

## OPTIMAL SEISMIC DESIGN OF FRICTION DAMPED BRACED FRAMES BASED ON EXISTING EARTHQUAKE RECORDS

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### SUMMARY

Friction dampers that are designed to act at a certain predetermined level have been applied successfully to steel structures as braces. This slippage action causes a major portion of the seismic energy to be dissipated, thus leaving the building intact without its members having to yield or buckle. Basically, two parts of the brace with slotted holes are connected by high strength bolts and may have a lining pad in-between. Slotted Bolted Connections (SBC's) are practically effort free to construct and implement and use commercially available materials. They are, thus, attractive to use in seismic design of new buildings and in retrofitting of existing structures. This paper is concerned with their design. A dynamic model is developed to describe the behavior of an SDOF steel frame that uses bilinear hysteretic behavior for the damper. This model is generalized to MDOF's. A design procedure is then applied to a 10-story steel frame. It attains the stiffness of the individual braces and the displacement at the threshold of activation. The process is a two-phase iterative procedure that converges rather fast.

As for the actual earthquake motion, a set of 35 earthquake records carefully grouped according to their ratio of peak ground acceleration (PGA) to peak ground velocity (PGV) were used for the design. The preferred choice of normalizing to  $PGV = 0.4\text{m/sec}$  produces designs that may be regarded as "autonomously optimal" for Zone 4.

As anticipated the designed braces and slip points effectively produced simultaneous slippage. The design procedure attained in this paper uses an optimality criterion of equal drifts. This criterion is evidently satisfactory when limited to first mode behavior. Care has to be exercised when higher modes are dominant.

### INTRODUCTION

The heavy damage caused by the 1994 Northridge and the 1995 Kobe earthquakes has led engineers to examine the time honored philosophy that life safety should be the only criterion for the design of structures under strong earthquakes. Damage limitation is now becoming a design objective. The need to contain damage also leads to a re-examination of the traditional design approach in which dissipation of seismic energy is achieved through limited damage in the post-elastic range of selected structural elements. Hence the recent interest in mechanical energy dissipating devices. The need for such devices is more acute in concentrically braced frames, since the ability of the braces to dissipate energy in severe earthquakes is limited mainly by tensile yielding and by the buckling capacity of the diagonal braces. The fundamental underlying principles for the design of energy dissipating devices of various kinds and their actual implementation may be found in the comprehensive book by Soong and Dargush (1997).

Friction dampers, which are designed to slip at a certain predetermined level, have been successfully applied to

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steel structures as braces (Filiatrault and Cherry, 1987, 1990; Pall *et al* 1996).

Friction devices that utilize slotted bolted connections (SBCs) in braced connections as proposed by Fitzgerald (1989) are discussed in this work. Basically, high strength bolts connect two parts of the brace with slotted holes and the faying parts are lined with steel or brass pads. SBCs are relatively easy to construct and implement, and use commercially available materials. They are, thus, attractive for use in the seismic design of new buildings and in retrofitting of existing structures. Popov, Yang & Grigorian, (1993) also applied SBCs to an experimental model which they tested on a shaking table, and found them very effective in dissipating most of the input energy.

The present paper is concerned with the practical design of the friction braces i.e. with the evaluation of the stiffnesses of the individual braces and their elongation at the threshold of slip for a wide range of earthquakes according to a sound design procedure.

## THEORETICAL BACKGROUND

### Dynamic Behavior

The basic SDOF analytical model consists of a flexural frame, a linear spring and the Coulomb slip element. A stick stage and a sliding stage describe the motion of the brace. The stick stage has the velocity of the friction damper  $\dot{y}^*$  equal to zero, the stiffness of the frame equal to the initial stiffness of the braced frame, and the friction force equal to zero. Linear behavior is assumed. The sliding stage has the velocity of the friction damper  $\dot{y}^*$  equal to the velocity of the frame  $\dot{y}$ . The stiffness of the frame is that of the stiffness of the frame without a brace and the friction force is equal to the slip force in the brace. When the direction of the horizontal movement of the frame is reversed the system returns to the first stage.

The equations of motion of the single-degree-of-freedom bilinear hysteretic system that is used to describe a MDOF system is expressed as follows:

$$\ddot{y}(t) + 2\xi\omega\dot{y}(t) + \alpha\omega^2 y(t) + (1 - \alpha)\omega^2 (y(t) - y^*(t)) = -\Gamma a(t) \quad (1)$$

$$\dot{y}^*(t) = \dot{y}(t)\varphi_1(t)\varphi_2(t) \quad (2)$$

$$\varphi_1(t) = 0.5(1 + \text{sign}(|y(t) - y^*(t)| - u_0)) \quad (3)$$

$$\varphi_2(t) = 0.5(1 + \text{sign}(|\dot{y} + s\dot{u}| - s|\dot{u}|)) \quad (4)$$

$$u = \dot{y}_q(t) - \dot{y}_{q-1}(t) \quad (5)$$

where:  $y(t), \dot{y}(t), \ddot{y}(t)$  = displacement, velocity and acceleration of the frame respectively;  $y^*(t), \dot{y}^*(t)$  = slip displacement and velocity of friction damper respectively;  $a(t)$  = earthquake acceleration at the base;  $\alpha$  = secondary slope ratio in the hysteretic model;  $u_0$  = maximum pre-slip brace deformation (spring only);  $\xi$  = viscous damping ratio;  $\omega$  = initial natural frequency of the structure;  $\Gamma$  = modal participation factor;  $u$  = differential velocity between two consecutive calculation steps;  $s$  = number of calculation steps at reversal of motion direction.

