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STRUCTURAL DAMAGE DETECTION AND PHYSICAL STATE OF FRAME BUILDINGS

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SUMMARY

A method to detect analytically structural damage in building frames is proposed. The structural damage is expressed as the loss of stiffness. It is assessed using the transformation matrix that relates the initial condensed stiffness matrix of the structure and that perturbed, due to the structure damage. Both matrices, initial and perturbed, can be obtained using the vibration modal shapes of the structure. The theoretical location and assessment of damaged structural elements in a plane frame due to an earthquake excitation is presented.

INTRODUCTION

Nowadays, the dynamic response of a real structure can be measured and compared with the one computed mathematically. If no great differences are found, it is reasonable to assume that the analytical model is able to represent the real structure. On the contrary, the theoretical model must be modified arising the question of what of the structural parameters of the model should be modified in order to get a better representation of the real structure. On the other hand, current techniques for measuring the dynamic properties of structures allow for a permanent monitoring of their behavior making possible to obtain a large amount of data [Murià-Vila and Toro-Jaramillo, 1998] and to detect any change in the parameters that define the dynamic response, such as the structural stiffness. If changes are due to deterioration or damage, the process of adjusting the theoretical models in order to get response values similar to those of the real one is complicated.

The objective of this study is to determine the location and magnitude of damaged elements in building structures using known vibration frequencies and modal shapes. To attain this aim the transformation matrix method [Escobar *et al.*, 1999] is evaluated.

STRUCTURAL DAMAGE DETECTION

In a recent study [Hassiotis and Jeong, 1995] it was established that damage detection in structures can be reduced to the development of a mathematical model of the structure that correctly reproduce its dynamical characteristics (modal shapes and frequency vibrations) before damage, and the updating of that model in order to reproduce the new dynamical characteristics after damage. In that work it was also established that the first part of the problem had produced identification procedures developed to adjust structural parameters such as mass and stiffness in order to reproduce the measured data, and that the second part of the problem is still under development. The present paper deals whit the latter part of the problem.

To establish the combination of structural changes that must be made to adjust the structural model to new values of the known modal configurations and vibration frequencies, several methods have been developed [Stubbs and Osegueda, 1985; Lin, 1990; Ricles and Kosmatka, 1992; Peterson et al, 1995; Ferregut et al, 1995; Kahl and Sirkis, 1996; Sohn and Law, 1997, among others]. The transformation matrix method, proposed in the

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present work is based on the relationship between the terms of the condensed stiffness matrix of two and three-dimensional frames, and the stiffness reduction of the structural elements.

THE TRANSFORMATION MATRIX METHOD

The static condensation of the degrees of freedom of the *global stiffness matrix* $[K]$ of a structure produces a *condensed stiffness matrix* $[\overline{K}]=[T]^T[K][T]$, which represents a geometric transformation of the global stiffness matrix [Ghali and Neville, 1989]. In this equation $[T]$ is the *transformation matrix,* and is a function of the primary and secondary degrees of freedom of the global stiffness matrix.

On the other hand, the global stiffness matrix of a plane frame is calculated as the contribution of the stiffness matrices, in global coordinates, of each one of its structural elements. Considering each element as a substructure, the stiffness matrix $[K_d]_j$ of the *j-th* plane frame corresponding to a damage state can be written as

$$[K_d]_j = \sum_{i=1}^{nej} (1 - dk_i)[K]_{ij} \tag{1}$$

where *nej* is the number of elements or substructures in the frame; dk_i is a non dimensional parameter that represents the decrease in the contribution of the stiffness matrix of the *i-th* element to the global stiffness matrix ($0 \le dk_i \le 1$); $[K]_{ij}$ is the global stiffness matrix of the *i-th* element, of the *j-th* frame without damage. The term ($1-dk_i$) allows to determine damage states in the *i-th* substructure, which are defined as those states for which the value of dk_i is greater than a specific value, normally zero. Developing the previous expression it is obtained

