ASEISMIC DESIGN OF SIMPLE PLASTIC STEEL STRUCTURES FOUNDED ON FIRM GROUND.

by M. RODRIGUEZ C.*

SYNOPSIS

A probabilistic method of aseismic design of one-degree of freedom plastic steel structures without damping and founded on firm ground is presented. The method is based on a statistical analysis of responses of such structures to idealized fictitious earthquakes essentially similar to true strong motion temblors. It is shown that the probabilities of failure through excessive deformation are smaller for ductile structures than for analogous elastic structures subjected to the same earthquakes.

ASSUMPTIONS

This paper is based on the following assumptions:

a) Structures considered have one degree of freedom in a horizontal direction, Fig. 1.

b) The mass of the structure is concentrated at the heads of the columns supporting it.

c) There is no damping in the structures under consideration.

d) The material of which the structure is made has a stress-strain diagram of the type shown in Fig. 2.

e) In any bar of the structure subject to bending plane sections orthogonal to the axis of the bar remain plane.

f) The stress-strain diagram in bending is the same as the stress-strain diagram in tension or compression.

g) The ground upon which the structure is founded has a stiffness comparable to that of sites at which reliable earthquake accelerograms have been recorded.

h) From the viewpoint of aseismic design, an earthquake may be idealized as a series of instantaneous changes in ground velocity distributed at random both in time and magnitude, Fig. 3.

The last hypothesis is due to G. W. Housner.

CHARACTERISTICS OF PROPOSED IDEALIZED EARTHQUAKES

A set of 19 fictitious motions, essentially similar to true strong-motion earthquakes, were constructed . They were assigned the following characteristics:

a) The magnitude of each pulse is small compared to the sum of the magnitudes of all pulses.

* Professor of Applied Mechanics, University of Mexico.

b) The magnitudes of the pulses are distributed in time and according to intensity, in three phases of which the second one is the most intense.

c) The algebraic sum of all the pulses is zero (at the end of a true earthquake the ground velocity is zero).

d) The duration and sign of the pulses are chosen at random.

Constant pulse magnitudes were used in each phase. This idealization does not materially alter the statistical behavior of structural responses[1].

DYNAMIC BEHAVIOR OF SIMPLE DUCTILE STRUCTURE

Consider a structure of the type shown in Fig. 4 under the action of a lateral force F of varying intensity. Let δ_o be the relative displacement in the columns of the structure before applying F, and δ the relative displacement at any instant measured from the initial position of equilibrium. It will be assumed in this paper that diagram (F,δ) is similar to that of Fig. 5.

It can be shown that the value of F causing yielding in the columns is

$$F_y = \frac{24\,EI\Delta}{l^3} \tag{1}$$

and that the value of Δ when yielding begins is

$$\Delta = \frac{\sigma_y S l^2}{6EI} \tag{2}$$

RESPONSE OF SIMPLE DUCTILE STRUCTURE TO IDEALIZED EARTHQUAKES

It is well known that a structure of the type considered may be idealized as a mass, equal to that concentrated at the tops of the columns of the structure, supported by a spring with the same force-deflection diagram as the (F,δ) curve, Fig. 5, of it.

Consider the motion of the mass depicted in Fig. 6 during the interval $t_i \angle t \angle t_{i+1}$ between to successive pulses of the idealized earthquake. Two cases are possible: a) Elastic behavior if $|\delta-\delta_i| \leq \Delta$ and $F = k\delta$. b) Plastic behavior if $|\delta-\delta_i| > \Delta$ and $F = k\Delta$.

Case a) Elastic Behavior.

From Newton's second law of dynamics,

$$m\ddot{x} = -k\delta, \tag{3}$$

which after integration and proper arrangement gives

$$\xi = A\cos p(\tau + B) + \delta_i \tag{4}$$

where

$$pA = -\frac{\dot{\xi}_i}{\sin pB}, \quad \tan pB = -\frac{\dot{\xi}_i}{p(\xi_i - \delta_i)}, \quad \tau = t - t_i \qquad (5),(6),(7)$$

From Eq. (4) it is possible to obtain

$$\dot{\xi}^2 + (p\xi - p\delta_i)^2 = p^2 A^2, \tag{8}$$

which in a coordinate system $(p\xi, \dot{\xi})$ represents a circle whose center is at $(p\delta_i, 0)$ and whose radius is pA, Fig. 7. It is evident from Eqs. (5) and (6) that pA depends only upon the initial conditions of motion at instant t_i, as represented by point Q_i in Fig. 7. It is easily seen from Eq. (4) that point Q_{i+1} represents the state of motion of the mass at instant t_{i+1}, and that any other point Q between Q_i and Q_{i+1} represents the state for $0 \angle \tau \angle t_{i+1} - t_i$ or $t_i \angle t \angle t_{i+1}$.

