STUDIES ON THE NONLINEAR VIBRATIONS OF STRUCTURES SUBJECTED TO DESTRUCTIVE EARTHQUAKES

By RYO TANABASHI [1]

PREFACE

In this paper the writer discusses the resistance of elasto-plastic structures against earthquake forces.

It is unnecessary for building structures to be so resistant that no structural members of a building suffer damage. Rather structural damage should be prevented which would result in disintegration of those portions of the building affecting public safety and cause excessive costs of rehabilitation. Should the damage be curbed at a level which, even for the very disastrous earthquakes expected to occur once or twice in a century, would result in collapse of the structure? This is a matter of economics. It must be realized that failure of subordinate members could work actually towards the preservation of the main structure.

To portray clearly the effect of earthquake shocks on buildings, the dynamic behavior of structures, and particularly of elasto-plastic structures, when subjected to impressed displacements is of great importance.

No great attention was given to the analyses of nonlinear vibration problems of building structures until recent years due to the mathematical difficulties involved. Attention has been paid to the importance of this consideration by several writers in the past, and included is a paper by the writer published in 1937 (1).

A brief historical review of the studies involved in this field of earthquake engineering is presented herein.

HISTORICAL REVIEW AND THE OBJECTIVE OF STUDIES

During the thirty years since the 1923 Kanto earthquake, there have been several shocks of equal or larger intensity and as a result seismologists in Japan have been able to develop and maintain intensive study. Strong-motion seismological records in Japan are too limited, however, to apply them to our research in earthquake resistant design. Our program to obtain instrument records of ground motions during strong earthquakes has just been recently initiated.

From many measurements of the natural period of buildings and other structures constructed following the building codes of Japan, it has become known that the natural period is usually in the range of from 0.3 to 1.3 secs. It can be stated conservatively from this information and past experience that earthquake pulses with large accelerations having a period range between 0.3 and 1.3 secs would be the most undesirable for building structures in Japan. Therefore, it is assumed that structures must be appropriately designed for the possibility of resonance.

---

1 Prof. of Structural Engineering, Kyoto University, Kyoto, Japan

This does not mean that the occurence of the so-called "resonance phenomena" is the expected behavior of structures during an earthquake. An important characteristic of earthquake shocks is the difference between a time series of ground pulses and the sinsusoidal external forces employed in engineering vibration problems. Earthquake motions are random and every successive ground pulse has an individual period. The writer believes that the structural engineer must design a structure for the most unfavorable case which would consist of the structure synchronizing with a small number of the most destructive pulses in a time series.

Such considerations were outlined in the writer's paper published in 1937 (1). The writer maintained, if the natural frequency of the building coincides with the earthquake period, that, "the destructive force of the earthquake is proportional to the square of the maximum velocity and not to the acceleration", and further that, "the resistance of the structure against the earthquake force is proportional to the potential energy conserved in the structure until failure rather than the measure of the force safely applied horizontally".

This theory may be expressed in a different manner. For instance, when an idealized oscillating mass system, m, is subjected to a ground pulse with a maximum velocity, v, the kinetic energy of the system, T, is equal to $\frac{1}{2}mv^2$. The deformation of the system will be maximum when the kinetic energy transmitted by the shock is fully changed to potential energy of deformation. In cases where the mechanical properties of the mass system is elastic but brittle, if the kinetic energy, T, is larger than the potential energy, L, that can be stored before the destruction limit of the system, then the system must collapse. However, in case the system is so elasto-plastic that it can be classified as ductile, the potential energy stored before the collapse is L + M, where M is the potential energy of the plastic deformation range, and a larger value of T compared with L does not always cause collapse. In the latter case, the destruction of the system depends upon whether the value of T is larger than that of L + M.

In his paper (1), the writer did not state that collapse of structures could depend upon the action of only one pulse but that the damage would be developed by a number of the seismic pulses. At that time, further research was expected on the characteristics of and the relationship between: 1. A small number of displacement pulses with a large acceleration value, 2. A large number of pulses with a small acceleration value and, 3. Pulses of the same period and which appear in succession.

The writer proposed (1) to define the destructiveness of an earthquake by the maximum velocity of ground motions. This concept also was based esentially on the empirical facts that in earthquakes with short periods every pulse generally has a small displacement but has a large acceleration, and vice versa.

Such an idea in aseismic design of structures may be translated generally into the conventional design use of seismic coefficients as follows: Rigid or low buildings with short periods should be designed using large values for the seismic coefficient and, on the contrary, flexible or tall buildings with long periods may be designed to withstand lateral forces corresponding to a small seismic coefficient. An example of this idea would be:

Period of Building	Seismic Coefficient, K
0.2 sec	0.5 - 0.8
0.5	0.2 - 0.32
1.0	0.1 - 0.16

Studies by M. A. Biot in 1943 (2), on the spectrum analysis of earthquake motion were appropriate to this concept advanced by the writer in 1937. However, for the numerical determination of coefficients, broad and extensive investigation of earthquake characteristics is necessary.

The natural period of a building lengthens with increasing height and it would seem that less hazards would be encountered for structures with a long period. From our experiences on the 1923 Kanto earthquake, we have no evidence that tall buildings were more dangerous and conversely no definite evidence that they were safer. It was concluded by the writer (1) that, in the future, extensive study was necessary to determine the elasto-plastic behavior of structural materials. Along this line much research has been undertaken, and particularly in recent years.