$$[K_d]_j = \sum_{i=1}^{nej} [K]_{ij} - \sum_{i=1}^{nej} dk_i [K]_{ij} \tag{2}$$

Because the first sum of this equation corresponds to the original global stiffness matrix $[K_{sd}]_j$, of the frame without damage, it can be written

$$[K_d]_j = [K_{sd}]_j - \sum_{i=1}^{nej} dk_i [K]_{ij} \tag{3}$$

This equation shows that the global stiffness matrix, corresponding to a damage state, can be computed as the difference between the global stiffness matrix of the structure without damage, and a matrix containing elements whose magnitude has been modified as a consequence of structural damage. The lateral stiffness matrix $[\overline{K}_d]_j$ corresponding to a damage state of the frame is calculated as

$$[\overline{K}_d]_j = [T_d]_j^T [K_d]_j [T_d]_j \tag{4}$$

where $[T_d]_j$ is the transformation matrix associated to the damage state of the *j-th* frame. For a three-dimensional model of a building structure, to compute structural damage using the stiffness matrix that corresponds to the primary degrees of freedom (rigid body displacements of the slabs), obtained from the stiffness matrices of the plane frames that compose it (coupled plane frames), the procedure is as follows:

The global stiffness matrix of each frame is computed. The global stiffness matrix corresponding to a damage state in the *j-th* frame is

$$[K_d]_j = [K_{sd}]_j - \sum_{i=1}^{nej} dk_{ij} [K]_{ij} \tag{5}$$

where dk_{ij} is the stiffness degradation of the *i-th* element of the *j-th* frame. The lateral stiffness matrix of each damaged frame and the transformation matrices associated with a damage state are computed from the global stiffness

$$\left[\overline{K}_d\right]_j = \left[T_d\right]_j^T \left[K_d\right]_j \left[T_d\right]_j \tag{6}$$

Substituting equation (5) in (6), it is obtained

$$\left[\overline{K}_d\right]_j = \left[T_d\right]_j^T \left[K_{sd}\right]_j \left[T_d\right]_j - \sum_{i=1}^{nej} dk_{ij} \left[T_d\right]_j^T \left[K\right]_{ij} \left[T_d\right]_j \tag{7}$$

Using compatibility conditions [Ghali and Neville, 1989], the local lateral stiffness matrix of the *j-th* damaged frame is transformed to the global coordinates system. In this reference the displacements transformation matrix $[C]_j$ relates the lateral degrees of freedom of the *j-th* frame to the primary degrees of freedom of the three-dimensional structure; thus, applying this procedure to equation (7)

$$\left[C\right]_j^T \left[\overline{K}_d\right]_j \left[C\right]_j = \left[C\right]_j^T \left[T_d\right]_j^T \left[K_{sd}\right]_j \left[T_d\right]_j \left[C\right]_j - \sum_{i=1}^{nej} dk_{ij} \left[C\right]_j^T \left[T_d\right]_j^T \left[K\right]_{ij} \left[T_d\right]_j \left[C\right]_j \tag{8}$$

For a damage state, the stiffness matrix of the three-dimensional model of the structure is obtained adding the lateral stiffness matrices of each frame associated to the global displacements. This is

$$\left[\overline{Kt}_d\right] = \sum_{j=1}^{Nm} \left[C\right]_j^T \left[T_d\right]_j^T \left[K_{sd}\right]_j \left[T_d\right]_j \left[C\right]_j - \sum_{j=1}^{Nm} \sum_{i=1}^{nej} dk_{ij} \left[C\right]_j^T \left[T_d\right]_j^T \left[K\right]_{ij} \left[T_d\right]_j \left[C\right]_j \tag{9}$$

where *Nm* it is the number of frames in the structure. In this equation, the double sum represents the loss of global stiffness of the structure as the contribution of all the elements of each frame. It is convenient to change the double sum such that the contribution of all the elements of the structure is taken into account. Thus, for each structural element, no matter it belongs to one or more frames, a unique factor *dk* will be associated. Equation (9) is transformed as