Case b) Plastic Behavior.

Suppose that at instant t_Δ within the interval $t_i \angle t \angle t_{i+1}$ plastic yielding starts in the spring. From this instant on, force F in the spring has a constant value, $F = k\Delta$, and Newton's second law gives

$$m\ddot{x} = -k\Delta, \tag{9}$$

which shows that motion of the mass m has constant acceleration and hence, after yielding occurs in the spring mass m no longer vibrates.

Integration of Eq. (9) and proper substitutions give

$$\xi = -\frac{\Delta}{2}(p\tau)^2 + \dot{\xi}_\Delta \tau + \xi_\Delta, \tag{10}$$

where

$$\tau = t - t_\Delta, \quad p\tau = \frac{\dot{\xi}_\Delta - \dot{\xi}}{p\Delta}. \tag{11),(12}$$

From Eq. (10) it is possible to obtain

$$\dot{\xi}^2 = -2p\Delta\left[p\xi - (p\xi_\Delta + \dot{\xi}_\Delta^2/2p\Delta)\right], \tag{13}$$

which in a coordinate system $(p\xi, \dot{\xi})$ represents a parabola whose axis coincides with the $p\xi$- axis, whose vertex is at $(p\xi_\Delta + \dot{\xi}_\Delta^2/2p\Delta,\ 0)$ and which opens towards the left if $\Delta > 0$, Fig. 8. It should be observed that this parabola does not depend upon the state of motion of the mass m when plastic yielding starts in the spring. In fact, if in Fig. 8 the origin of the coordinate system is shifted to O' and if $(p\xi)'$ denotes the new abscissas, Eq. (13) becomes,

$$\dot{\xi}^2 = -2p\Delta(p\xi)', \tag{14}$$

which clearly shows that the parabola which it represents depends only upon the product $p\Delta$ for the dynamic system under consideration. Therefore, in the graphical method of dynamic analysis which will be described later, it is only necessary to draw a parabola for a given mass-spring system when it is subjected to a fictitious earthquake.

From Fig. 5 it is concluded that plastic behavior of the spring will cease **at** instant t_u when δ starts to decrease, i. e., when ξ attains a maximum or $\dot{\xi}$ vanishes. The state of motion is then represented by the vertex of the parabola, Eq. (14). After this state is reached the structure will behave elastically again. The duration of the plastic behavior is $t_u - t_\Delta$ if $t_u \angle t_{i+1}$, or $t_{i+1} - t_\Delta$ if $t_u > t_{i+1}$. In any case, this duration can be computed from Eqs. (11) and (12).

Now if the mass-spring system is subjected to a fictitious idealized earthquake, the method discribed above permits the determination of the state of motion at any instant using the following procedure: curve $(p\xi, \dot{\xi})$ is drawn for the first time interval, between the first and second pulses, starting from point O representing the state of rest of the system. Next, curve $(p\xi, \dot{\xi})$ is drawn for the second time interval, between the second and third pulses, starting from the state reached at the end of the first interval, and so on. The diagram resulting from this construction is known as "gyrogram". The maximum value of p can be obtained from this. Fig. 9 is an example of a gyrogram[2],[3].

DATA FOR STATISTICAL ANALYSIS

In order to obtain data which could permit the use of statistical methods, a set of 19 idealized fictitious earthquakes were constructed, and the maximum value of $p\xi$ that they produce in one-degree of freedom mass-spring systems of natural periods 0.2, 0.4, 0.6,... 4.0 sec, and different values of Δ found from the graphical construction described above.

STATISTICAL ANALYSIS OF DATA OBTAINED

Data obtained may be regarded as coming from observations of the effects of true earthquakes on certain systems representing real structures. Therefore, it is necessary to treat them statistically to arrive at general conclusions.

