Response Spectrum Techniques in Engineering Seismology

by D. E. Hudson*

Abstract: Several types of response spectra of use in engineering seismology problems are defined, and the relationships between these spectra and other basic quantities such as energy inputs and seismic coefficients are given. The use of the response spectrum to reveal significant characteristics of ground motion is discussed, and the role of the response spectrum in establishing seismic coefficients for structural behavior is illustrated by some experimental data. Various methods for determining response spectra are compared, and an electric analog spectrum analyzer is briefly described.

Introduction: One of the central problems of engineering seismology is the calculation of the behavior of a structure subjected to a given ground motion. An exact solution of this problem in transient dynamics is seldom possible because of the great complexity of the ground motions associated with earthquakes, and because of the complicated nature of many of the structures of interest to the engineer. One of the attempts to simplify this problem has involved the introduction of the so-called "response spectrum". Since this name has been used in the literature with a number of different meanings, and since several other names have been applied to essentially the same concept, it would seem to be desirable to review the subject in general, and to indicate the kind of results which can be expected from its application to engineering seismology problems.

In the present paper we shall first define the response spectrum in a form which has been found to be most suitable for engineering seismology problems. We shall then show the relationship between this response spectrum and other quantities of basic interest. Several ways in which the response spectrum can be used, with some examples of applications that have been made, will then be given. Finally, some ways of determining the response spectrum from the ground acceleration records will be indicated.

Definition of the Response Spectrum: Consider a mechanical system consisting of a single concentrated mass m, a linear spring of spring constant k, and a viscous damping element having a damping force proportional to the relative velocity through the constant c. (Fig. 1a). Suppose that the motions are possible only in the direction of the spring, so that the system has but one degree of freedom.

The absolute displacement of the mass m will be denoted by x, and the absolute displacement of the ground will be denoted by y. The extension of the spring will thus be $(x-y)$, and the relative velocity between the mass and the ground is $(\dot{x}-\dot{y})$. The absolute acceleration of the ground is $\ddot{y}$ and the absolute acceleration of the mass is $\ddot{x}$. It will usually be

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found convenient to describe the damping in the system in terms of fraction of critical damping, n, which in terms of the parameters of Fig. 1a is $n = c/2\sqrt{km}$. The spring-mass characteristics will be described by the undamped natural period of oscillation $T = 2\pi\sqrt{m/k}$.

The given data for the problem includes also the ground acceleration-time record, i.e., the curve as obtained from a strong-motion accelerometer showing the relationship between $\ddot{y}(t)$ and the time t, as in Fig. 1b.

The mathematical problem of the determination of the behavior or response of the mass m when acted upon by a particular $\ddot{y}(t)$ function is in principle a simple one, and it can be shown that the following expression forms a complete solution to the problem. (Refs. 1, 2)

$$(x - y) = \frac{-T}{2\pi\sqrt{1-n^2}}\int_0^t \ddot{y}(\tau)\, e^{-\frac{2\pi n}{T}(t-\tau)} \sin\frac{2\pi}{T}\sqrt{1-n^2}\,(t-\tau)\,d\tau \tag{1}$$

where τ is a time parameter which disappears in the course of the integration. For small damping of the magnitude encountered in most building structures ($n < 0.2$), this expression can be simplified by putting $\sqrt{1-n^2} \approx 1$.

$$(x - y) = \frac{-T}{2\pi}\int_0^t \ddot{y}(\tau)\, e^{-\frac{2\pi n}{T}(t-\tau)} \sin\frac{2\pi}{T}(t-\tau)\,d\tau \tag{2}$$

By differentiation of (2) expressions for velocities and accelerations can be obtained which will be useful for various applications. Making again the usual approximations for small damping, we obtain:

$$(\dot{x} - \dot{y}) = -\int_0^t \ddot{y}(\tau)\, e^{-\frac{2\pi n}{T}(t-\tau)} \sin\frac{2\pi}{T}(t-\tau)\,d\tau \tag{3}$$

$$\ddot{x} = \frac{2\pi}{T}\int_0^t \ddot{y}(\tau)\, e^{-\frac{2\pi n}{T}(t-\tau)} \sin\frac{2\pi}{T}(t-\tau)\,d\tau \tag{4}$$

For engineering design work, the maximum values of the forces and strains are usually of most concern. Noting that the same integral expression occurs in all of the above relationships, we denote the maximum value of this integral by the symbol S_v.

$$S_v = \left[\int_0^t \ddot{y}(\tau)\, e^{-\frac{2\pi n}{T}(t-\tau)} \sin\frac{2\pi}{T}(t-\tau)\,d\tau\right]_{max} \tag{5}$$

We use the subscript "v" because the physical dimensions of this quantity are those of velocity.