$$\left[\overline{Kt}_d\right] = \sum_{j=1}^{Nm} \left[C\right]_j^T \left[T_d\right]_j^T \left[K_{sd}\right]\left[T_d\right]_j \left[C\right]_j - \sum_{r=1}^{Nr} dk_r \sum_{\substack{j=1 \\ r\in j}}^{Nm} \left[C\right]_j^T \left[T_d\right]_j^T \left[K\right]_{rj} \left[T_d\right]_j \left[C\right]_j \tag{10}$$

where *Nr* is the number of elements in the structure. Making transformations

$$\left[\overline{Kt}_{sd}\right] = \sum_{j=1}^{Nm} \left[C\right]_j^T \left[T_d\right]_j^T \left[K_{sd}\right]\left[T_d\right]_j \left[C\right]_j \tag{11}$$

$$\left[\overline{K_d}\right]_r = \sum_{\substack{j=1 \\ r\in j}}^{Nm} \left[C\right]_j^T \left[T_d\right]_j^T \left[K\right]_{rj} \left[T_d\right]_j \left[C\right]_j \tag{12}$$

where $[K_{sd}]$ is the original condensed stiffness matrix of the structure without damage; $\left[\overline{K_d}\right]_r$ is the stiffness contribution of the *r-th* element of the structure, obtained adding the stiffness of all the frames that include the element. Substituting equations (11) and (12) in (10) it is obtained

$$\left[\overline{Kt_d}\right] = \left[\overline{Kt_{sd}}\right] - \sum_{r=1}^{Nr} dk_r \left[\overline{K_d}\right]_r \tag{13}$$

From this expression it is possible to establish a linear system of equations writing an equation for the *t-th* term different of zero of each matrix. In matrix notation this idea is expressed as

$$\left\{\overline{kt_{sd}}\right\} - \left\{\overline{kt_d}\right\} = \left[S_k\right]\left\{dk\right\} \tag{14}$$

where $[S_k]$ is a matrix formed by the $\bar{k}_{dr}$ terms. The displacement transformation matrices are independent of the damage state of a frame. As initial approximation for the solution of equation (14), it can be considered that the transformation matrix corresponds to the non-damage state.

The condensed stiffness matrix corresponding to the known vibration frequencies and modal shapes can be adjusted using the Baruch and Bar Itzhack equations [Baruch and Bar Itzhack, 1978]. The condensed stiffness matrix can have terms with zero values [Sosa *et al.*, 1998], therefore, the number of equations is a function of its order, of the connectivity of the structural elements and of the selection of the structural degrees of freedom. Thus, to obtain the theoretical damage state of a structure, it is necessary to establish the specific conditions of the problem that include the elimination of equations and/or unknowns, as well as the modification of the damage interval, and to follow the above developed procedure [Escobar *et al.*, 1999].

It is worth to mention that the stiffness contribution of each structural element, obtained using equation (10), considers implicitly that the degradation of its stiffness is the same in the local reference systems of the frames and to those which belongs to the elements. However, an element common to two frames (normally orthogonal) such as a column can present different degradation stiffness in two directions [Wilson *et al.*, 1995]. To consider this effect with the proposed methodology, it is necessary to change from the location of damage in each element, to the location of damage in the local directions of each one of them.

APPLICATION

The proposed methodology to locate and assess structural damage was applied to the ten-storey reinforced concrete building show in fig 1. It was designed using the Mexico City Building Code [NTC, 1987] considering soil-structure interaction effects. Its properties were selected in such a way that its fundamental vibration period was equal to 1.0 s. To evaluate the damage state of the frame subjected to seismic loads, it was excited with real earthquakes recorded in Mexico City: September 19, 1985 (SCT-85), April 25, 1989 (SCT-89), and September 14, 1995 (SCT-95); and with the simulated earthquake records [Grigoriu *et al.*, 1986]: AX15, AX39 and AX120. Characteristics of these records are shown in fig 2. These records were selected in order to get an idea about the effect of high and middle magnitude earthquakes on the frame structure.