Let A be the event defined by: "pX exceeds $p\Delta$", and B the event defined by: "pX exceeds pX_a". We can then say that the sample space of A consists of all values of $pX \geq p\Delta$, and that the sample space of B consists of all values of $pX \geq pX_a$. Therefore, if AB represents the event: "both A and B occur", its sample space will consist of all values of $pX \geq pX_a \geq p\Delta$. Finally, if B|A represents the event: "B occurs when A has occurred", it can be shown that[4],[5],[6]

$$P(AB) = P(A)\,P(B|A) \tag{15}$$

Since event B implies plastic behavior of the structure, Eq. (15) should be written

$$P_p(AB) = P_e(A)\,P_p(B|A) \tag{16}$$

On the other hand, if we had elastic behavior even after pX exceeds $p\Delta$, the probability of AB would be $P_e(AB)$. Hence the ratio

$$\frac{P_p(AB)}{P_e(AB)} \tag{17}$$

will describe the influence of plastic behavior of the structure.

It has been shown[7],[8] that, approximately,

$$P_e(A) = P_e(p\Delta) = 2\sum_{i=0}^{\infty}(-1)^i \operatorname{erfc}\frac{(2i+1)\sqrt{\pi}}{2}\frac{p\Delta}{E_1} \tag{18}$$

$$P_e(AB) = P_e(pX_a)_{pX_a \geq p\Delta} = 2\sum_{i=0}^{\infty}(-1)^i \operatorname{erfc}\frac{(2i+1)\sqrt{\pi}}{2}\frac{pX_a}{E_1} \tag{19}$$

where
$$E_1 = E(pX_e) = \sqrt{\Sigma u_i^2} \tag{20}$$

Then, in order to compute (17) we have to determine $P_p(B|A)$.

As a first step two variables were formed
$$x = \frac{E_2}{p\Delta}, \quad y = \frac{pX_p}{E_2}, \tag{21),(22}$$
in which[2,7,8]
$$E_2 = [E(pX_e)]_{pX_e > p\Delta} = \frac{4}{\pi} E_1 \sum_{i=0}^{\infty} \frac{(-1)^i}{2i+1} e^{-\frac{\pi}{4}(2i+1)^2 \left(\frac{p\Delta}{E_1}\right)^2}. \tag{23}$$
Points of coordinates (x,y) were plotted in a graph that suggested the possibility of representing the expectation of y for a given x by a function of the form
$$E(y|x) = \alpha + \beta x^{-1} \tag{24}$$
where α and β are constants. To verify this, values of x^{-1} were grouped in intervals, means $(\overline{x^{-1}}, \bar{y})$ of (x^{-1}, y) for each interval were computed and points $(\overline{x^{-1}}, \bar{y})$ were plotted. As the points in this graph lie approximately on a straight line, Eq. (24) holds and the problem now is to compute α and β. Linear regression analysis[6] can be applied, but we have to make a hypothesis about the form in which the dispersion of y for a given x varies. Therefore, coefficients of variation[6] of y were computed in all intervals and compared. It was found that it could reasonably be assumed constant throughout the range of values of x at hand. Accepting this and using Markoff's theorem of linear regression analysis, the following result was found
$$E(pX_p/E_2) = 0.3455 + 0.9823(E_2/p\Delta)^{-1}, \tag{25}$$
from which
$$E(pX_p) = 0.3455\, E_2 + 0.9823\, p\Delta \tag{26}$$

Now to obtain the distribution function of pX_p, that is, the function $P_p(B|A)$, a new variable was formed
$$\theta = \frac{pX_p}{E(pX_p)}. \tag{27}$$
After the values of θ had been computed, they were grouped into adequate intervals and the frequency of the values of θ greater than the upper limit θ_u of each interval were determined using the relationship
$$f(\theta > \theta_u) = \frac{\text{Number of values of } \theta > \theta_u}{\text{Total number of values of } \theta} \tag{28}$$
The frequencies thus obtained are estimates of the probabilities that $\theta > \theta_u$. It remains now to fit a distribution function to the computed frequencies. After several trials the form
$$P\left(\frac{pX_p}{E(pX_p)} > \theta\right) = \frac{2}{\pi}\int_0^{g(\theta)} e^{-v^2} dv \equiv \operatorname{erf} g(\theta), \tag{29}$$
with
$$g(\theta) = \alpha(1+\theta)^{-\beta}, \ \beta > 0, \tag{30}$$
α and β being constants, was found convenient. It should be noted that functions (29) and (30) satisfy the following conditions

a) If $\theta \to \infty$, $g(\theta) \to 0$ and $\operatorname{erf} g(\theta) \to 0$, (31)

b) If $\alpha \geq 4$ and $\theta \rightarrow 0$, erf $g(\theta) \rightarrow 1$, (see Ref. 9) (32)