RECENT STUDIES

In 1954 studies were made by mathematical analysis of the dynamic behavior of building structures subjected to earthquakes, by assuming that the period of both the structures and ground pulses were in the same range (3). The study was an attempt to identify the most destructive element of seismic waves for structures. For the purpose, an arbitrary series of constant acceleration pulses, equivalent to complicated patterns of ground motion, were used. The structure was represented as a continuous shear beam in one dimension, under the assumption that the structure is subjected to a shearing vibration wherever the structure is slender.

Under an impressed ground displacement pulse, no important relationship could be found between the maximum deflection of the structure and the ground acceleration value. It was confirmed, however, that the deflection grew large whenever the period of the displacement pulses coincided with the natural period of the idealized structure. This was so if every pattern of pulses had a constant mean velocity which is defined as a ratio of the pulse amplitude to it's half period. When the period of ground motion approached closely to that of the natural period of the structure, there was little difference in the structural behavior from any arbitrary pattern of ground motion as long as the mean velocity was the same.

The results indicated that if little difference exists between the period of structure and the period of pulses, the maximum structural deflection in a transient state most likely depends on the pulse velocity.

The above analysis shows the appropriateness of the proposed design curve for the lateral force coefficient, $C = K/T$, in the Joint Committee Proposal, ASCE (4).

ANALYSIS OF STRUCTURAL RESPONSE

Structural materials generally employed in building frames do not have purely elastic properties but rather elasto-plastic properties. Many analyses, including that mentioned above (3), have been made assuming the structure as purely elastic. It can be appreciated that these results do not correspond always to actual structural behavior in strong earthquakes.

These results would be applicable only when the stresses in the structure are not large. If it was intended that the structure act elastically, even in case of strong earthquakes, very large structural members would have to be provided. Special emphasis on linear analyses of structures allows no appropriate consideration of the plasticity or ductility of materials. The plasticity of materials is not only associated with the ductility (resistance capacity to shock) but also plays a vital role in energy dissipation.

A large degree of design confidence can be had by an engineer only if he knows both the effects of earthquake shocks on the structure and the resistance thereto. In this connection, the analysis of the elasto-plastic structure to non-linear vibrations is indispensable.

Non-linear problems, particularly in stationary vibrations, have been studied intensively in the fields of mechanical and electrical engineering since the beginning of this century. The advances that have been made in these fields are almost always concentrated to obtaining responses to systems acted upon by sinsusoidal external forces or displacements, both in steady or transient states, in which regularity is recognized. For this reason these advances and results can hardly be applied to the problem in the field of earthquake engineering.

Our problem is to estimate the earthquake resistance due to the ductility of a structure by means of dynamic analysis in which distinctive features are the restoring and dissipative forces of the system and the irregularity of the time-varying forces. The non-linearity of the system is associated with a specific property of elasto-plastic materials which commonly displays a distinct hysteresis in a wide variety of stress-strain relationships.

ESSENTIAL EARTHQUAKE FACTORS

Every earthquake record presents remarkable complications due to peculiar ground characteristics and the restraint of existing buildings. It is improbable that the pattern of ground motion, as it affects any one structure can be predicted for future earthquakes. There must be, however, the one most unfavorable element which has a direct connection to structural damage among the several factors involved in the time-displacements curves of ground motion. The determination of this element appears to be of great importance.

It is possible to substitute, with fair accuracy, a series of straight lines and/or quadratic curves for the complicated time-displacement curves at very small time intervals. Basically it is convenient in obtaining the dynamic behavior of structures, subjected to such ground motions, to use graphical procedures described later or to use step-by-step numerical computations. A series of constant acceleration pulses also can be substituted easily for any pattern of seismic displacements, as shown in Fig. 5.

Special attention is required to clarify the relationship between the periods of structure and ground, and such factors as the acceleration patterns, velocity patterns and displacement patterns of the ground motion.

Several types of elasto-plastic structures considered in this paper have been analyzed subjected to the following types of pulse series with a variety of time durations:

1. Quadratic displacement pulses of a constant acceleration.

2. Quadratic pulses of a constant mean velocity ("mean velocity" is a ratio of the amplitude of pulses to the corresponding period).

3. Quadratic pulses of a constant displacement (a constant amplitude).

PRELIMINARY EXPLANATIONS

The results of the analyses are given later in the paper. A preliminary discussion of procedure is presented.

Structural Characteristics.

In general, the dynamic analysis of an n-story framed structure is simplified and is made by assuming the structure as a system of n-degrees of freedom. However, the elasto-plastic dynamic restoring forces of the system have wide variety dependent upon such factors as the construction material, wall openings and partitions and the quality of workmanship, and will differ from the statical restoring forces. The dynamic elasto-plastic restoring force features were assumed to have the characteristics as shown in Fig. 1 (1). The effects of variety in such elasto-plastic restoring forces have been studied to show their consequent influences on the dynamic behavior.

Fig. 1 shows the responses of single-mass systems with differing restoring forces characteristics, namely, pure elastic, elasto-plastic and ideal-plastic. The single-mass systems were subjected to a series of ground pulses of constant period coinciding with the initial periods of the systems. There is little difference between the dynamic behaviors corresponding to Figs. 1 (1) and 1 (2). As noted, some slight modification in the restoring force curve shows little influence on the dynamic behavior of the structure. Consequently, for simplcity, the restoring force diagram was assumed to be ideal elasto-plastic.

Consideration on the Continuity of Earthquake Shocks.

