

ELECTRICAL ANALOG FOR EARTHQUAKE SHEAR STRESSES
IN A MULTI-STOREY BUILDING

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1. ABSTRACT

An electrical analog for the solution of the motion of a multi-degree of freedom resonator when subjected to irregular impulses of the nature of an earthquake is described. The frequency band is swept very closely and the spectra produced show that vibrations other than the fundamental have, in general, little significance.

2. INTRODUCTION

The analysis of the effects of an earthquake on a building is very complex, in that a building has infinite degrees of freedom. Bending, shear and torsion will exist in all the components. The full solution of such a problem is not practical. It is reasonable then to consider a much simplified model. The ground motion of an earthquake is substantially horizontal and the motion of many types of buildings, especially steel frame construction, may be taken as that of a compound oscillator consisting of rigid masses connected by shear springs. The rigid masses correspond to the mass of each floor and the shear springs to the shear stiffness between storeys. The analog represents electrically this model.

In other types of building construction, such as the bearing wall type, bending of the walls between storeys is more significant than shearing and a different analog circuit would be necessary.

Much work has been done previously on preparing spectra of single storey structures. Assuming reasonable values for the damping of a structure these spectra indicate a great magnification of the earthquake acceleration due to the partial resonance built up. Biot (1941) used a mechanical torsional pendulum and Housner (1951) an electronic computer to prepare spectra of many American earthquakes. The indicated accelerations which may appear to arise from these spectra, even with 10% damping, are so large that failure would appear certain. Damping has a large effect on the maximum accelerations produced and the authors considered it a possibility that partial yielding of metal frames and slight cracking of concrete might well dissipate sufficient energy to keep relative displacements reasonable.

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With this view in mind the analog was constructed to incorporate yield characteristics. However preliminary runs without yield indicated that the harmonics do not build up as anticipated and this paper is concerned with the problem without yield.

The analytical solution is perfectly straightforward but the computation involved for a series of building types and a range of earthquakes is laborious and an analog solution is more practical.

3. THE ANALOGY

The mechanical system assumed is shown in Fig. 1. Both viscous damping between storeys and also between the storey and fixed space is incorporated. It has been found from tests on buildings (Alford, Housner, 1953) that the percentage damping of the modes of a building is approximately the same and it is shown later that a certain combination of both the dampings mentioned above will roughly fulfil this condition. The analogy is now established for a three-storied building but obviously relates to any number of storeys.

- m_n = mass of the n^{th} storey.
- k_n = stiffness between the n^{th} and $(n-1)^{th}$ storey.
- b_n = damping coefficient between n^{th} and $(n-1)^{th}$ storey.
- λ_n = damping coefficient between n^{th} storey and absolute space.
- x_n = horizontal displacement of the n^{th} storey from the base or ground.
- $v_0(t)$ = velocity of the base.
- $\alpha_0(t)$ = acceleration of the base.

The following may be derived easily from the equations of motion of the system.

$$\begin{aligned}
 (1) \quad & k_3(x_3 - x_2) + b_3(\dot{x}_3 - \dot{x}_2) + m_3 \ddot{x}_3 + m_3 \alpha_0(t) + \lambda_3(\dot{x}_3 + v_0) = 0 \\
 (2) \quad & -k_3(x_3 - x_2) - b_3(\dot{x}_3 - \dot{x}_2) + k_2(x_2 - x_1) + b_2(\dot{x}_2 - \dot{x}_1) \\
 & + m_2 \ddot{x}_2 + \lambda_2(\dot{x}_2 + v_0) + m_2 \alpha_0(t) = 0 \\
 (3) \quad & -k_2(x_2 - x_1) - b_2(\dot{x}_2 - \dot{x}_1) + k_1 x_1 + b_1 \dot{x}_1 \\
 & + m_1 \ddot{x}_1 + \lambda_1(\dot{x}_1 + v_0) + m_1 \alpha_0(t) = 0
 \end{aligned}$$

Fig. 2 shows the analogous electrical circuit with resistances in both the inductive and capacitive arms.

- L_n = inductance of each coil.
 C_n = capacity in bridging arm.
 R_n = resistance of bridging arm.
 τ_n = resistance of inductive arm.
 $I(t)$ = input current.
 i_n = circulating currents.
 q_n = charge corresponding to current i_n .

A loop analysis of the circuit yields the following equations.

$$(4) \quad R_3 \left(\frac{dq_3}{dt} - \frac{dq_2}{dt} \right) + L_3 \frac{d^2 q_3}{dt^2} + L_3 \frac{d^2 Q}{dt^2} + \frac{1}{C_3} (q_3 - q_2) + \tau_3 \left(\frac{dQ}{dt} + \frac{dq_3}{dt} \right) = 0$$

$$(5) \quad R_3 \left(\frac{dq_2}{dt} - \frac{dq_1}{dt} \right) + \frac{1}{C_3} (q_2 - q_3) + R_2 \left(\frac{dq_2}{dt} - \frac{dq_1}{dt} \right) + \frac{1}{C_2} (q_2 - q_1) + L_2 \frac{d^2 q_2}{dt^2} + L_2 \frac{d^2 Q}{dt^2} + \tau_2 \left(\frac{dQ}{dt} + \frac{dq_2}{dt} \right) = 0$$

$$(6) \quad R_2 \left(\frac{dq_1}{dt} - \frac{dq_2}{dt} \right) + \frac{1}{C_2} (q_1 - q_2) + R_1 \frac{dq_1}{dt} + \frac{q_1}{C_1} + L_1 \frac{d^2 q_1}{dt^2} + L_1 \frac{d^2 Q}{dt^2} + \tau_1 \left(\frac{dQ}{dt} + \frac{dq_1}{dt} \right) = 0$$