Therefore function (29) can be accepted as a form of distribution function. Moreover, a plot of log $(1+\theta)$ vs log $g(\theta)$ gives a straight line approximately. Computing constants α and β by the method of least squares then gives

$$P\left(\frac{pX_p}{E(pX_p)} \geq \theta\right) = \operatorname{erf}\left[670\,(1+\theta)^{-10.860}\right] \tag{33}$$

To test the goodness of fit of distribution function (33) to the **frequencies** computed, the χ^2-test was used. The amplitudes of the intervals of variable θ were modified so that each contained more than 5 values of $pX_p/E(pX_p)$. The total number of these intervals was $k = 15$. Now the number N of values of $pX_p / E(pX_p)$ within each interval **was** computed using Eq. (33). The ratio $(N - n)^2/N$ was computed for each interval and its sum obtained for all intervals. The number of constants estimated in Eq. (33) is $c = 2$; then the value of fractile 0.95 of χ^2 with $f = k - c - 1 = 12$ degrees of freedom is found [6] to be

$$\chi^2_{0.95} = 21.0, \quad f = 12 \tag{34}$$

Hence

$$\sum \frac{(N-n)^2}{N} = 17.79 < 21.0 \tag{35}$$

and function (33) is statistically adequate for the range under consideration.

From Eqs. (33) and (26) it can be written

$$P\left(\frac{pX_p}{E(pX_p)} \geq \frac{pX_a}{E(pX_p)}\right) = P(pX_p \geq pX_a) = P_p'(B|A) \tag{36'}$$

$$P_p'(B|A) = \operatorname{erf}\left[670\left(1 + \frac{pX_a}{0.3455\,E_2 + 0.9823\,p\Delta}\right)^{-10.86}\right] \tag{36}$$

Now since

$$\left[P_p(B|A)\right]_{pX_a = p\Delta} = 1 \tag{37}$$

and expression (36), which implicitly has assumed this fact, does not give 1 for $pX_a = p\Delta$, it is concluded that it is necessary to apply a correction factor to expression (36), that takes into account the specific value of $p\Delta$ in each particular case, to obtain $P_p(B|A)$. From condition (37)

$$P_p(B|A) = \frac{P_p'(B|A)}{\left[P_p'(B|A)\right]_{pX_a = p\Delta}} \tag{38}$$

Substituting (38) in (16) when $pX_a = p\Delta$ there results

$$\left[P_p(AB)\right]_{pX_a = p\Delta} = P_e(A), \tag{39}$$

as it should be according to the physical conditions involved.

Eqs. (18), (19), (20), (23), (36) and (38) permit the computation of expression (17) for a given value of $p\Delta$ and a given value of E_1, which, according to Ref. (7), can be considered as a measure of the intensity of the earthquake, and is given by (20).

It is difficult to show in the general case that for given values of $p\Delta$ and E_1, the ratio (17) is lower than 1. However, it is possible to verify this numerically. Table 1 constitutes an example thereof.

From the foregoing discussion it can be concluded that plastic behavior is beneficial in the sense that, given a certain structural response, the probability of its being exceeded is lower than the corresponding probability in the case of purely elastic behavior. Conversely, for a given probability of failure, responses of ductile structures are lower than those of perfectly elastic systems.

PROPOSED METHOD OF DESIGN

The following method of design is proposed:

a) Choose trial section of columns.

b) Establish the maximum relative displacements in the columns which may be allowed, pX_a, according to functional requirements.

c) Compute $p\Delta$.

d) If $pX_a \leq p\Delta$, the structure should be designed by the probabilistic methods described in Refs. (1), (7) and (8).

e) If $pX_a > p\Delta$, plastic action is allowed in the columns of the structure and the following steps are necessary.

f) Ascertain the earthquake intensity, E_1, for which the structure should be designed, from the records of earthquakes and from past structural experience in the region where the structure is to be built.

g) Fix the probability, $P(pX_a)$, that pX_a be exceeded for the value of E_1 chosen, according to these factors: type, nature and importance of the structure, damage that its failure would cause, etc.

h) Compute the probability $P_p(AB)$ with the known values of $p\Delta$ and E_1, using expressions (16), (18), (23), (36) and (38).

i) Compare $P_p(AB)$ with $P(pX_a)$. If $P_p(AB) < P(pX_a)$, the design is safe. If $P_p(AB) > P(pX_a)$, the design errs on the unsafe side.