Using the above notation, we obtain:

$$(x - y)_{max} = \frac{T}{2\pi} S_v \tag{6}$$

$$(\dot{x} - \dot{y})_{max} = S_v \tag{7}$$

$$\ddot{x}_{max} = \frac{2\pi}{T} S_v \tag{8}$$

We shall now plot this quantity S_v, which evidently plays an important part in our problem, versus the undamped natural period of vibration T, for several values of the damping factor n, thus producing a family of curves as shown in Fig. 1c. This set of curves will be referred to as the <u>maximum relative velocity response spectrum</u> of the system, or for brevity, the <u>response spectrum</u>.

It will be evident that there are many other possible definitions for the response spectrum, which could be expressed in terms of displacements or accelerations, and could involve the use of either absolute or relative motions. There are also other definitions which would be appropriate when the input to the system is given in the form of a force rather than as a motion. The reasons for preferring the present definition of the response spectrum for applications in engineering seismology are as follows:

(1) Expressions (6) or (8), which might also be used as fundamental definitions of the response spectrum, involve the period of the system as a multiplier or as a divisor. This means that at one end or the other of the period scale, these expressions would have small values, and hence detail in the curves would be obscured. The velocity spectrum (7), however, gives approximately a horizontal plot for earthquake records, and is thus convenient for plotting and reading the curves.

(2) The velocity has a simple relationship to the maximum energy in the system, which may be shown as follows:

$$\text{maximum strain energy} = \frac{1}{2} k(x - y)_{max}^{2} \tag{9}$$

$$\text{maximum strain energy per unit mass} = \frac{1}{2}\frac{k}{m}(x-y)_{max}^{2}; \text{ also} \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

$$\text{maximum energy per unit mass} = \frac{1}{2}\left[\frac{2\pi}{T}(x-y)_{max}\right]^2 = \frac{1}{2} S_v^{2}$$

Thus S_v measures directly the energy per unit mass in the system. Since energy methods involving the equating of energy inputs and energy dissipations in a structure represent one important method of calculating the dynamic response of structures, S_v thus becomes a parameter of significance.

(3) For certain types of exciting motions the S_v spectrum assumes a particularly simple shape. For example, if $\ddot{y}(t)$ consists of a series of

impulses randomly oriented in time and having a random amplitude distribution, the S_v spectrum curves become straight horizontal lines* (Ref. 3). The use of S_v for the earthquake plot thus may enable one to draw some general conclusions about the earthquake exciting motion from an examination of the shape of the spectrum curves.

It will be observed from expressions (6) and (8) that the relative displacement response spectrum and the absolute acceleration response spectrum are easy to determine once S_v is known. The relative displacement spectrum is of importance for design problems since the strains in the structure are directly proportional to these relative displacements. The absolute acceleration response spectrum is of importance in any situation in which experimental measurements are to be taken as a check on the calculations, since the structural response which is easiest to measure directly is the absolute acceleration. A second reason for the importance of the absolute acceleration response spectrum is the fact that the so-called seismic coefficient or lateral force coefficient is directly proportional to the absolute acceleration response spectrum, which may be seen as follows: The maximum force on the mass is the spring constant multiplied by the maximum relative displacement, thus,

$$F_{max} = k(x-y)_{max} = k\frac{T}{2\pi}S_v$$

The acceleration acquired by the mass under the action of this F_{max} is $F_{max}/m = F_{max}\,g/W$ where W is the weight (gravity force) of the mass, and g is the acceleration of gravity. The maximum force can also be expressed by means of a "seismic coefficient" or "lateral force coefficient" C, where $F_{max} = CW$. Thus:

$$F_{max} = \left(k\frac{T}{2\pi}S_v\right)\left(\frac{W}{mg}\right) = CW; \quad \text{so, } C = \frac{kTS_v}{2\pi mg}$$

putting in the fact that $\frac{T}{2\pi} = \sqrt{\frac{m}{k}}$, we obtain

$$C = \frac{2\pi}{T}\cdot\frac{S_v}{g} \tag{10}$$

This can be checked from expression (8) above, since the seismic coefficient should be just the maximum acceleration of the mass expressed as a fraction of the acceleration of gravity:

$$C = \frac{\ddot{x}_{max}}{g} = \frac{\frac{2\pi}{T}S_v}{g} = \frac{2\pi}{T}\frac{S_v}{g} \quad \text{as above.}$$

The maximum absolute acceleration response spectrum thus gives directly the seismic coefficient corresponding to a particular exciting motion.

<u>Applications of the Response Spectrum</u>: The use of the response spectrum has frequently been criticized on the grounds that it is not possible to represent a complex structure by the very simplified model containing only a single mass, spring, and dashpot. It is important that it be clearly understood that no exact equivalence between the response spectrum

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* See Appendix

and the behavior of an actual structure is necessarily assumed nor implied in the response spectrum technique. The practical usefulness of the response spectrum method is based on the following considerations.

(1) Some general conclusions concerning the relative importance of various factors in the earthquake problem can sometimes be deduced from a study of the response spectrum. Perhaps the most important example of this is the contribution made by the response spectrum to a more complete understanding of the influence of damping in limiting the dynamic stresses in structures subjected to earthquake ground motions. If one examines a typical undamped response spectrum, as in Fig. 2, $n = 0$, it will be seen that there are many irregular peaks, and that the idea of certain dominant periods is a natural one. From this same figure, however, it will also be noted that an amount of damping such as is likely to exist in most structures, ($n = 0.10$) effectively removes most of these peaks. A study of the damped response spectra for a number of earthquakes leads to the conclusion that there are actually no dominant periods as far as most earthquakes, and most structural situations, are concerned, but that the energy input to the system is essentially constant over a broad band of periods. These are conclusions that could not have been reached by a study of the acceleration records alone, and which are valid for structures of any complexity.

(2) For complex structures in which system response must be calculated by considering motions in a number of modes of vibration simultaneously, the response spectrum may be used directly to get approximate solutions by the principle of superposition. The response spectrum as defined above, since it is concerned with maximum values only, loses some information concerning the phases of the motions in various modes. A superposition of the response values from the response spectrum corresponding to the periods of the modes of vibration does, however, give an approximate value of total system response. This approximation is always on the safe side, since the assumption that the maximum responses in the various modes will always occur at the same time, will give a total value somewhat higher than the actual maximum response. (Ref. 4)

(3) Many structures may, in spite of their complexity, behave under some circumstances essentially as single-degree-of-freedom systems, and for these situations the response spectrum can be applied directly to give system behavior. Because of the way in which motion is excited, even very complicated structures may move in essentially one mode of vibration. Ground accelerations, for example, since they are equivalent to a distributed set of inertia forces acting always in the same direction at all points in a structure, tend to excite motion in which the fundamental mode of vibration predominates.

(4) A realistic evaluation of seismic coefficients or lateral force coefficients cannot be obtained without the use of the response spectrum. The maximum accelerations to be expected in a structure are not those which are recorded by the ground motion accelerometer, since dynamic amplification effects can occur which may make the structural accelerations considerably larger than the ground accelerations. From the

response spectrum, the maximum value of the total base shear force is directly obtained.

(5) As has been shown above the velocity spectrum gives directly the energy input into the system. By equating this energy input to the sum of the various energy dissipations which occur in the structure, a determination of over-all system behavior can be made. Such energy methods, used in connection with a limit design technique, will permit the establishment of rational dynamic strength criteria for many structures subjected to earthquake loading (Ref. 5).