It is felt in elastic problems that when the periods of the ground pulses coincide with the natural frequency of the structure, the structure is liable to severe damage. Is this presumption also true in the case of non-linear behavior of elasto-plastic structures? In Fig. 2, it is noted that after the yield point stress is exceeded, the period of the system elongates and the amplitude of oscillation remains constant due to energy dissipation. From Fig. 2, it is to be noted further that for an elasto-plastic system, if the yield point stress is exceeded within one cycle of oscillation, this maximum deflection of the system is maintained regardless of the number of succeeding pulses.

Some seismologists feel that a small number of pulses with large amplitude appear repeatedly in the secondary wave (distortional) of destructive earthquakes. After a system has become non-linear, subsequent ground motion of intensity equal to or less than in previous pulses will not develop larger deformation due to energy dissipation from a hysteresis of restoring force and to period elongation.

From results of these initial studies, the ground motion has been simplified for use in analysis; and as representative of this motion, a quadratic displacement pulse pattern is considered.

RESULTS OF ANALYSIS

Figs. 3 and 4 show the time-displacement curves for each of the two levels of a two-mass system acted upon by an acceleration pulse and a displacement pulse with a duration of one period of the system.

In an attempt to clarify the most decisive factor in the effects of the displacement pulse on a structure, a number of the dynamic behaviors of elasto-plastic one-mass systems are compared with each other in regard to variation both of the acceleration values and the durations of the pulses.

Fig. 7(a) presents the time-displacement response curves of an ideal-plastic one-mass system subjected to four kinds of displacement pulses having the same acceleration value but different durations.

In Figs. 7(b), 7(c), 9 and 11, the response curves of one-mass systems having specific elasto-plastic behaviors are presented. In this case, every displacement pulse has the same mean velocity.

Figs. 8, 10 and 12 are the time-displacement response curves of one-mass systems corresponding to several different kinds of ground pulses having the same amplitude.

In these figures, the number of each time-displacement response curve and of the corresponding elasto-plastic behavior of one-mass systems shows that the duration of the ground motions are:

1. Two times the period of the system.
2. One and a half of the system period.
3. A full system period, and
4. A half of the system period.

CONSIDERATIONS AND CONCLUSIONS

The ability to estimate the most decisive factor of ground shocks affecting a structure and leading to possible collapse would be most significant. In an attempt to evaluate this most destructive damage factor, the influence of displacement, velocity, acceleration and period of ground shocks are considered.

From the time-displacement response curves shown in Fig. 7(a) of a one-mass system acted upon by impressed displacements of identical acceleration value but with various durations, it is noted that the maximum deflection of the system increases with increasing pulse duration. Fig. 7(a) presents the case of an ideal-plastic system only; but some difference in the elasto-plastic behavior of the system does not affect the general trend shown.

Figs. 7(b), 7(c), 9 and 11, indicate that as the period of the impressed displacements approaches the natural period of the system, larger deflections are effected.

Figs. 8, 10 and 12 show the existence of an apparent peak of the amplitudes of each dynamic behavior when the period of the impressed displacements coincides with the natural period of the system. It is to be noted, however, that if the system is acted upon by ground pulses of a period less than that of the system, then the response of the system is affected by the amplitude of the ground motion.

From the results of the analysis, it may be postulated that the most destructive element of earthquake motion on a structure is dependent on the ratio of the period of the ground motion to the period of the structure. If the ratio is far less than unity, namely, the period of ground motion is far less than that of the structure, the most destructive element is the displacement value of a dominant pulse. When the ratio approaches unity, the most destructive element is associated with the ground velocity. Only when the ratio is far above unity, will the acceleration value of the ground pulses be the most destructive element.

Thus it may not be a rational procedure to follow the building code requirements of this country which assume the structure is subjected to constant accelerations.

Based upon the range of the period of both the ground motion and the structure, it is concluded that a more logical design basis would be: a. Short period structures should be designed for constant accelerations, b. Structures with medium period ranges should be designed for the constant velocities, and c. The longer period structures should be designed for constant displacements.

As a result, the writer proposes that lateral force coefficients for structures with a height H, and at a level y, should be determined by use of equation 1.

$$K = K_1 \sin \frac{\pi Y}{2H} \qquad \text{.......} \ (1)$$

where Y is the height to y above the base and K_1, is the coefficient as shown in Fig. 6(a) to be determined for varying periods, T_o , of structure. Fig. 6(b) shows an identical relationship for K_1 with varying frequencies, F, of structures.

This lateral force coefficient is not to be used with conventional building codes but for that design procedure which considers the elasto-plastic properties of the structural materials. The estimation of the real resistance of structures to earthquake shocks should be based upon the dynamic analysis of elasto-plastic structures subjected to strong-motion earthquakes.

On the basis of results obtained thus far, the writer believes that further research along these lines will lead to the development of more useful design procedures.

STEP-BY-STEP GRAPHICAL PROCEDURES

For the analysis of linear vibration problems, it is well known that the responses developed in the multi-mode "elastic" system by arbitrary base motion can be developed by superposition if we know the response due to an impuse. In the case of the analysis of non-linear problems, the preceding method of superposition loses its validity. Therefore, such methods that make use of the torsion analyzer or the linear analog computer will not be applicable to the analysis. Thus, there are no procedures now available for the purpose except step-by-step graphical constructions or lengthly numerical computations.

