SHEARS IN A TALL BUILDING
SUBJECTED TO STRONG MOTION EARTHQUAKES

T. P. Tung* and N. M. Newmark*

1. Introduction

Results are given of a digital computer analysis for the dynamic shears in a 10-story structure subjected to the horizontal ground motions for each of twelve different strong-motion earthquakes recorded by the U. S. Coast and Geodetic Survey on the West Coast. The building is considered as a shear beam. Two different configurations of the 10-story building are considered:

(a) a building having a shear stiffness in the lowest story of 2,000 kips per in., with stiffness varying uniformly to 200 kips per in. for the top floor. The weight of the building varies uniformly from a value of 760 kips at the lowest floor to 400 kips at the top floor;

(b) a similar building having a uniform stiffness of 2,000 kips per in. for all floors, and a uniform weight of 582 kips at each floor level.

The method of analysis used was the same as that described in a previous paper by the authors(1). The analysis was carried out on the electronic digital computer at the University of Illinois, the ILLIAC, with each of the accelerograms reproduced by a series of polygonal lines. The twelve accelerograms used in the analyses are identified in Table 1.

The time interval in the numerical integration process was 0.0075 sec. Each problem used 20 to 30 minutes of machine time with about one third of the time used in punching results on the output tape. Only maximum shears were printed. The calculations were carried out for three different damping conditions corresponding to: (a) no damping; (b) 2 per cent of critical damping; and (c) 10 per cent of critical damping. The damping coefficient, n, was computed on the basis of the fundamental frequency and average weight of the building. All of the frequencies and mode shapes for the building with variable stiffness are given in Table 2 of Ref. (1). The fundamental period of this structure is 1.32 sec.

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* University of Illinois, Urbana, Illinois.

2. Results of Analysis

Only a part of the results of the analyses can be presented here because of shortage of space. The characteristics of the accelerograms are not given here. However, from integration of the accelerograms, the maximum velocities of the ground are determined and are reported in Table 1. In general, these maximum velocities occurred from one to ten seconds after the onset of the earthquake.

The maximum dynamic shears in all stories of the building with variable stiffness, for four selected accelerograms, is given in Table 2. In general, the maximum shears at different points in the building, for a given earthquake, occurred at different times, ranging from a little over 1 sec. to as much as 30 sec. after the onset of the earthquake. A similar tabulation for the building of uniform stiffness and mass is given in Table 3 for the same accelerograms. These accelerograms correspond to the least severe, the most severe, and two intermediate cases.

In order to complete the presentation of the most useful data, the maximum base shears, and the maximum shears in the top story, are given, respectively, in Tables 4 and 5 for both buildings and for all of the accelerograms.

3. Interpretations of Data

The relative shear distribution over the height of the buildings, as taken from Tables 2 and 3, are shown for selected accelerograms in Figs. 1(a) and (b). In general, the shear distribution is nearly parabolic, with the exception that for the lower stories a somewhat linear departure from the parabolic curve is noted. However, there are large differences in the distribution among the various accelerograms and between the two buildings, but other things being equal, only small differences for different degrees of damping.

Statistical analyses of shear distributions in a uniform building based on a random excitation of the base of the building, when the building is considered to be a shear beam, indicate a parabolic distribution of shear(2). The departures from the parabolic curve are due not only to the fact that one of the buildings considered herein is not uniform, but also because the excitation is not entirely random.

A plot of the base shear in each of the buildings for different degrees of damping is shown in Figs. 2(a), 2(b), and 2(c) as a function of the maximum ground velocity. In general, there is a fairly good correlation. The severity of the different earthquakes appears to be roughly in proportion to the maximum velocity reached in the ground motion of each of the quakes. However, the intensity of the maximum base shear decreases sharply as the damping coefficient increases from zero to 10 per cent.

An interpretation of the dynamic shears can be given in terms of either shear coefficients or seismic coefficients. The shear coefficient, C, is defined as that part of the total tributary weight above a certain level which, applied statically in a horizontal direction, would account for the maximum dynamic shear at that level.

The local seismic coefficient, c, is defined as that proportion of the weight at any level which, applied statically in a horizontal direction, would account for the shears at any elevation below that level. The two coefficients are equal for the top story.

The base shear coefficient, in terms of the total mass of the building, for both buildings and for all degrees of damping, is given in Table 6. It can be seen that the magnitude of the base shear is considerably larger than the quantity usually considered as applicable in the design of tall buildings and exceeds by a large factor the base shear coefficients used in the Joint Committee recommendations(3) or the Uniform Building Code specifications. The discrepancy, however, should not be interpreted as a lack of conservatism in present design procedures. The present analysis neglects certain factors in the resistance of the building. The actual conditions may not be as severe as those indicated by the analytical results.