With a suitable choice of coefficients the two sets of equations are analogous; charge in the electrical circuit representing displacement relative to the base in the mechanical system. It is necessary, however, to work at a much higher frequency in the electrical circuit. Change the time base of equations (4), (5) and (6) by putting,

$$(7) \quad t = \rho T$$

Equation (4) takes the form,

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$$(8) \quad \frac{R_3}{\rho} \left(\frac{dq_3}{dT} - \frac{dq_2}{dT^2} \right) + \frac{L_3}{\rho^2} \frac{d^2 q_1}{dT^2} + \frac{L_3}{\rho^2} \left(\frac{d^2 Q}{dT^2} \right) \\ + \frac{1}{C_3} (q_3 - q_2) + \frac{\tau_3}{\rho} \left(\frac{dQ}{dT} + \frac{dq_3}{dT} \right) = 0$$

It is necessary for the analogy then that the following hold:-

$$(9) \quad \frac{R_n}{\rho} \propto b_n$$

$$(10) \quad \frac{L_n}{\rho^2} \propto m_n$$

$$(11) \quad \frac{1}{C_n} \propto k_n$$

$$(12) \quad \frac{\tau_n}{\rho} \propto \lambda_n$$

These relations mean that the resonant frequencies are increased by the same ratio as the increase in speed of passing the earthquake record.

The percentage damping remains the same in both systems. It is shown later that the earthquake record is fitted to a drum which is allowed to run down in speed. This run down in speed effectively changes the factor ρ and thus analyses a set of buildings. The ratios between the masses, between the stiffnesses and between the damping factors, are the same for each member of the set but the frequencies go through a range as the wheel decreases in speed. Each run down of the wheel thus allows a spectrum to be drawn of a particular type of structure.

It is seen from equations (1), (2), (3) and (8) that the following equations hold.

$$(13) \quad \frac{d^2 Q(T)}{dT^2} \equiv \alpha_o(t)$$

$$(14) \quad \frac{dQ(T)}{dT} \equiv v_o(t)$$

$$(15) \quad q_n(T) \equiv x_n(t)$$

Equation (13) if written in terms of amps and seconds becomes

$$(16) \quad \rho^2 \frac{d^2 Q(T)}{dt^2} = \rho^2 \frac{dI}{dt} \equiv \alpha_o(t)$$

i.e. the current fed in is to be such that equation (16) holds. However it was found more convenient to integrate the voltage ordinate corresponding to the accelogram ordinate and feed in a current proportional to this integrated voltage. This means that instead of equation (16) we have

$$(17) \quad \frac{dI(T)}{dt} \equiv \alpha_o(t)$$

Consequently, instead of equation (15) we have,

$$(18) \quad x_n(t) \equiv \frac{q_n(T)}{\rho^2}$$

We wish to obtain finally the spectrum in the form of the equivalent static acceleration of the building which would produce the same stress at the various storeys, i.e. the force at each storey divided by the total mass above it. This follows simply from equations (10), (11), (18) as

$$(19) \quad \begin{aligned} S_n &= \frac{k_n (x_n - x_{n-1})}{(m_n + m_{n+1} + \dots + m_p)} \\ &= \frac{\rho^2 [q_n(T) - q_{n-1}(T)]}{C_n \rho^2 (L_n + L_{n+1} + \dots + L_p)} \\ &= \frac{[q_n(T) - q_{n-1}(T)]}{C_n (L_n + L_{n+1} + L_{n+2} + \dots + L_p)} \end{aligned}$$

This result is independent of the factor ρ and thus the spectrum, as defined above, may be measured directly, with suitable calibration, from the voltages appearing on the condensers.

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4. DAMPING

The nature of the damping of a structure is not really known. However, indications are that the first few modes have approximately the same percentage damping. This may be arranged by the combination of the two dampings already mentioned. It may be easily shown that the percentage damping of any mode is equal to the ratio of the energy loss in that mode to the maximum potential energy of the mode. The following estimation is general in principle for any relation between the masses and stiffnesses but is illustrated here for the case of a uniform building, i.e. all the storeys identical.

- (20) p_m = angular frequency of the m^{th} mode.
- (21) p_i = individual storey frequency.
- (22) α_m = ratio of the damping in the m^{th} mode to the damping in an individual storey.
- (23) $\beta = \frac{b}{\lambda}$
- (24) $x_{m,n}$ = displacement of the n^{th} storey in the m^{th} mode.

A consideration of the energy loss and the potential energy of the system shows that,

$$(25) \quad \alpha_m = \frac{p_m \left[\beta + \frac{\sum_{n=1}^{n=p} x_{m,n}^2}{\sum_{n=1}^{n=p} (x_{m,n} - x_{m,n-1})^2} \right]}{p_i (\beta + 1)}$$

This result is plotted in Fig. 3 for a uniform building and it is noticed that if β is chosen to be four, the damping in the first few harmonics is approximately the same.

The shape of the modes are determined experimentally but in the uniform case may be determined theoretically as shown in the appendix.