As an example of an application in which it was possible to make direct measurements to verify the calculations based upon the response spectrum, consider the situation shown in Fig. 3a (Ref. 6,7). In this case a large quarry blast was detonated a small distance from a building, and it was possible to measure simultaneously the ground acceleration and the acceleration of an upper story in the building. The ground motions excited by the blast were fortunately quite similar to those of a moderately large earthquake, as far as amplitudes and periods are concerned. From the ground acceleration record of Fig. 3b a maximum absolute acceleration response spectrum was determined (Fig. 3c). From this acceleration response spectrum and the calculated period of the building, the maximum acceleration to be expected at the upper story location was determined. The comparison between the acceleration calculated from the response spectrum and that actually measured during the blast is shown in Fig. 3d. It will be noted that a very satisfactory agreement was obtained. It will also be noted that the building accelerations were larger than the ground accelerations, showing that a dynamic amplification effect existed in this case. The seismic coefficient would thus have to be determined from the response spectrum.

The blast ground acceleration is different from the typical earthquake motion in one important respect, which does not, however, affect the validity of the comparison for our purpose. It will be seen that the blast response spectrum shows a definite dominant period, whereas typical damped earthquake spectra, as shown in Fig. 2, do not exhibit this feature. For the present purpose this was perhaps fortunate, since at the building period, which was somewhat removed from the peak, the response is relatively insensitive to damping. An accurate determination of building response was thus possible without an equally accurate knowledge of structural damping. For earthquake problems this simplification would very seldom be possible, and damping would always be expected to be an important factor in building behavior.

<u>The Determination of the Response Spectrum</u>. Assuming that a ground acceleration record is available, the determination of the response spectrum involves an evaluation of the integral of equation (5) for a series of values of the parameters T and n. For typical earthquake excitations this involves the determination of several hundred points on the curves. These calculations can be done in several different ways.

One method would be a direct numerical integration, which could be carried out by any of the standard techniques. Various high speed digital com-

puting machines could probably be used to advantage for such calculations. A second method would involve some graphical integration system, or the use of an instrument such as an integraph. The most commonly used techniques, however, have involved analog computing methods, several of which are well suited to problems of this kind.

The first analog technique to be applied was the torsion pendulum direct mechanical analog used by Biot (Ref. 8), who calculated some earthquake spectra in this way. The main difficulties involved in the torsion pendulum are the inconveniences of changing the basic parameters of period and damping in the mechanical system, and the relatively long time required to obtain the complete spectrum curves in the desired detail. It is also difficult to control the damping with sufficient accuracy and to reduce it to as low a value as might be desired for some applications.

These difficulties were overcome by a much improved system developed by R. Takahasi , of the Earthquake Research Institute of Tokyo University, in which a moving coil galvanometer element was used as the mechanical torsional system (Ref. 9). In this system, the torsional element has a fixed frequency, and period changes are obtained by changing the speed of the film drive mechanism in the ground motion function generator. By arranging for an energy input into the torsional system through a feedback system, the effective damping can be reduced to zero if desired.

The operational type of electric analog computer has also been applied to the problem. Crede, Gertel, and Cavanaugh (Ref. 10) have shown one way of applying a commercial electronic differential analyzer to the determination of response spectra. A special purpose spectrum analyzer using electronic operational techniques has been described by Morrow and Riesen (Ref. 11).

The method which has so far been most extensively used for earthquake response spectrum determination employs the direct electric analog computer (Ref. 12). In this type of computer a series electrical circuit consisting of an inductance, a capacitance, and a resistance, forms a direct analogy to the mechanical single-degree-of-freedom system. The ground acceleration record is reproduced as an electric signal by means of a variable width film and photocell arrangement, and response characteristics are measured as voltages at various points in the circuit. The advantages of this method are: (1) parameter changes can be made with ease; (2) the speed of operation is such that complete spectra can be determined in a matter of minutes; (3) zero damping can be obtained by the use of negative resistance devices, and any amount of damping can be easily introduced and measured; (4) various types of response spectra can be obtained with the same device. In Fig. 4 the basic elements of a special purpose direct electric analog spectrum analyzer designed for earthquake response spectrum determination are shown (Ref. 13). Using this electric analog technique, response spectra have been obtained for a series of strong motion earthquakes in the Western United States (Ref. 14) and general studies of the characteristics of such earthquakes have been made (Ref. 3, 15).