To carry out the non-linear analysis of the structural models, after which the dynamic characteristics of the damaged structure were computed, the program CANNY-D [Li, 1995] was used. To simulate the non-linear behavior of the structural elements, the tri-linear Takeda model with stiffness and strength deterioration was utilized. In order to increase the correspondence between the theoretical model and the real structure, rigid zones in the elements ends and shear deformation were included in the analytical model. To simulate soil-structure interaction, additional springs in the model supports were added.

In figures 3 to 5, the fundamental vibration period history, maximun storey drift, structural damage after the earthquakes, and structural damage estimated for the frame subjected to the earthquake records are presented. In general, it can be seen that damage computed values in the structural elements are consistent with the earthquake magnitude; the frame subjected to the SCT-85 record presents the greatest damage values in the structural elements. For real earthquakes, results obtained show the effect of strong column-weak beam target in the current seismic design philosophy of the [NTC, 1987]. In these cases, the damage distribution in the structure shows the greatest values for the beams than for the columns in the same storey, and the maximum computed damage value is 70.5%. All of the simulated earthquake records, cause practically same damage values to the structural elements of the building frame.

CONCLUSIONS

In this paper, the transformation matrix method to locate and estimate structural damage in structural elements of frame buildings using known modal shapes and vibration frequencies, has been proposed and evaluated. The results obtained show that the maximum computed damage values, when the frame was subjected to real earthquake records, are consistent with the maximum stiffness degradation observed in reinforced concrete structures, around 70% [Sakai *et al.*, 1989], and between $0.3E_cI_g$, and $0.8E_cI_g$ for columns [MacGregor, 1993], where E_cI_g product is the flexure stiffness, E_c is the elastic modulus of concrete and I_g the inertia moment of the original cross section. The structural response is sensible to the uncertainties about the structural parameters and to the soil-structure interaction effect for structures built on soft soil. These parameters should be taken into account through the stiffness matrix in order to get a realistic estimation of the dynamic characteristics of structures that are the basis for the proposed damage detection methodology.

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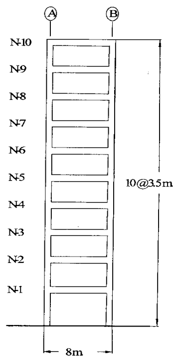