5. DESCRIPTION OF THE ANALOG

(a) Method of Excitation

From the sharply-peaked response curves which have been published from results on simple one-storey torsional pendulum analysers it appeared that a large number of readings would have to be taken to define these peaks on the response curves of a multi-storey resonator. To take the required number of points by a point-by-point method, i.e., using a fixed wheel speed for the application of the earthquake impulse, would have taken a very considerable time. It was decided, therefore, to fix the earthquake record to a wheel which could be motored up to some required top speed and then take recordings as the wheel slowed down exponentially under the influence of an eddy current brake.

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It can be seen from equation (19) above that if the condenser voltage is recorded and plotted against building period, then no correction for ρ is required.

A suitable wheel exists in this Laboratory as part of the Admiralty Harmonic Analyser and is shown in Fig. 6. It is approximately 30 inches in diameter with a 5 inch wide rim and can be driven up to 600 r.p.m. An adjustable eddy-current brake will bring it to rest in times ranging from 50 minutes to about 15 minutes. For our purposes the brake was left in its least effective position taking about 50 minutes to run down from 600 r.p.m. to 3 r.p.m.

The earthquake records are enlarged so that their maximum double amplitude is about 3 inches and their length about 30 inches, i.e., not longer than one-third of the circumference of the wheel. The remaining two-thirds of the wheel is used for timing pulses and allows time for relays and camera to operate before the next application of the earthquake impulse. It also allows the networks to become quiescent.

(b) Display and Recording Method

The method of display and recording are arranged to give the response curves ready plotted to ordinates of length suitable for publication without further re-plotting. This is done by displaying the required voltage on a Cossor Type 1049 cathode ray oscilloscope with the time base switched off and by recording on a Cossor Type 1428 camera suitably modified. The modifications are to the film transport mechanism. A standard Post Office unisector switch replaces the normal motor on the side of the camera and transports the film through appropriate gearing, the unisector being pulsed by means of a valve-operated relay from a mark on the wheel.

Thus the film is moved 0.002 inch for each revolution of the wheel, the movement of the film taking place very shortly after the end of the earthquake impulse to the building networks. One bank of the unisector switch contacts are connected to the beam-brightening terminals of the oscilloscope so that the film is exposed once every five revolutions of the wheel. Time marks are also put onto the film at one minute intervals by brightening the spot for a short period and thus drawing black lines across the trace. These timing marks, together with the number of revolutions of the wheel, plainly visible on the film, enable ρ to be evaluated, and the equivalent building period to be calculated and printed on the film as a time base. The wheel speed and therefore the film speed decays exponentially and consequently this time base is linear. A typical record is shown in Fig. 5.

(c) Driving Amplifier (Fig. 4)

The earthquake record after being enlarged and fixed to the wheel is blacked-in on one side. This is scanned through an illuminated slit by two photo-cells mounted in the box shown on the right of the photograph (Fig. 6). Also mounted in the box are two amplifier stages and a cathode follower feeding into the connecting cable. The signals from this cable are then amplified in three fairly low-gain stages and then integrated for the reasons given in para. (17) above. This integrator has a comparatively short time constant of 0.1 seconds, in order that slight unbalance of the

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black and white sides of the earthquake record due to uncertainty of the base-line position, is not added up. The integrated signal is then amplified in two stages and fed to the grid of the 6AG7 driving valve. The impedance of this valve is high compared with the variations of load impedance and hence the current flowing into the networks is always proportional to the integral of the earthquake acceleration as required in Equation (17) above.

(d) Networks

The ten networks are arranged in the same manner as in the basic circuit (Fig. 2) with the addition only of switchedappings on the inductances and switched capacity values at each condenser point.

The inductances are air-cored, having a maximum inductance of 3 Henries and are tapped at ten points. They are mounted in line with the axis of each alternate coil at right angles to its neighbours to minimise pick-up. Below the coils are mounted the condensers, switches and resistors on two-storey unit chassis. All the necessary measuring points are brought out to sockets at the front of these panels.

The switching is arranged so that any number of stories up to ten can be selected readily, and also it is possible to open-circuit all the condensers to produce an infinitely stiff building for calibration purposes.

Provision is also made for biased diodes to be connected across the condensers to simulate plastic yield in the springs.

6. RESULTS

At the present time only one earthquake (Cook Strait, 13 January 1950, North and South) has been used and this has been applied to two types of building. The first building known as the Uniform Type is of ten storeys, each storey being equal in mass, stiffness and damping.

The second building known as the Graded Type comprises ten storeys, each being equal in mass and damping, but the inter-storey stiffnesses varying in proportion to the total mass above, i.e. under the influence of a static acceleration it deflects in a straight line.

Both buildings have been checked at two damping values, $c/c_0 = 4\%$ and 10% .

The buildings were set up by selection of the appropriate inductance, condenser and resistance settings, the latter being found from the curves shown in Fig. 3. Practical tests were carried out on the networks to check the theoretical damping ratios. This was done by exciting the networks with an audio oscillator, the frequency of which was slowly raised through the range of the harmonics, while the voltage across condenser No. 1 was noted. The input current was maintained constant. The damping at each harmonic was then calculated using the relationship

$$(26) \quad c/c_0 = \frac{\Delta f}{f_r}$$

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where

(27) C/C_0 = damping ratio,

(28) Δf = width of resonance peak at 90% of maximum amplitude,

(29) f_r = resonant frequency.

Good agreement with the theoretical values was obtained.

The shapes of the modes of oscillation had previously been noted, using the audio oscillator, in order that these computations could be carried out.