Conclusions. Response spectrum methods have been found to be useful not only for the determination of structural responses in particular cases of ground motion, but also as a means of investigating the general character of earthquake ground motion. Instrumentation and techniques are now available which will permit the determination of response spectra in a form suitable for many applications in engineering seismology. What is now most needed to further such studies are more strong-motion accelerometer records of earthquakes and of other ground motions, so that an increased amount of basic data can be prepared in the form of response spectra.

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NOMENCLATURE

m = mass

k = linear spring constant (force/length)

c = viscous damping force coefficient (force = c x velocity)

x = absolute displacement of mass m

$\dot{x}$ = absolute velocity of mass m

$\ddot{x}$ = absolute acceleration of mass m

y = absolute displacement of ground

$\dot{y}$ = absolute velocity of ground

$\ddot{y}$ = absolute acceleration of ground

n = damping factor = $c/2\sqrt{km}$ = fraction of critical damping

T = natural undamped period = $2\pi\sqrt{m/k}$ (sec.)

t = time (sec.)

τ = time parameter in integration

S_v = maximum relative velocity response spectrum

F_{max} = maximum force acting on mass

g = acceleration of gravity

W = gravity force on mass m = weight of mass m

C = seismic coefficient = lateral force coefficient

Appendix: The Response Spectrum of a Random Exciting Motion

Suppose that the ground motion consists of a large number of impulses randomly distributed in time. Consider first the response of an undamped single-degree-of-freedom oscillator having a natural period T to a sequence of such pulses. Each impulse will excite a harmonic motion whose velocity is described by

$$v = v_i \cos \frac{2\pi}{T} (t - t_i)$$

where v_i is the maximum velocity caused by one impulse occurring at the time t_i. The velocity after a sequence of N impulses is obtained by

superposition:

$$v_N = \sum_{i=1}^{N} v_i \cos\frac{2\pi}{T}(t - t_i) = \sqrt{\left(\Sigma v_i \cos\frac{2\pi}{T} t_i\right)^2 + \left(\Sigma v_i \sin\frac{2\pi}{T} t_i\right)^2} \cos\left(\frac{2\pi}{T} - \psi\right)$$

which can also be written as:

$$v_N = \sqrt{\Sigma v_i^2 + 2\sum_i \sum_j v_i v_j \cos\frac{2\pi}{T}(t_i - t_j)} \quad \cos\left(\frac{2\pi}{T} - \psi\right)$$

Since the values of t_i and t_j are randomly distributed, the $\cos\frac{2\pi}{T}(t_i - t_j)$ term will for any particular T assume a random sequence of values, both positive and negative, and hence for a large number of impulses the double summation term will be small compared to the first term, which increases steadily with the number of impulses. We then obtain for the maximum velocity attained:

$$v_{max} = \sqrt{\sum_{i=1}^{N} v_i^2}$$

The maximum velocity attained after a given sequence of impulses will thus be independent of the period T. It is true that for any particular finite sequence of impulses, v_{max} will probably vary slightly with period because of occasional fortuitous conjunctions of the time at which an impulse is applied and the motion of the oscillator. If a large number of such random sequences were to be applied, however, the average of all the v_{max}-T curves would be a horizontal straight line, and it is in this sense that v_{max} can be said to be independent of T. As was shown above

$$E_{max} = \frac{1}{2} m v_{max}^2 \text{ and } E_{max}/m = \frac{1}{2} S_v^2$$

Thus $S_v = v_{max}$, and the response spectrum S_v is also independent of T. The undamped response spectrum for a series of random impulses thus consists simply of a horizontal straight line.

From the above expressions we see that each impulse contributes its part to increasing the maximum velocity of the system, and this maximum velocity will thus depend upon the number of impulses that occur. This conclusion would be true for any amplitude distribution of the pulses having any combination of positive and negative directions.

