic multiplier. To simulate the gradual change of post-yield stiffness from linear elastic stiffness K_1 to asymptotic stiffness K_2, the bounding surface theory is applied. The bounding surface for F_p is defined as

$$\mathrm{B}(F_p)=\left|F_p\right|-Q_D\ . \tag{7}$$

As displacement increases, the hysteretic force F_p increases asymptotically to Q_D and the post-yield stiffness decreases asymptotically from K_p^0 to K_2. The proposed nonlinear hardening rule is as follows:

$$\dot{\alpha}=(K_p^0-K_2)\cdot\left[1-\left(1-\frac{\delta}{\delta_{in}}\right)^n\right]\cdot\dot{U} \tag{8}$$

in which, K_p^0 is the initial post-yield stiffness, δ is the distance between current post-yield hysteretic force, F_p, to the bounding surface B in the loading direction, δ_{in} is the δ at initial yield (see figure 7), and the exponent n controls how rapidly the hysteretic force, F_p, increases asymptotically to Q_D. The initial distance to the bounding surface, δ_{in}, is updated whenever the hysteretic force, F_p, hits the opposite side of yield surface $\Phi(F_p,\alpha)$. Following the standard plasticity procedures, the restoring force can be obtained from equations 1, 3, and 5-8 for a given displacement history.

Six parameters, K_1, K_2, Q_D, K_p^0, F_y and n, are required to define the improved model. By choosing $K_p^0=K_1$ and $F_y\cdot(1-K_2/K_1)=0.2\cdot Q_D$, the number of parameters is reduced to four, one parameter more than the bilinear plasticity model, with a good fit to the experimental data as described in the next section.

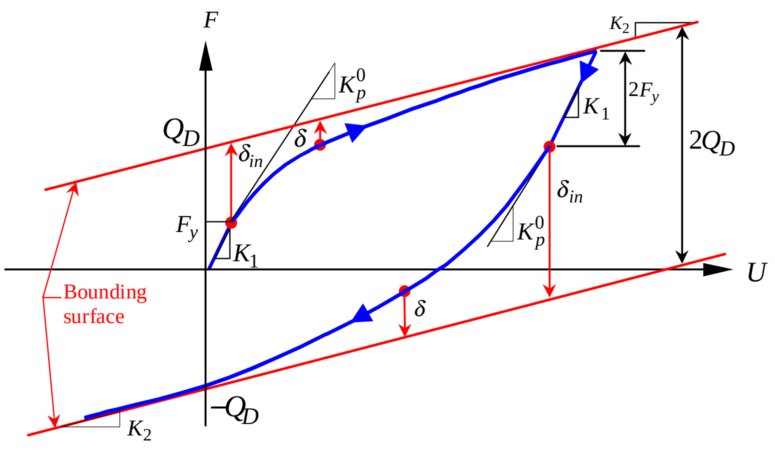

Figure 7: Schematic of Uni-directional Force-Displacement Relationship for Improved Model

EXPERIMENTAL CALIBRATION OF IMPROVED MODEL

Uni-directional displacement controlled tests were performed on the lead-rubber bearings in a single-bearing test machine. The lead-rubber bearings were subjected to cyclic displacement histories with 25, 50, 100, 150% shear strain levels under a constant vertical load of 16 kips. Figure 8 shows the force-deformation response from the experiment and models with best-fit parameters. It can be clearly observed in figure 8 that the size of hysteretic loop increases and the "post-yield stiffness" decreases as the shear strain increases, as indicated by the calibrated bilinear parameters in figure 5. The best-fit parameters for the improved model are $K_1 = 6.846$ kip/in., $K_2 = 0.8986$ kip/in., $Q_D = 0.925$ kip, and $n = 0.224$, with a normalized residual of 0.061 kip (from equation 4). The bilinear models are calibrated separately at strain levels of 25, 50, 100, 150% with residuals equal to 0.059, 0.070, 0.087, and 0.088 kip respectively. The plot in figure 8 for the bilinear model has different sets of parameters at each strain level. It is not possible to select a single bilinear model that will be accurate for all strain levels. Nor is it possible to modify the bilinear model with strain-dependent properties and produce correct hysteretic loops. In contrast, the improved model represents the strain-dependent hysteretic behavior of the lead-rubber bearings. Comparing the best fit at each strain level, the improved model gives a smaller residual for all but the smallest strain level.