Before each spectrum was recorded a calibration run was made by open-circuiting all the condenser arms, i.e. making the building infinitely stiff, running the wheel up to 300 r.p.m., and making a short recording of the response. This calibration can be seen on the right of Fig. 5.

The amplitude of the left-hand trace represents the ratio

$$\frac{\text{max. inter-storey shear force}}{\text{total mass above}}$$

as explained by Equation (19), whereas the right-hand calibration trace represents the same ratio when the building is infinitely stiff. If the response value at any particular period is divided by the calibration value, the amplification factor plotted in Figs. 7, 8, 9 and 10 is obtained. This value is essentially a design factor, in that the equivalent static acceleration to give the same stresses at any storey is obtained by multiplying the maximum value of the earthquake accelerogram by the amplification factor.

7. CONCLUSIONS

Drawing conclusions from the results obtained is somewhat difficult unless the difference between stiffness and strength is kept clearly in mind.

It is seen that for a uniformly stiff structure the upper storeys require a larger design "g" value than the lower, particularly when oscillating predominantly in the second harmonic (1.1 sec period). For the "graded type no. 1" structure the upper storeys require an even further increase in design "g" values relative to the lower, both for resonance in the fundamental as well as in the second harmonic mode (0.2 sec and 0.4 sec fundamental, and 0.9 sec for the second harmonic). The influence of the second harmonic is not so pronounced in this case.

If we make the assumption that for practical reasons stiffness and strength are proportional, it follows that for the uniform structure if the bottom storey is just strong enough the upper storeys are unnecessarily strong. For the graded structure, if the bottom storey is just strong enough, the upper storeys have inadequate strength. On this basis the second harmonic has very little influence, except in the upper storeys of the graded structure.

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It would appear that the ideal structure which gives a uniform stress distribution throughout, lies somewhere between these two types. The authors suspect that most actual structures also lie between these two types. This is regarded as being extremely fortuitous.

It is intended that in future work all available earthquakes will be fed through 10 storey structures of three types (the two already chosen and a third intermediate type) and an attempt made to define a grading of design "g" value up the building and to relate this to the fundamental period of the structure. The absolute value of "g" to be catered for will remain an open question until the damping of actual structures and their possible yield has been thoroughly assessed. It would appear however that a tall structure being as it is relatively free from harmonic excitations has quite a bit to recommend it if the fundamental period is long enough to place it on the reduced part of the simple spectrum.

8. BIBLIOGRAPHY

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9. NOMENCLATURE

- m_n = mass of the n^{th} storey.
- k_n = stiffness between the n^{th} and $(n-1)^{th}$ storey.
- b_n = damping coefficient between the n^{th} and $(n-1)^{th}$ storey.
- λ_n = damping coefficient between the n^{th} storey and absolute space.
- x_n = horizontal displacement of the n^{th} storey from the base.
- $v_0(t)$ = velocity of the base.
- $\alpha_0(t)$ = acceleration of the base.
- L_n = inductance of the n^{th} coil.
- C_n = capacitance of the n^{th} coil.

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- R_n = resistance of the bridging arms.
 T_n = resistance of inductive arm.
 i_n = circulating current in the n^{th} circuit.
 q_n = charge corresponding to current i_n .
 S_n = spectrum magnitude.
 p_m = angular frequency of the m^{th} mode.
 p_i = angular frequency of an individual storey.
 α_m = ratio of the percentage damping in the m^{th} mode to that in an individual storey.
 $\beta = \frac{b}{\lambda}$
 $x_{m,n}$ = displacement of the n^{th} storey in the m^{th} mode.

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APPENDIX

The case of an undamped uniform building yields relatively simply to analysis. In this case the equations of motion assume the form,

$$(1) \quad m \ddot{x}_n = k(x_{n+1} - x_n) + k(x_{n-1} - x_n), \quad n = 0, 1, \dots, (p-1).$$

$$(2) \quad m \ddot{x}_p = k(x_{p-1} - x_p)$$

where m is the mass of each floor, k is the stiffness between floors, x_n is the absolute displacement of the n^{th} floor, the coordinate x_0 is the ground displacement, and p is the number of storeys. A Laplace transformation of these equations gives,

$$(3) \quad f_n = A(f_{n+1} + f_{n-1})$$

$$(4) \quad f_p = B f_{p-1}$$

where,

$$(5) \quad A = \frac{1}{2 + \frac{m\omega^2}{k}}, \quad B = \frac{1}{1 + \frac{m\omega^2}{k}}$$

and f_n is the Laplace transform of x_n .

This difference equation may be solved to give,

$$(6) \quad \frac{f_n}{f_0} = \frac{\cos(p-n+\frac{1}{2})\theta}{\cos(n+\frac{1}{2})\theta}$$

where,

$$(7) \quad 2 \sin \frac{\theta}{2} = -i\omega \sqrt{\frac{m}{k}}$$

If $F_0(\omega)$ is the transform of $d_0(t)$ then,

$$(8) \quad f_n = \frac{\cos(p-n+\frac{1}{2})\theta}{\cos(p+\frac{1}{2})\theta} \frac{F_0(\omega)}{\omega^2}$$

The critical frequencies follow from,

$$(9) \quad \cos(p+\frac{1}{2})\theta_m = 0, \quad 2 \sin \frac{\theta_m}{2} = -i\omega_m \sqrt{\frac{m}{k}}$$