If we now further assume that the impulses have a random distribution of magnitude and direction, then on the average any large group of impulses occurring in a particular time interval will produce the same maximum velocity. Considering the total time at any instant as the sum of a number of equal time intervals, each of which contains a similar set of random impulses, then the maximum velocity would increase with a constant average rate. Thus the maximum velocity, and hence the maximum energy input to the system, varies linearly with time. We therefore conclude that for the system acted upon by a series of random impulses, the energy input per unit time is a constant.

For a viscously damped system, the energy loss may be easily calculated. Consider the harmonic motion:

$$x = x_o \sin\frac{2\pi}{T} t \quad ; \quad \dot{x} = \frac{2\pi x_o}{T} \cos\frac{2\pi}{T} t$$

The energy dissipated per cycle in such a harmonic motion is

$$\int_0^T c\left(\frac{2\pi x_o}{T}\cos\frac{2\pi}{T}t\right)dx = \int_0^T c\left(\frac{2\pi x_o}{T}\cos\frac{2\pi}{T}t\right)^2 dt = \frac{2\pi^2 c\, x_o^2}{T}$$

Multiplying this by 1/T cyc/sec, and noting that $v_{max} = 2\pi x_o/T$, we have

$$\text{energy loss per unit time} = \frac{1}{2}\, c v_{max}^2$$

We have seen above that the velocities attained after any particular sequence of random impulses is on the average independent of T, and it thus appears that the energy dissipated per unit time is also independent of period as is the energy input to the system. Thus, even for the damped system, if c is a constant, the damped velocity response spectrum will also be a straight horizontal line.

If the damping in the system is measured by the fraction of critical damping $n = \frac{c}{2\sqrt{km}} = \frac{cT}{4\pi m}$, and n is kept constant for the response spectrum curves, as is the usual practice, then since n involves T, the damped spectrum curves will no longer be straight horizontal lines, as may be seen in the following way.

Let P be the energy input per unit time by the random impulses, which we have seen above is constant. The net energy input to the system is thus $(P - \frac{1}{2} c v_{max}^2)$ or $(P - \frac{c}{m} E_{max})$. Thus we may write the equation:

$$\frac{dE_{max}}{dt} = P - \frac{c}{m} E_{max}$$

for which the solution is:

$$E_{max} = \frac{m}{c} P\left(1 - e^{-\frac{c}{m}t}\right).$$

We also remember that $E_{max}/m = \frac{1}{2} S_v^2$. So we have, for a sequence of impulses lasting a total time t_o:

$$S_v = \sqrt{\frac{2P}{c}\left(1 - e^{-\frac{c}{m}t_o}\right)}$$

Again we note that if c is a constant, S_v is constant. Putting in $\frac{c}{m} = \frac{4\pi n}{T}$, however, we obtain:

$$S_v = \sqrt{\frac{PT}{2m\pi n}\left(1 - e^{-\frac{4\pi n}{T}t_o}\right)}$$

For a particular series of impulses, and for n = constant >0, this becomes

$$S_v = \sqrt{c_1 T\left(1 - e^{-\frac{c_2}{T}}\right)}$$

The response spectrum thus falls off at the low period end, and approaches zero at T = 0. For large values of T the spectrum curve becomes asymptotic to the horizontal line describing the constant c spectrum.

The velocity spectrum curve which shows an approximate horizontal section *with an exponential fall-off at the short period end is typical of* many earthquake spectra that have been obtained, as may be seen in Fig. 2. It appears that for certain purposes it should be possible to consider some typical earthquakes as a sequence of random impulses.