Columns 50x90 cm
Beams 40x90 cm
f'c= 250 kg/cm^2
f'y=4200 kg/cm^2
w=3.78 T/m

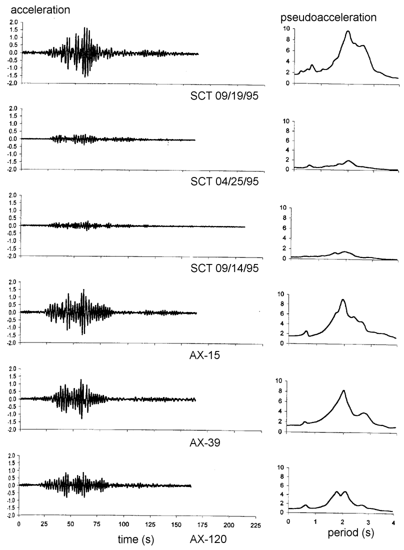

Figure 1. Frame studied

Figure 2. Earthquake records considered

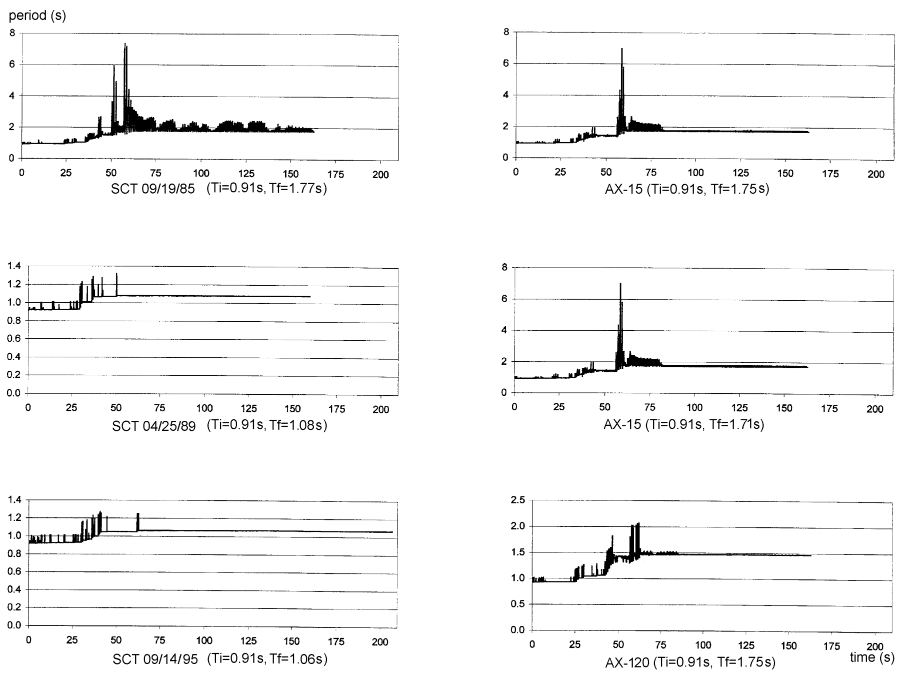

Figure 3. Fundamental period history

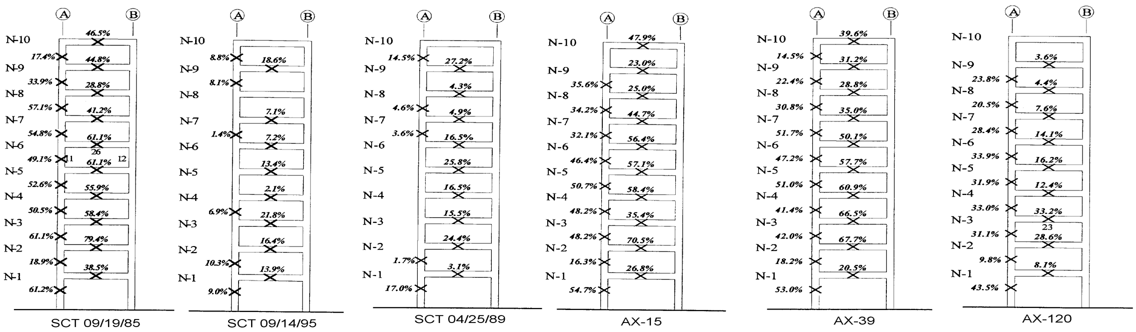

Figure 4. Computed damage using the proposed method

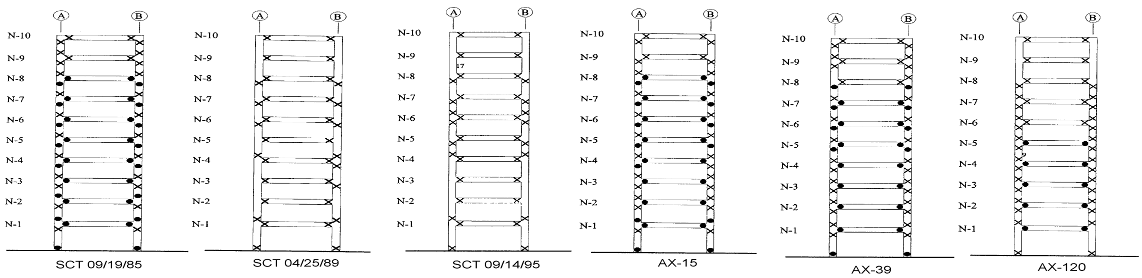

Figure 5. Computed cracked sections (×), and plastic hinges formation (•)