The non-linear vibrations of building structures have been studied in the behavior of idealized single-mass structures for which case the differential equation of a one-mass system subjected to a ground motion can be written as shown in Eq. 2.

$$m\ddot{x} + f(x, \dot{x}\,; t) = -m\ddot{X} \qquad \cdots\cdots\cdot \ (2)$$

where
- m = idealized single mass
- f = non-linear restoring and dissipative forces
- X = distinct ground displacement, and
- x = relative displacement of the mass with respect to the ground

Graphical solutions of Eq. 2 are obtained by either Messner's method or the Phase-Plane-Delta method (6 and 7). These methods, however, have been applied generally to one-mass systems. An attempt has been made to extend these methods to multi-mass problems to be used in our studies.

For the sake of simplicity, it is assumed that the nonlinear system has two masses (m_1, m_2), two restoring and dissipating forces (f_1, f_2), and their coordinates (x_1, x_2), respectively. Consider the differential equation of motion

$$\begin{cases} m_1\ddot{x}_1 + f_1(x_1) - f_2\{(x_2 - x_1)\} = -m_1\ddot{X} & \cdots\cdots\cdot \ (3) \\ m_2\ddot{x}_2 + f_2\{(x_2 - x_1)\} = -m_2\ddot{X} & \cdots\cdots\cdot \ (4) \end{cases}$$

which can be written

$$\begin{cases} m_1\ddot{x}_1 + f_1(x_1) = -m_1\ddot{X} - m_2(\ddot{X} + \ddot{x}_2) & \cdots\cdots\cdot \ (5) \\ m_2(\ddot{x}_2 - \ddot{x}_1) + f_2\{(x_2 - x_1)\} = -m_2\ddot{X} - m_2\ddot{x}_1 & \cdots\cdots\cdot \ (6) \end{cases}$$

1. Extension of Meissner's Method to Two-mass Problems.

Transform Eqs. 5 and 6 into

$$\begin{cases} m_1\ddot{x}_1 + kx_1 = kx_1 - f_1(x_1) - (m_1+m_2)\ddot{X} - m_2\{(\ddot{x}_2-\ddot{x}_1)+\ddot{x}_1\} & \cdots\cdots\ (7) \\ m_2(\ddot{x}_2-\ddot{x}_1) + k(x_2-x_1) = k(x_2-x_1) + f_2\{(x_2-x_1)\} - m_2\ddot{X} - m_2\ddot{x}_1 & \cdots\cdots\ (8) \end{cases}$$

Introducing the quantities

$$\begin{cases} u_1 = p_1 t, \quad p_1 = \sqrt{k/m}, \quad x_1 = q_1 & \cdots\cdots\ (9) \\ u_2 = p_2 t, \quad p_2 = \sqrt{k/m}, \quad x_2 - x_1 = q_2 & \cdots\cdots\ (10) \end{cases}$$

Eqs. 7 and 8 become

$$\begin{cases} q_1'' + q_1 = \{q_1 - \frac{1}{k} f_1(q_1)\} - (\frac{1}{p_1^2} + \frac{1}{p_2^2})\ddot{X} - (q_2'' + q_1\frac{p_1^2}{p_2^2}) & \cdots\cdots\ (11) \\ q_2' + q_2 = \{q_2 - \frac{1}{k} f_2(q_2)\} - (\ddot{X}/p_2^2) + q_1''(p_1^2/p_2^2) & \cdots\cdots\ (12) \end{cases}$$

Thus, from Eqs. 11 and 12 graphical solutions of q_1 and q_2 are obtained. Accordingly, two construction planes are necessary.

Supposing that the structure is at rest at time zero so that the initial displacements, x_1 and x_2, are zero, respectively, then the initial conditions are

$$[-X = x_2]_{t=0} \qquad \cdots\cdots\ (13)$$

or

$$[-\ddot{X} = \ddot{x}_2 = q_1'' p_1^2 + q_2'' p_2^2]_{t=0} \qquad \cdots\cdots\ (14)$$

at the time Eq. 11 becomes

$$q_1'' + q_1 = \{q_1 - \frac{1}{k} f_1(q_1)\} - (\ddot{X}/p_1^2) \qquad \cdots\cdots\ (15)$$

Then, a part of the solution corresponding to the short time interval from zero to dt is given as the first step of the construction. By Meissner's method, the values of the displacement, q_1, and the acceleration, q_1'', at the time dt are obtained simultaneously, so that the value of q_1'' can be put into Eq. 14. Thus Eq. 14 is written as follows, and we get a part of the solution of this equation for a short time interval, 0 -- dt.

$$q_2'' + q_2 = \{q_2 - \frac{1}{k} f_2(q_2)\} - (\ddot{X}/p_2^2) + [q_1'']_{1st\ step} \times (p_1^2/p_2^2) \cdots\cdots\ (16)$$

As the second step of construction, setting both values of $(q_1'')_{1st.}$ and $(q_2'')_{1st.}$ into Eq. 12, we obtain new values of q_1 and q_1''.

An alternate application of Eqs. 11 and 12 gives the step-by-step graphical solutions of elasto-plastic systems corresponding to an unstationary state.

In Figs. 3 and 4, there are shown the examples obtained by this method when $m_1 = m_2 = m$, and $f_1 = f_2 = f$.