Average values, for all the accelerograms, of shear coefficients for all stories and local seismic coefficients are given in Table 7 for both buildings and all degrees of damping. Considerable variation from these values is indicated by the results for individual accelerograms. In many cases, one would compute a negative local seismic coefficient at some of the intermediate floor levels from the results for a particular accelerogram.

4. Concluding Remarks

These remarks should be regarded as preliminary. Further calculations will be made to explore more systematically the general problem of earthquake response of structures. A systematic study of the effect of various distributions of mass and stiffness over the height of the building can be made with the techniques that have been explored in this paper. Some better means of characterizing an earthquake accelerogram in terms of its effect on the building must be sought.

There is a discrepancy between the results of the present analysis and that reported in Ref. (1). In the previous work, maxima were tabulated at too coarse a time interval, and some of the peak values were apparently lost. In the present calculations, the shear at each iteration interval was computed and compared with the previous maximum to insure the recording of the absolute maximum value.

5. Acknowledgment

The program for the digital computer calculation was prepared by Dr. Tung. However, since his departure from the University of Illinois, the calculation of a number of cases has been carried out by Mr. T. C. Hu, Research Assistant in Civil Engineering, who assisted also in the interpretation of the results. Grateful acknowledgment is made to Dr. G. W. Housner of the California Institute of Technology, for the loan of the original records of the accelerograms.

6. Bibliography

(1) "Numerical Analysis of Earthquake Response of a Tall Building," by T. P. Tung and N. M. Newmark, Bulletin of the Seismological Society of America, Vol. 45, No. 4, October 1955, pp. 269-278.

(2) "A Statistical Estimate of Relative Distribution of Extreme Shear in a Tall Building Subjected to Random Earthquake Shocks," by T. P. Tung and N. M. Newmark, Structural Research Laboratory Report No. 116, Civil Engineering Department, University of Illinois, March 1956.

(3) Report of a Joint Committee on Lateral Forces, "Lateral Forces of Earthquakes and Wind," Transactions ASCE, Vol. 117, 1952, pp. 717-754.

TABLE 1

LIST OF ACCELEROGRAMS USED

No.	Location	Date	Direction	Max. Velocity in./sec.
2	L. A. Subway Term.	Oct. 2, 1933	N-39°E	2.94
3	L. A. Subway Term.	Oct. 2, 1933	N-51°W	3.81
4	L. A. Subway Term.	Mar. 10, 1933	N-39°E	5.43
5	L. A. Subway Term.	Mar. 10, 1933	N-51°W	3.74
6	El Centro, Calif.	May 18, 1940	E-W	17.85
7	El Centro, Calif.	May 18, 1940	N-S	17.44
9	El Centro, Calif.	Dec. 30, 1934	N-S	10.58
10	El Centro, Calif.	Dec. 30, 1934	E-W	13.41
15	Vernon	Mar. 10, 1933	E-W	9.49
16	Vernon	Mar. 10, 1933	N-S	9.83
17	Vernon	Oct. 2, 1933	S-82°E	3.83
19	Vernon	Oct. 2, 1933	N-08°E	2.14

TABLE 2

MAXIMUM DYNAMIC SHEARS-VARIABLE STIFFNESS BUILDING

Story	Damping Factor	Max. Shear, kips, for Accelerogram No. 6	16	5	19
Top	0	1094	431	114	159
9		1608	645	175	165
8		1962	793	228	193
7		2020	809	298	293
6		1798	853	360	276
5		2000	863	417	285
4		2397	1081	437	320
3		2907	1331	510	271
2		3262	1532	544	332
1		3563	1649	562	430
Top	0.02	488	226	51	118
9		737	345	79	113
8		755	416	114	126
7		950	438	149	185
6		943	442	170	183
5		1024	504	178	181
4		1199	531	195	207
3		1340	625	206	181
2		1586	717	223	231
1		1739	797	239	289
Top	0.10	256	164	30	76
9		330	249	46	87
8		316	283	61	72
7		354	282	70	117
6		457	349	84	108
5		488	417	104	108
4		559	439	124	114
3		624	461	144	113
2		673	481	160	113
1		720	506	173	157