By an application of the convolution theorem and a contour integration the displacements become,

$$(10) \quad x_n = x_0 + \sum_{m=1}^p \frac{2\sqrt{\frac{k}{m}} \cos \frac{\theta_m}{2} \sin(n\theta_m)}{(p+\frac{1}{2}) \omega_m'^2} \int_0^t \alpha_d(\tau) \sin \omega_m'(t-\tau) d\tau$$

where,

$$(11) \quad -j \omega_m = \omega_m'$$

We wish to compare the maximum relative displacements that occur in the dynamic case with those which would occur if a static acceleration were applied to the structure. If the static displacements are denoted by dashes then,

$$(12) \quad x_n' - x_{n-1}' = \left[p - (n-1) \right] \frac{m \alpha_0(t)}{k}$$

The ratio we are interested in is then,

$$(13) \quad \frac{x_n - x_{n-1}}{x_n' - x_{n-1}'} = \sum_{m=1}^p \frac{\cos \frac{\theta_m}{2} [\sin n\theta_m - \sin(n-1)\theta_m] \omega_m'}{4 (p+\frac{1}{2})(p-n+1) \sin^3 \frac{\theta_m}{2} \alpha_0(t)} \int_0^t \alpha_d(\tau) \sin \omega_m'(t-\tau) d\tau$$

The coefficients,

$$(14) \quad D_{m,n} = \frac{\cos \frac{\theta_m}{2} [\sin n\theta_m - \sin(n-1)\theta_m]}{4 (p+\frac{1}{2})(p-n+1) \sin^3 \frac{\theta_m}{2}},$$

represent the "weighting factors" to be given to each harmonic at the various storeys. The rest of the expression is the spectrum of the earthquake divided by the static acceleration. The same coefficients hold approximately if the undamped spectrum is replaced by the damped one. The following table gives these coefficients for three positions on a ten-storey structure for the first few harmonics.

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m	n = 1 $D_{m,1}$	n = 8 $D_{m,8}$	n = 10 $D_{m,10}$
1	0.840	1.120	0.008
2	0.094	0.320	0.588
3	0.030	0.084	0.220
4	0.014	0.000	0.144
5	0.008	0.025	0.092
6	0.004	0.020	0.062

Fig. 11 shows the experimental and theoretical spectra for the eighth storey of a four per centum damped uniform building. The theoretical curve is computed from the coefficients in the above table but no allowance was made for phasing of the components and consequently the computed values should everywhere be slightly higher than the experimental ones.