2. Extension of the Phase-Plane-Delta Method to Two-mass Problems.

Eqs. 5 and 6 are rewritten in a form of the Phase-Plane-Delta method as follows:

$$\begin{cases} \ddot{x}_1 + p_1^2[x_1 + \delta_1] = 0 & \text{....... (17)} \\ (\ddot{x}_2 - \ddot{x}_1) + p_2^2[(x_2 - x_1) + \delta_1] = 0 & \text{....... (18)} \end{cases}$$

where

$$\begin{cases} \delta_1 = \frac{1}{k}[f_1(x_1) - kx_1] + \frac{\ddot{X}}{p^2} - \frac{m_2}{k}(\ddot{X} + x_2), & p^2 = k/m_1 \\ \delta_2 = \frac{1}{k}[f_2\{(x_2 - x_1)\} - k(x_2 - x_1)] + \frac{1}{p_2^2}(\ddot{X} + x_1), & p_2^2 = k/m_2 \end{cases}$$

Eqs. 17 and 18 show, respectively, the general form of the delta method. To simplify these equations, we let $m_1 = m_2 = m$ and

$$\begin{cases} \ddot{x}_1 + p^2[x_1 + \delta_1] = 0 & \text{....... (19)} \\ (\ddot{x}_2 - \ddot{x}_1) + p^2[(x_2 - x_1) + \delta_1] = 0 & \text{....... (20)} \end{cases}$$

where

$$\begin{cases} \delta_1 = \frac{1}{k}[f_1(x_1) - kx_1] + \frac{2\ddot{X}}{p^2} + \frac{\ddot{x}_2}{p^2} & \text{....... (21)} \\ \delta_2 = \frac{1}{k}[f_2\{(x_2 - x_1)\} - k(x_2 - x_1)] + \frac{\ddot{X}}{p^2} + \frac{\ddot{x}_1}{p^2} & \text{....... (22)} \end{cases}$$

These equations should be solved simultaneously on the two phase-planes, (x_1-- $\dot{x}_1/p$) plane and (x_2 -x_1) -- ($\dot{x}_2$ - $\dot{x}_1$)/p plane, by means of the step-by-step method. The initial condition is also given as follows:

$$[-X = x_2]_{t=0} \qquad \text{....... (23)}$$

From Eq. 21, the center of the circle describing the initial motion on one phase-plane (x_1-- $\dot{x}_1/p$) is locate at δ_{11} on the x_1-axis.

$$\delta_{11} = \frac{1}{k}[f_1(x_1) - kx_1] + \frac{\ddot{X}}{p^2} \qquad \text{..... 1st step}$$

Secondly, Eq. 19 can be written $\frac{\ddot{x}_1}{p^2} = -[x_1 + p]$

$$\delta_{21} = \frac{1}{k}[f_2\{(x_2 - x_1)\} - k(x_2 - x_1)] + \frac{\ddot{X}}{p^2} - [x_1 + \delta_{11}]_{1st\ step} \qquad \text{..... 1'st step}$$

and so

$$\delta_{1\cdot n} = \frac{1}{k}[f_1(x_1) - kx_1] + \frac{2\ddot{X}}{p^2} - [\{(x_1)_{(n-1)th\ step} + \delta_{1\cdot(n-1)}\} + \{(x_2 - x_1)_{(n'-1)th\ step} + \delta_{2\cdot(n-1)}\}] \qquad \text{..... N-th step}$$

$$\delta_{2\cdot n} = \frac{1}{k}[f_2\{(x_2 - x_1)\} - k(x_2 - x_1)] + \frac{\ddot{X}}{p^2} - [(x_1)_{n\,th\ step} + \delta_{1\cdot n}] \qquad \text{..... N'th step}$$

Thus, the phase trajectories of x_1 and $(x_2 - x_1)$ can be obtained by successive repetition of this process. As this method is essentially identical with the extension of Meissner's method, it is natural that both methods introduce the same solution as shown in Figs. 3 and 4.

ACKNOWLEDGEMENTS

The author wishes to express his sincerest thanks to Messrs. Takuji Kobori, assistant professor of Kyoto University, Kiyoshi Kaneta and Ryoichiro Minai, both graduate research assistants at Kyoto University, for their cooperation in making the dynamic analysis with extensive contributions in the development of the methods of analysis and in the organization and application of the graphical procedures presented.

BIBLIOGRAPHY

(1) "On the Resistance of Structures to Earthquake Shocks", by R. Tanabashi, Memoirs of the College of Engineering, Kyoto Imperial University, (English) 1937.

(2) "Analytical and Experimental Methods in Engineering Seismology", by M. A. Biot, Transactions, ASCE, Vol. 108, 1943, p. 365.

(3) "Vibration Problems of a Skyscraper. Destructive Elements of Seismic Waves for Structures", by R. Tanabashi, T. Kobori and K. Kaneta, Bulletin of the Disaster Prevention Research Institute, Kyoto University, (English) March, 1954.

(4) "Lateral Forces of Earthquake and Wind", by a Joint Committee of the San Francisco, California Section, ASCE, and the Structural Engineers Association of Northern California. Proceedings ASCE, April, 1951.

(5) "On the Nonlinear Vibration of Buildings by Violent Earthquakes", by R. Tanabashi, T. Kobori and K. Kaneta, Proceedings of the 4th Japan National Congress for Applied Mechanics, 1954, (English).

(6) "Graphische Analysis vermittelst des Linienbild einer Funktion", by E. Meissner, Schweizerische Bauzeitung 1931-1932, Zurich, (German).

(7) "On a General Method of Solving Second Order Ordinary Differential Equations by Phase-Phase-Plane-Displacements", by L. S. Jacobsen, Journal of Applied Mechanics, Vol. 19, No. 4, December, 1952.