TABLE 3

MAXIMUM DYNAMIC SHEARS-UNIFORM STIFFNESS BUILDING

Story	Damping Factor	Max. Shear, kips, for Accelerogram No. 6	16	5	19
Top	0	764	385	113	272
9		1274	678	217	403
8		1696	889	301	423
7		2059	1060	357	390
6		2241	1155	400	396
5		2551	1277	419	322
4		2968	1456	424	250
3		3425	1537	436	290
2		3640	1573	456	407
1		3801	1688	475	529
Top	0.02	526	271	72	160
9		862	473	136	263
8		1156	583	178	281
7		1420	743	206	288
6		1643	849	230	301
5		1731	926	251	259
4		1734	1010	279	176
3		1973	1074	303	220
2		2120	1105	316	249
1		2294	1179	326	296
Top	0.10	262	156	43	90
9		414	300	82	151
8		508	444	111	173
7		622	558	134	190
6		670	632	154	210
5		669	682	165	176
4		702	726	170	131
3		759	754	174	126
2		850	767	181	149
1		928	814	184	199

TABLE 4

MAXIMUM BASE SHEARS

Accel. No.	Max. Ground Velocity in./sec.	Base Shear, kips: Variable Stiffness n = 0	Variable Stiffness n = 2%	Variable Stiffness n = 10%	Uniform Stiffness n = 0	Uniform Stiffness n = 2%	Uniform Stiffness n = 10%
2	2.94	418	232	216	385	220	190
3	3.81	705	301	241	601	413	296
4	5.43	1305	1011	231	847	448	327
5	3.74	562	239	173	475	326	184
6	17.85	3563	1739	720	3801	2294	928
7	17.44	2593	1856	1202	3717	2404	1441
9	10.58	2132	1393	765	2240	1266	1041
10	13.41	1708	893	535	1424	856	551
15	9.49	1516	1040	820	1767	1541	1197
16	9.83	1649	797	506	1688	1179	814
17	3.83	638	492	404	975	666	509
19	2.14	430	289	157	529	296	199

TABLE 5

MAXIMUM TOP STORY SHEARS

Accelerogram No.	Top Story Shear, kips					
	Variable Stiffness			Uniform Stiffness		
	n = 0	n = 2%	n = 10%	n = 0	n = 2%	n = 10%
2	116	109	82	141	72	44
3	206	105	64	189	99	60
4	366	197	41	161	78	42
5	114	51	30	113	72	43
6	1094	488	256	764	526	262
7	825	565	377	901	561	385
9	741	429	295	609	361	297
10	513	297	122	602	367	123
15	367	321	193	430	313	218
16	431	227	164	385	271	156
17	197	149	90	286	181	156
19	159	118	76	272	160	90

TABLE 6

BASE SHEAR COEFFICIENTS

Accelerogram No.	Base Shear in Terms of Total Weight					
	Variable Stiffness			Uniform Stiffness		
	n = 0	n = 2%	n = 10%	n = 0	n = 2%	n = 10%
2	0.072	0.040	0.037	0.066	0.038	0.033
3	0.122	0.052	0.042	0.103	0.071	0.051
4	0.225	0.174	0.040	0.146	0.077	0.056
5	0.097	0.041	0.030	0.082	0.056	0.032
6	0.614	0.300	0.124	0.653	0.394	0.160
7	0.447	0.320	0.207	0.639	0.413	0.248
9	0.368	0.240	0.132	0.385	0.218	0.179
10	0.295	0.154	0.092	0.245	0.147	0.095
15	0.261	0.179	0.141	0.304	0.265	0.206
16	0.284	0.137	0.087	0.290	0.203	0.140
17	0.110	0.085	0.070	0.168	0.114	0.087
19	0.074	0.050	0.027	0.091	0.051	0.034

TABLE 7

AVERAGE SHEAR AND LOCAL SEISMIC COEFFICIENTS

Stiffness Distribution	Story	Story Shear Coeff., C, in Terms of Tributary Weight			Local Seismic Coeff., c, in Terms of Local Weight		
		n = 0	n = 2%	n = 10%	n = 0	n = 2%	n = 10%
Varying	Top	1.068	0.637	0.373	1.068	0.637	0.373
	9	0.688	0.412	0.260	0.344	0.208	0.157
	8	0.529	0.315	0.193	0.251	0.146	0.081
	7	0.442	0.263	0.153	0.222	0.132	0.053
	6	0.345	0.222	0.135	0.126	0.091	0.077
	5	0.292	0.185	0.117	0.086	0.034	0.048
	4	0.275	0.161	0.104	0.197	0.060	0.046
	3	0.257	0.151	0.094	0.164	0.099	0.040
	2	0.255	0.149	0.090	0.232	0.135	0.068
	1	0.248	0.148	0.086	0.201	0.132	0.058
Uniform	Top	0.695	0.438	0.269	0.695	0.438	0.269
	9	0.584	0.370	0.232	0.472	0.301	0.195
	8	0.498	0.316	0.203	0.328	0.210	0.146
	7	0.422	0.275	0.179	0.193	0.150	0.106
	6	0.357	0.249	0.164	0.099	0.143	0.101
	5	0.327	0.219	0.146	0.177	0.070	0.056
	4	0.302	0.195	0.129	0.153	0.052	0.032
	3	0.277	0.179	0.120	0.100	0.070	0.057
	2	0.269	0.174	0.114	0.209	0.132	0.066
	1	0.264	0.171	0.110	0.217	0.139	0.071