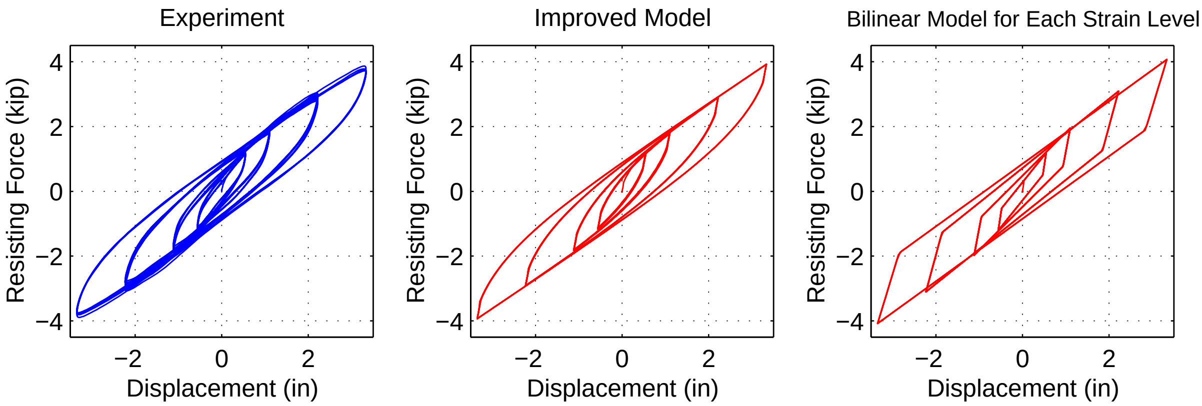

Figure 8: Comparison of Experiment and Models for Uni-directional Loading

To compare the bilinear and improved models further, figure 9 shows the best-fit parameters, calibrated from the uni-directional displacement controlled tests, for the improved model compared with the parameters for the bilinear model as a function of maximum shear strain. Similar to results from the bi-directional tests shown in figure 5, K_2 decreases and Q_D increases with maximum shear strain for the bilinear model. The strain-independent K_2 and Q_D for the improved model can be considered as asymptotes for the corresponding parameters of the bilinear model. Since the uni-directional tests were conducted after the large number of bi-directional tests, the values for Q_D in figure 9 correspond to the degraded values shown in figure 5 for the orbit 4 test at the fast loading rate.

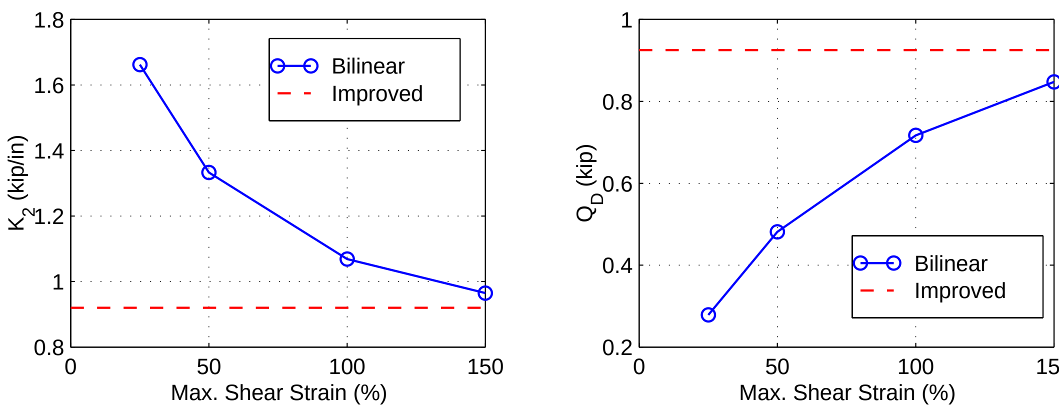

Figure 9: Best-Fit Values for K_2 and Q_D for Bilinear Model and Asymptotic Values for Improved Model

CONCLUSIONS

The rate-independent plasticity (bilinear) model provides a good characterization of the behavior of lead-rubber bearings when compared with the results of displacement-controlled, bi-directional test data. Further study is underway to determine the importance of bi-directional coupling, compared with the uncoupled bilinear model, depending on the type of earthquake, configuration of the bridge, and design of the isolation system. The experimental calibration of lead-rubber bearings shows that the parameters of the bilinear model are functions of strain level. An improved model, using bounding surface theory, has been developed with the objective of characterizing elastomeric bearings by a single set of parameters over a wide range of strain. The improved model has six independent parameters, but an excellent fit with the experimental data is obtained with two constraints on the parameters. The improved model then has four parameters, one more than the bilinear model. When compared with displacement-controlled uni-directional testing from 25% to 150% shear strains, the improved model with best-fit parameters matches the measured force-deformation hysteresis with a small residual error. The four-parameter improved model is conceptually similar to the four-parameter Bouc-Wen type of models [Wen, 1976]. However, the improved model is consistent in a plasticity sense, unlike the Bouc-Wen model, which is known to violate energy postulates [Thyagarajan and Iwan, 1990]. Extension of the improved model for bi-directional behavior is underway.

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