NOMENCLATURE

			Unit
δ.	=	Phase plane displacement	cm
F.	=	Natural frequency of vibration of a building structure	/sec
f.	=	Nonlinear restoring and dissipating forces	kg
H.	=	Height of building	cm

			Unit
i.	=	As a subscript, refers the symbol to its appropriate floor of building	None
k.	k_1. =	Lateral force coefficients	None
k.	=	Spring constant at initial state of oscillating system	kg/cm
L.	=	Potential energy stored in idealized oscillating system up to elastic limit of deformation of the system	kg-cm
M.	=	Potential energy stored in elasto-plastic system, corresponding to plastic state	kg-cm
m.	=	Mass	kg-sec^2/cm
p.	=	Circular frequency of oscillating system at its initial state	rad/sec
q.	=	Displacement defined by Meissner's graphical method	cm
T.	=	Kinetic energy of idealized system subjected to ground motion	kg-cm
T_o	=	Natural period of vibration of a building structure	sec
v.	=	Relative velocity of the mass with respect to the ground	cm/sec
X.	=	Distinct ground displacement	cm
x.	=	Relative displacement of the mass with respect to the ground	cm
Y.	=	Height of level at y above the base of building	cm

FIGURE CAPTIONS

Fig. 1 Displacement curves, of one-mass systems, of each specific elasto-plastic behavior due to quadratic displacement pulses of the foundation translation.

Fig. 2 Displacement curves, of one-mass systems, due to impressed quadratic displacements of a long duration.

Fig. 3 Displacement curves at each level, of a two-mass system, due to a constant acceleration pulse of the foundation translation.

Fig. 4 Displacement curves at each level, of a two-mass system, due to a quadratic displacement pulse of a duration equal to the natural period of the system.

Fig. 5 Ground motion patterns and an approximation of these patterns.

Fig. 6 Lateral force coefficients for buildings.

(a) Relation between fundamental periods, T_0, and the coefficient, K_1.
(b) Relation between fundamental frequencies, F, and the coefficient, K_1.

Fig. 7(a) One-mass system. Comparison of the displacements due to quadratic displacement pulses of the foundation translation, with an equal acceleration value but of various durations.

Figs. 7(b), 7(c), 9 and 11. Several elasto-plastic behaviors of one-mass systems. Comparison of displacements due to quadratic displacement pulses with an equal mean velocity.

Figs. 8, 10 and 12. Several elasto-plastic behaviors of one-mass systems. Comparison of displacements due to quadratic displacement pulses of an identical amplitude.