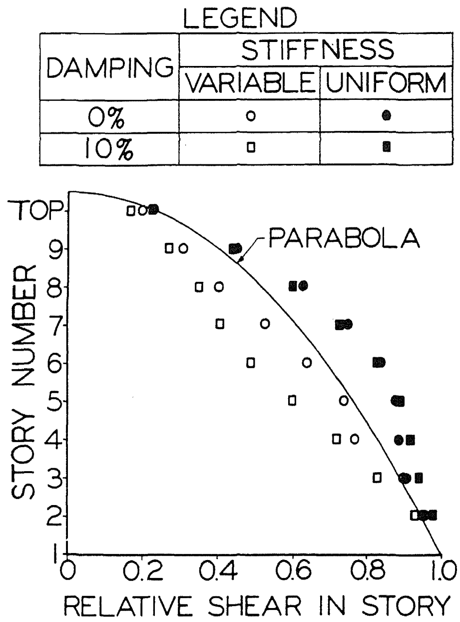


FIG. 1(a)

Relative Shear Distribution over Height of Building, Accelerogram 5

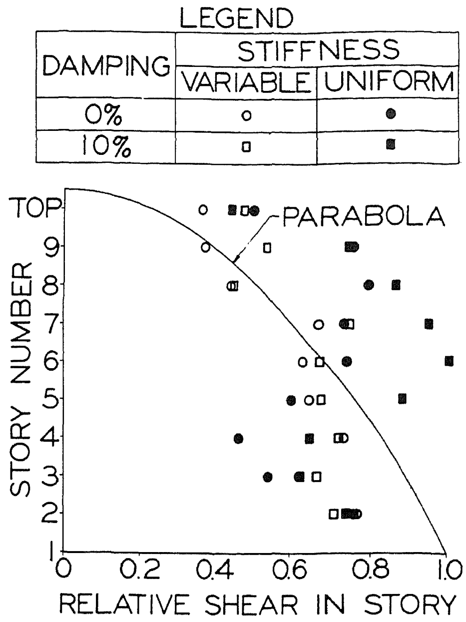


FIG. 1(b)

Relative Shear Distribution over Height of Building, Accelerogram 19

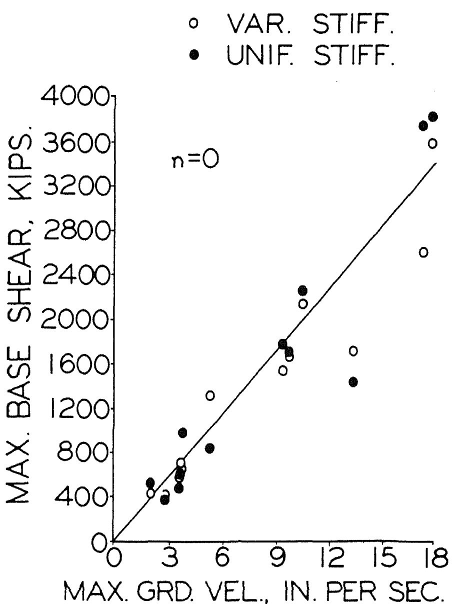
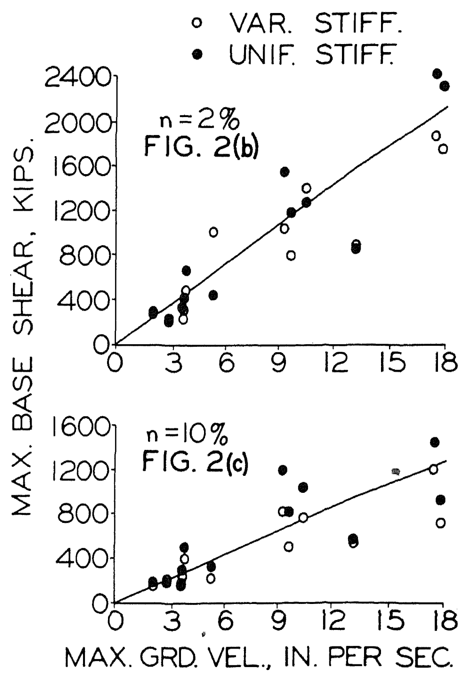


FIG. 2(a)

Base Shear versus Ground Velocity, n = 0



2(b) **Base Shear versus Ground Velocity, n = 2%**
2(c) **Base Shear versus Ground Velocity, n = 10%**