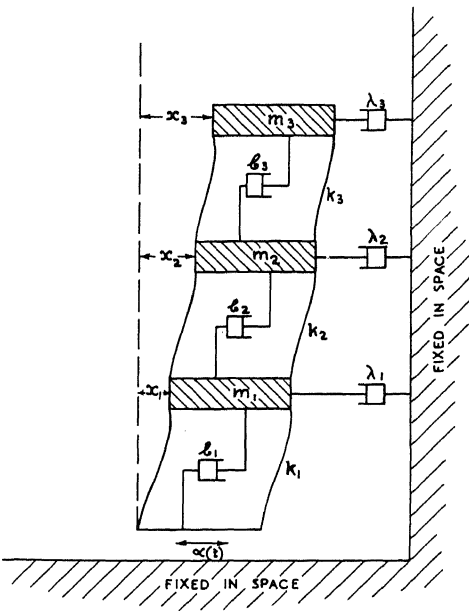


Fig. 1 Mechanical System

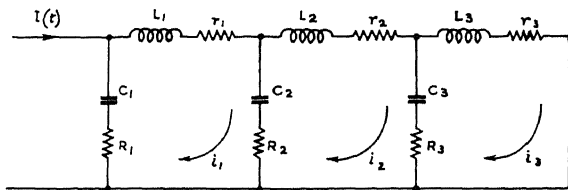


Fig. 2 Equivalent Electrical Circuit

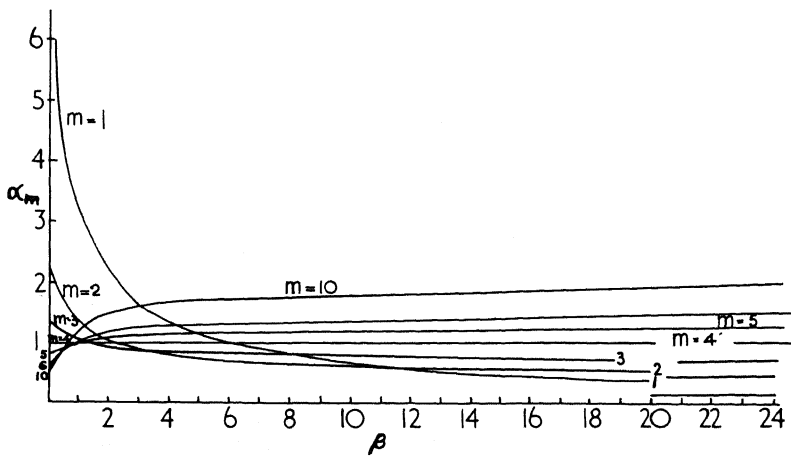


Fig. 3 Damping Curves

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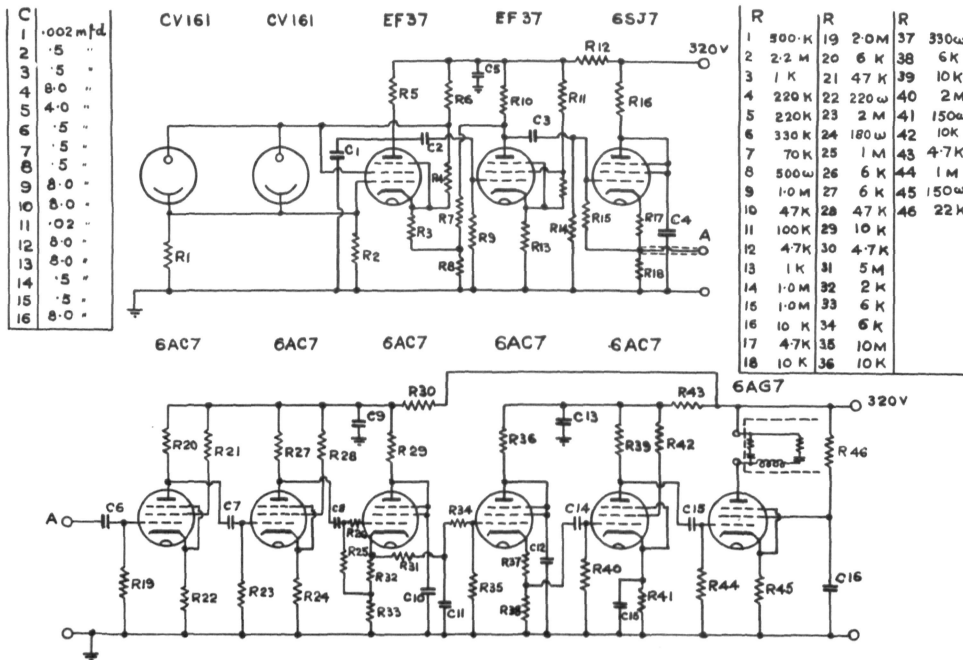