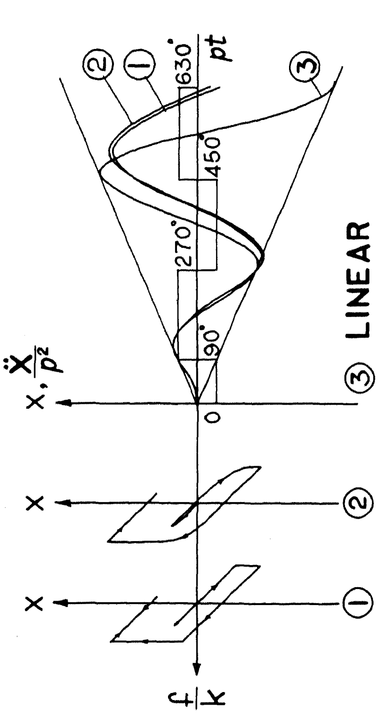

FIG. 1

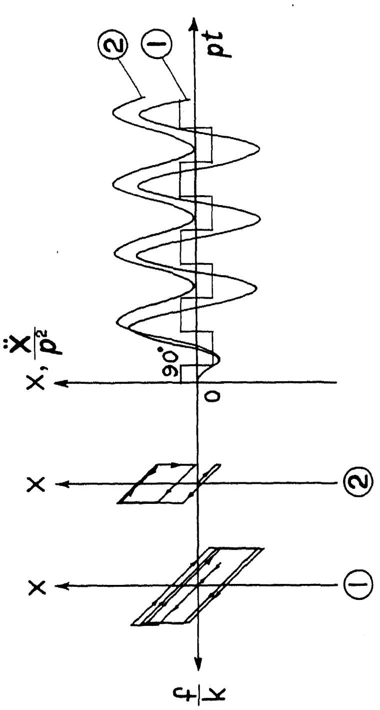

FIG. 2

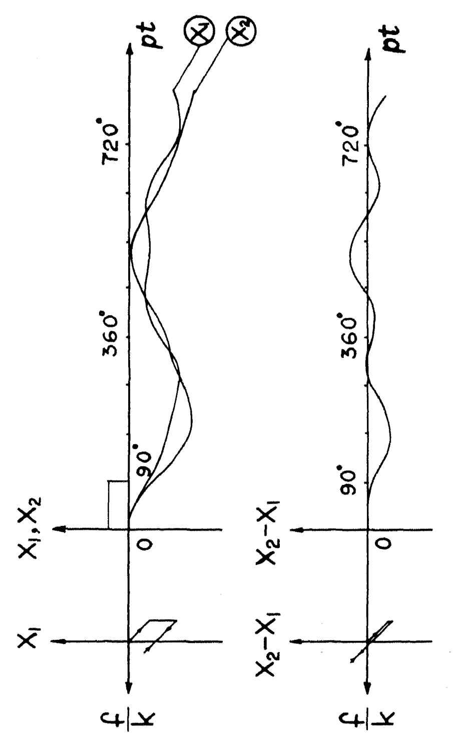

FIG. 3

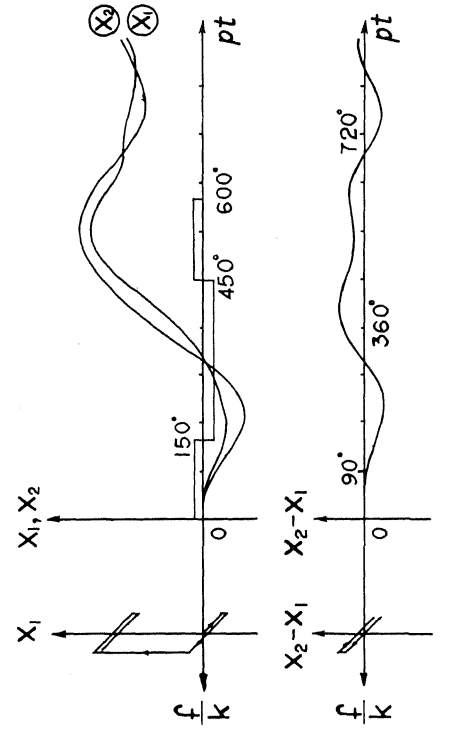

FIG. 4

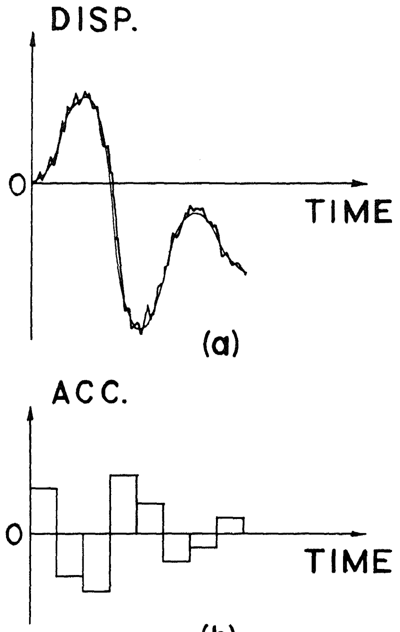

FIG. 5

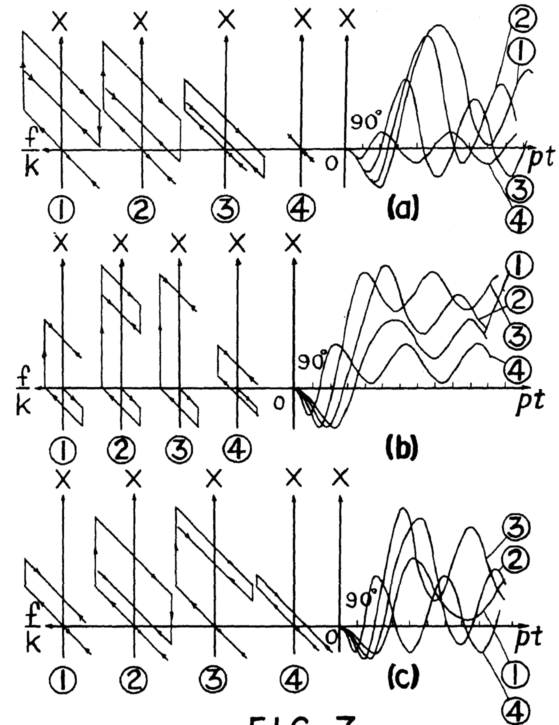

FIG. 7