It should be noted that apart from steps (f) and (g), which require a great deal of experience, the rest of the method involves simple computation.

ACKNOWLEDGMENT

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(9) "A Short Table of Integrals", B. O. Peirce, Ginn and Co., 1929, pp. 116 - 120.

NOMENCLATURE

$E(X)$, expectation of X.

E_1, expectation of pX_e

E_2, expectation of pX_e for values of pX_e greater than $p\Delta$.

$P_e(pX_a)$, probability that pX exceed pX_a in the elastic range.

$P_p(pX_a)$, probability that pX exceed pX_a when there is plastic action.

X_e, maximum value of ξ in the elastic range.

X_p, maximum value of ξ in the plastic range.

Δ, yield elongation of the spring in a mass-spring system.

ξ, relative displacement.- $p, \sqrt{k/m}$ in a mass-spring system.

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Δ , yield elongation of the spring in a mass-spring system.

ξ , relative displacement.- $p, \sqrt{k/m}$ in a mass-spring system.

TABLE 1

$pX \geqq p\Delta$	$pX_p/E(pX_p)$	$P'_p(B\|A)$	$P_p(B\|A)$	$P_p(AB)$	$P_e(AB)$	$\frac{P_p(AB)}{P_e(AB)}$
10	0.844	0.795	1.000	0.4190	0.419	1.00
12	1.015	0.365	0.456	0.1911	0.265	0.72
14	1.182	0.157	0.196	0.0821	0.158	0.52
16	1.353	0.068	0.085	0.0356	0.089	0.40
18	1.522	0.033	0.041	0.0172	0.048	0.36
20	1.691	0.018	0.024	0.0106	0.029	0.36

$p\Delta = 10$ cm-sec^{-1}
$E_1 = 10$ cm-sec^{-1}
$E_2 = 5.860$ cm-sec^{-1}
$E(pX_p) = 11.845$ cm-sec^{-1}
$P_e(A) = 0.419$

FIGURE CAPTIONS

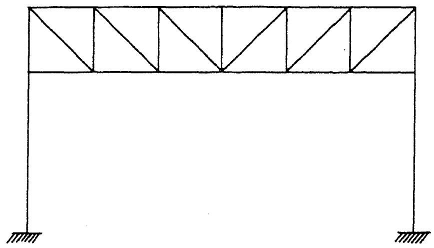

FIG. 1
Typical one-degree of freedom structure.

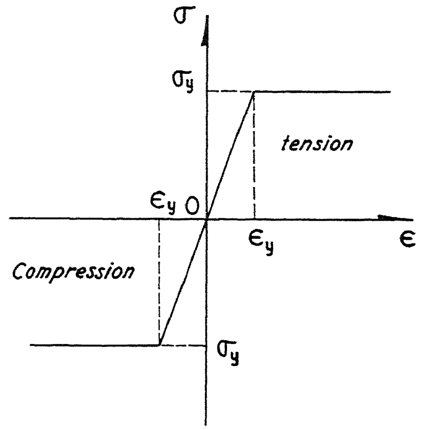

FIG. 2
Assumed stress-strain diagram.

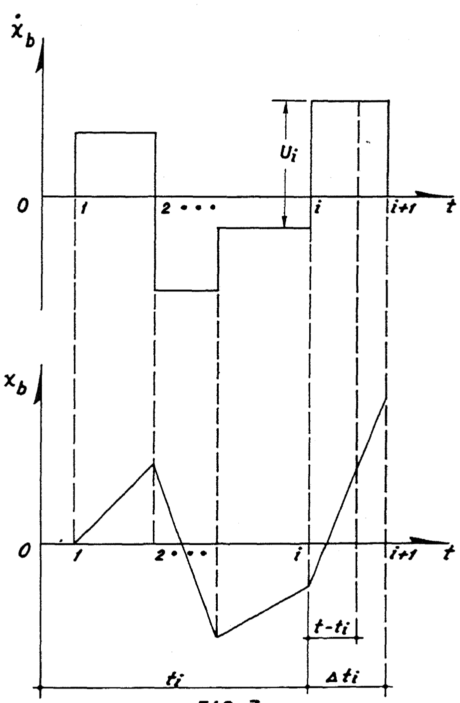

FIG. 3
Idealized earthquake.