Fig. 4 Driving Amplifier Circuit

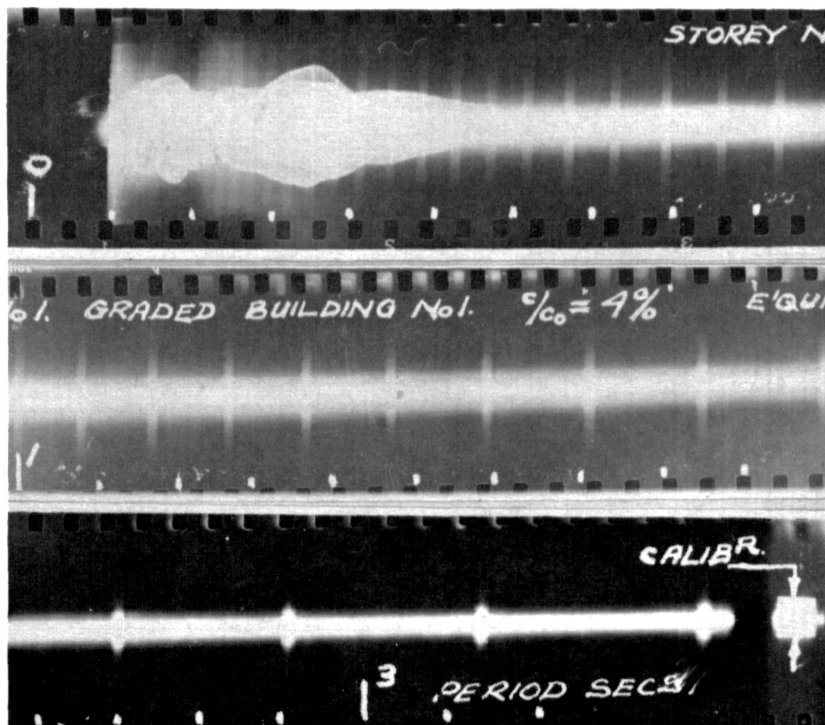


Fig. 5 Typical Record

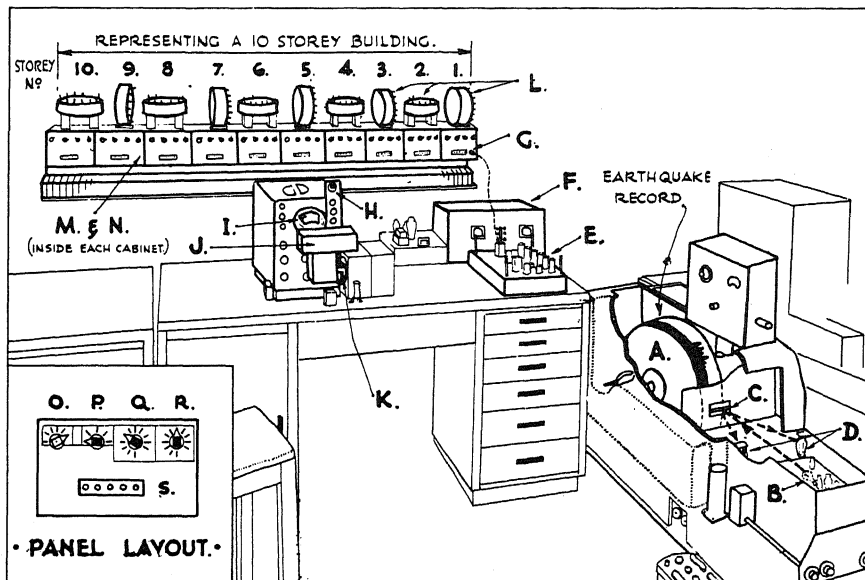


Fig. 6 Arrangement of 10 storey analog

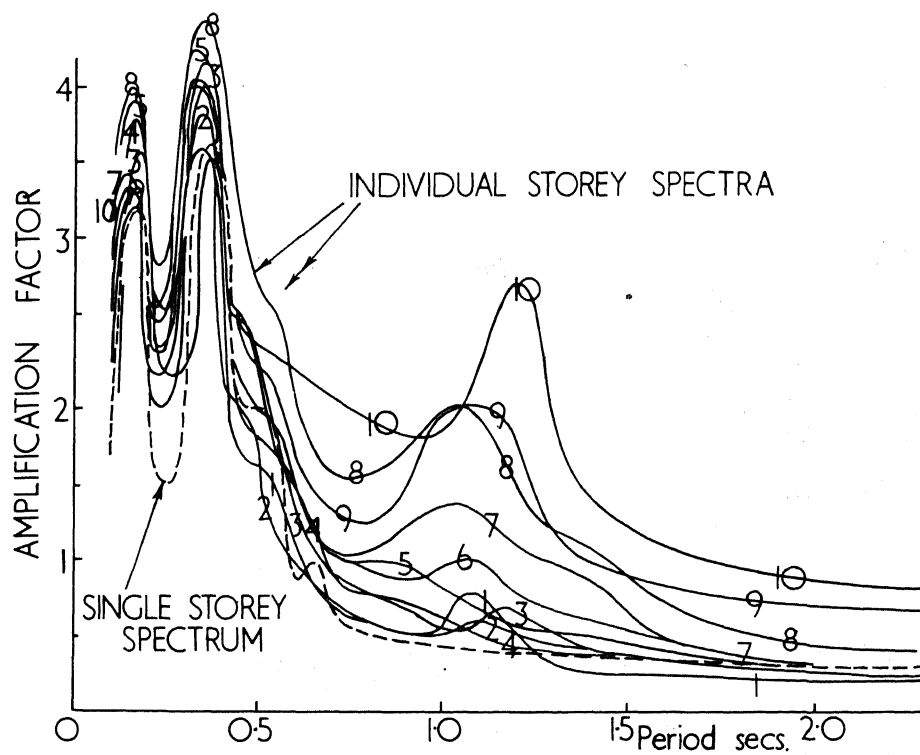


Fig. 7 Spectra: Uniform Building $\frac{c}{c_0} = 4\%$

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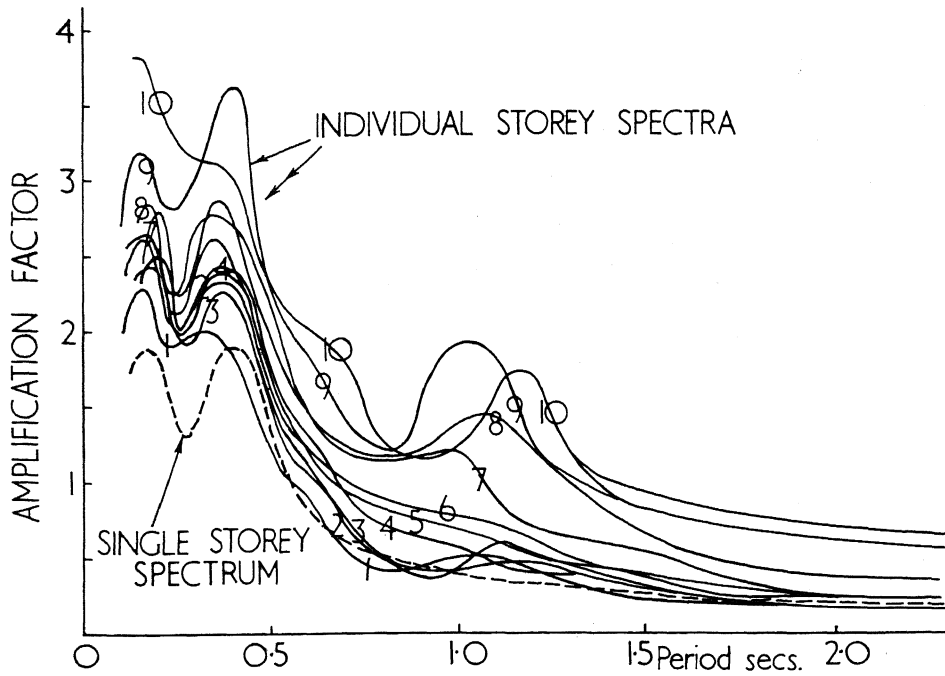