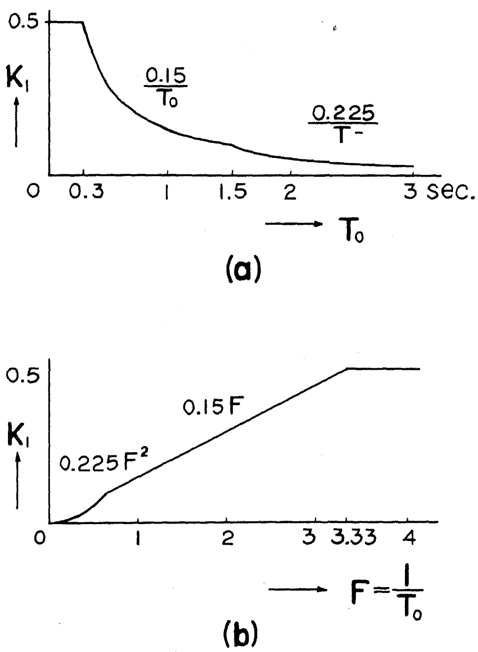

FIG. 6

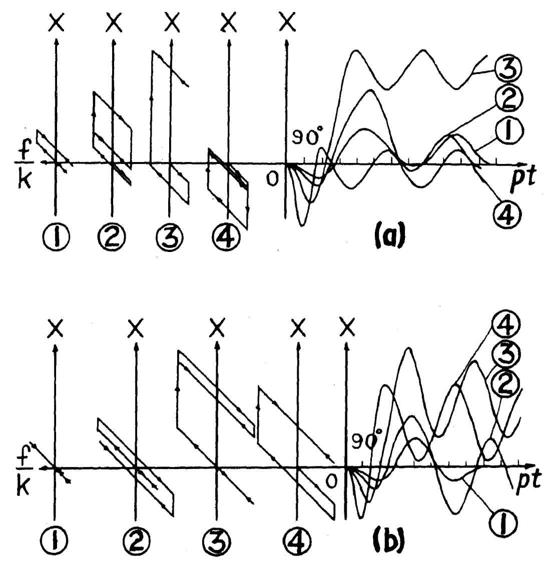

FIG. 8

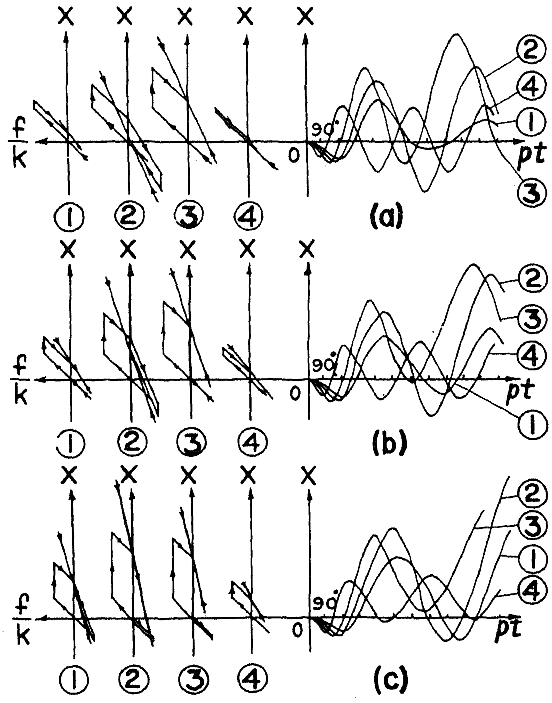

FIG. 9

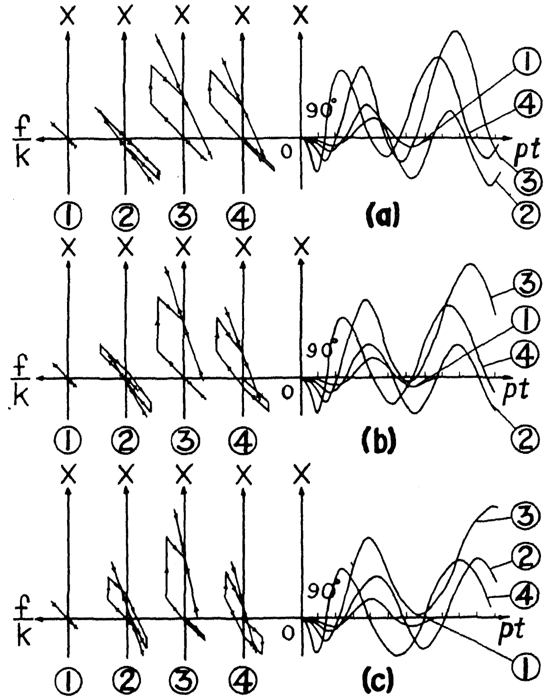

FIG. 10

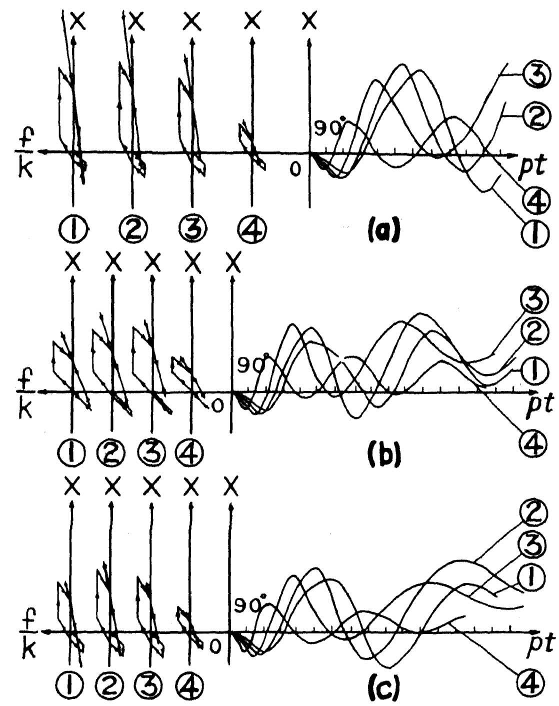

FIG. 11

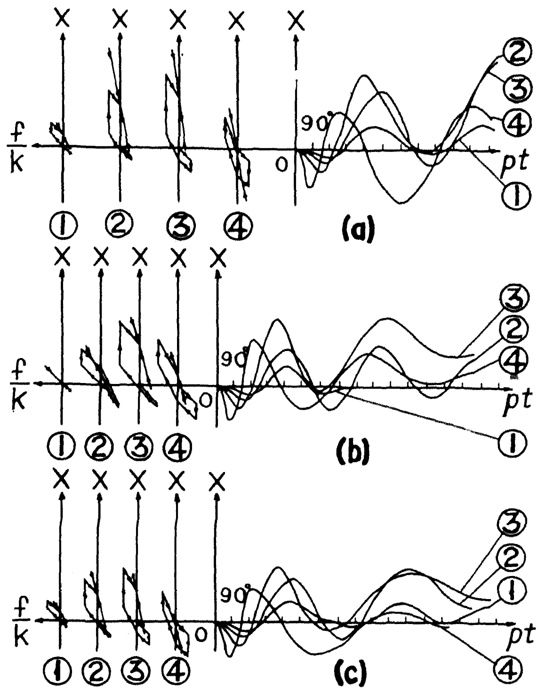

FIG. 12