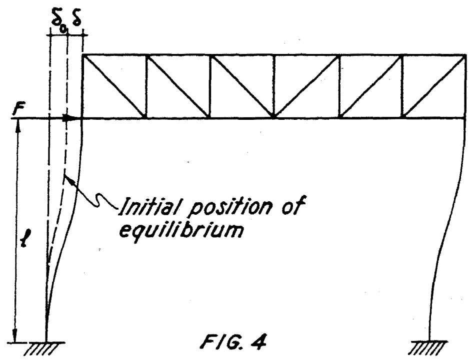

FIG. 4
Lateral force F. acting on a simple structure.

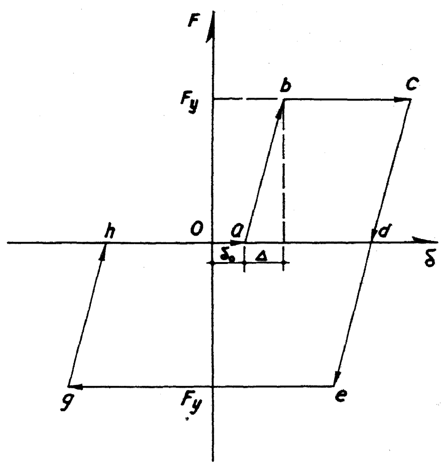

FIG. 5
Assumed force-displacement diagram of simple structure.

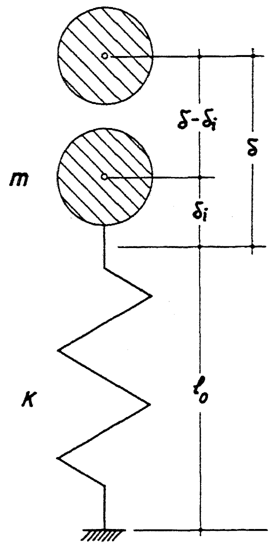

FIG. 6
Simple mass-spring system.

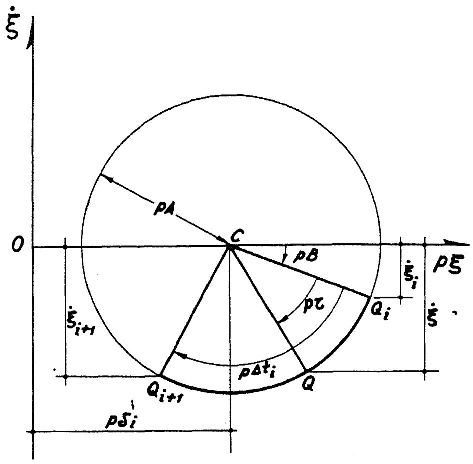

FIG. 7
Graphical representation of motion in the elastic range.

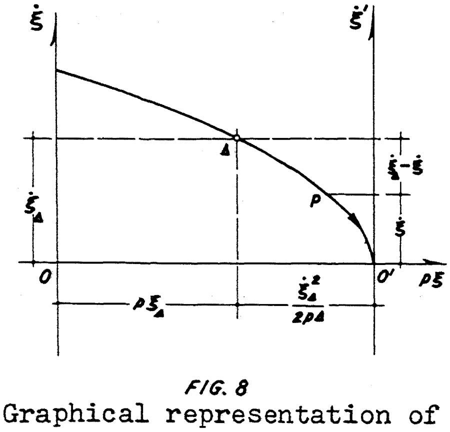

FIG. 8
Graphical representation of motion in the plastic range.

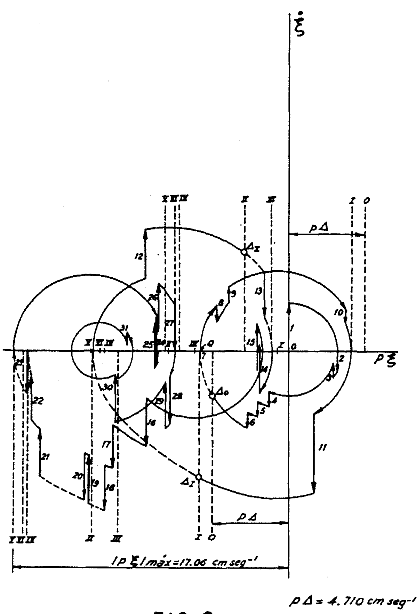

FIG. 9
Example of a gyrogram.