FIG 1 DEFINITION OF THE MAXIMUM RELATIVE RESPONSE SPECTRUM

FIG 2 TYPICAL EARTHQUAKE RESPONSE SPECTRUM

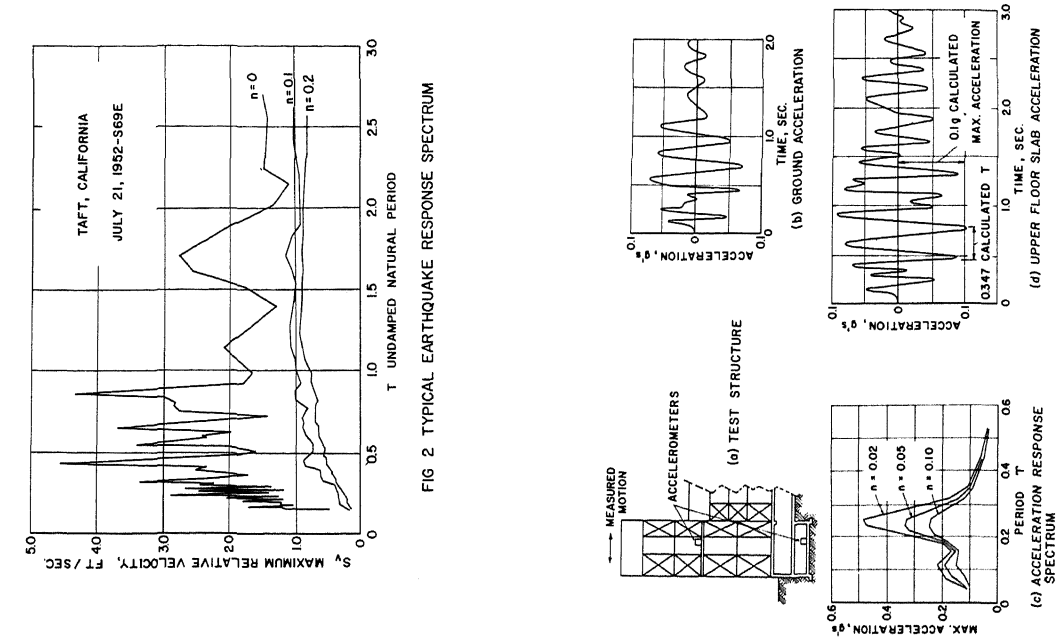

FIG. 3 QUARRY BLAST STRUCTURAL ACCELERATION TEST

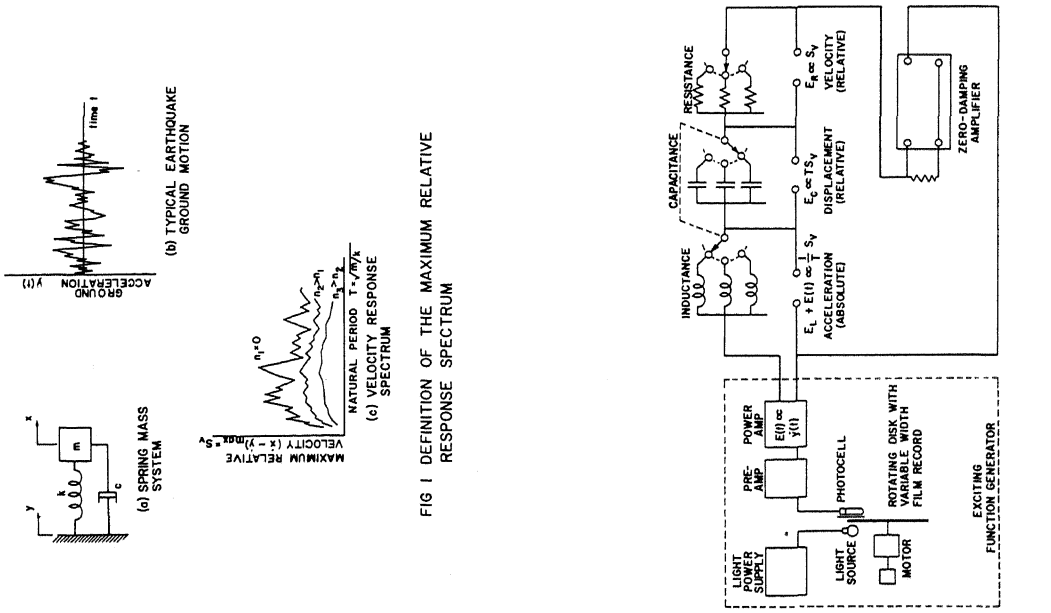

FIG 4 DIRECT ELECTRIC ANALOG RESPONSE SPECTRUM ANALYZER FOR EARTHQUAKE STUDIES