Fig. 8 Spectra: Uniform Building $\%c_o = 10\%$

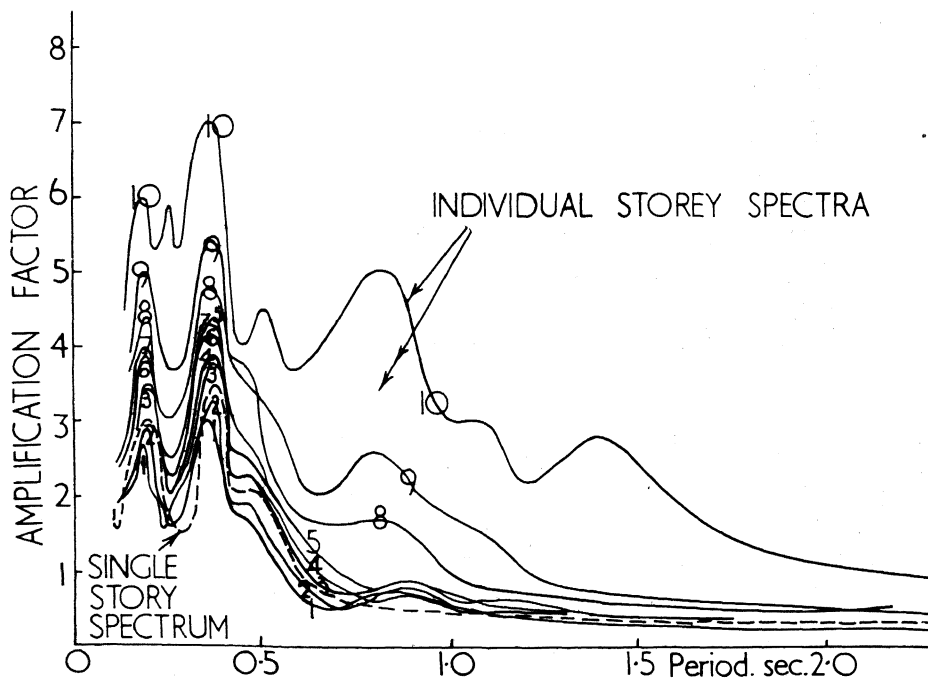


Fig. 9 Spectra: Graded No. 1 Type Building $\%c_o = 4\%$

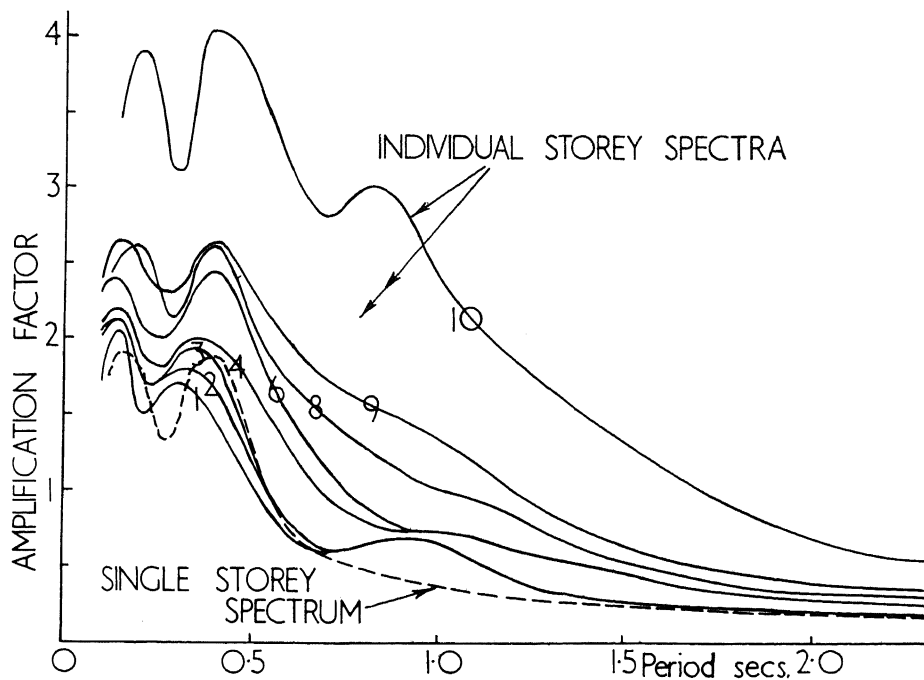


Fig. 10 Spectra: Graded No. 1 Type Building $\%c_o = 10\%$

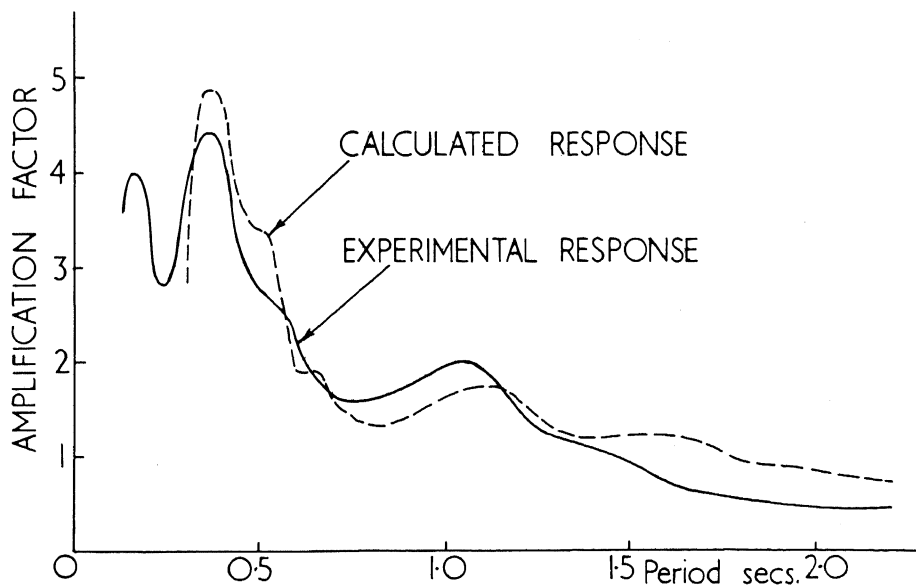


Fig. 11 Experimental and theoretical response on the eighth storey of a uniform building.