Equation 2 controls the motion along the hysteresis shape. The first function  $\varphi_1(t)$  of the equation expresses the fact that when the velocity of the friction damper  $\dot{y}^* = 0$  the relative displacement of the frame-to-damper displacement is less than  $u_0$ ,  $\dot{y}^* = \dot{y}$  in all other cases. The second function,  $\varphi_2(t)$ , of the equation expresses the fact that when the direction of motion is reversed, the velocity of the damper in the first  $s$  steps of the calculation is artificially held at zero ( $\varphi_2(t) = 0$ ). This is done for the purpose of numerically obtaining  $\varphi_1(t) = 0$ , and thus automatically diverting motion into the stick stage. This paper uses  $s=8$ . The displacement of the frame  $y(t)$  begins to decrease, but the displacement of the brace has not changed. After  $s$  steps of calculation, the displacement of the frame  $y(t)$  relative to the damper displacement  $y^*(t)$  is equal, or not significantly less, then  $u_0$ ,  $\varphi_2(t) = 1$ , but  $\varphi_1(t) = 0$  and the system moves from the sliding stage to the stick stage

The generalized differential equation of motions for the MDOF (multi degree of freedom) braced frame can be expressed as follows:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) + \mathbf{f}_y = -a(t)\mathbf{M} \quad (6)$$

where:  $\mathbf{y}(t), \dot{\mathbf{y}}(t), \ddot{\mathbf{y}}(t)$  = displacement, velocity and acceleration vectors for the story levels respectively;  $\mathbf{M}$  = structure mass matrix;  $\mathbf{C}$  = structure viscous damping matrix;  $\mathbf{K}$  = structure stiffness matrix (unbraced);  $\mathbf{f}_y$  = vector of horizontal components of friction forces.

For a more detailed description of the MDOF equations the reader is referred to Segal *et al* (1998).

## Design Procedure

Design of friction dampers constitutes the evaluation of the stiffness of the braces and their slip elongations for a given set of constraints on interstory drifts. This work uses equal drifts. Thus, at maximum response (after slippage), the displaced shape of the structure will have attained a straight line. Slip points are used to define the elongations of the braces up to the brink of slippage. The design approach has two main phases.

### Phase 1

Phase 1 condenses the MDOF model of the structure with no braces to an equivalent SDOF model. A brace is designed according to an iterative optimization scheme and, thus, an equivalent natural period of the SDOF model with a brace is achieved. This is the period that the MDOF will be designed for in Phase 2.

The equivalent mass, stiffness, and natural frequency of the SDOF model are calculated from the first mode behavior of the basic frame structure (without braces) as  $M_1 = \phi_1^T \mathbf{M} \phi_1$ ;  $K_1 = \phi_1^T \mathbf{K} \phi_1$ ;  $\omega = \sqrt{K_1 / M_1}$ . The interstory drifts are assumed equal for all stories. The maximum allowable displacement,  $\Delta^{all}$ , of the top of the structure for both the SDOF and MDOF models becomes  $\Delta^{all} = N d^{all}$  with  $N$  = number of stories;  $d^{all}$  = allowable interstory drift displacement.

Moreover, it is assumed that all the friction dampers are activated simultaneously and the slip elongations are the same for all the dampers. Thus, in the SDOF model the slip elongation,  $U_o$ , is taken as sum of the slips of the MDOF model. The slip elongation of the brace is given as:  $u_{o_{max}} = (f_s l \cos \theta) / EA$ . Here  $u_{o_{max}}$  = slip elongation of a single brace in the MDOF system;  $f_s$  = slip force;  $l$  = length of the brace;  $A$  = cross sectional area of brace.

The stiffness of the braced SDOF equivalent structure is obtained iteratively using an ‘‘analysis/redesign’’ procedure in structural optimization (Spillers, 1975) that takes the form of:

$$\alpha^{n+1} = \left( \Delta^{all} / \Delta^n \right)^p \alpha^n \quad (7)$$

where  $\Delta^n$  = maximum response at  $n$ -th,  $p$  is a convergence exponent (usually 0.5) and  $\alpha = (K_1) / (K_{1_{full}})$ , the already defined secondary slope. The analysis is actually a full nonlinear one using the dynamic model (Eq. 1). The term ‘‘full’’ refers to the frame with its brace. whereas the term ‘‘frame’’ refers to the frame without its brace. Finally, the period of the braced SDOF equivalent structure is obtained from  $T_{full} = 2\pi \sqrt{M_1 / K_{1_{full}}}$ .

Most of the analytical methods to estimate the earthquake response of non-linear hysteretic systems are based on defining a viscously damped linear system whose maximum response approximates that of the non-linear hysteretic system. The equation of motion of the effective linear SDOF system is given by:

$$\ddot{y}(t) + 2\xi_{eff} \omega_{eff} \dot{y}(t) + \omega_{eff}^2 y(t) = -\Gamma a(t) \quad (8)$$

where:

$$\omega_{eff}^2 = \frac{k_{eff}}{M_1}; \quad 2\xi_{eff} \omega_{eff} = \frac{c_{eff}}{M_1} \quad (9)$$

The parameters,  $k_{eff}$ ,  $\omega_{eff}$ ,  $c_{eff}$  and  $\xi_{eff}$ , represent effective system quantities. This paper proposes effective stiffness and damping,  $k_{eff}$  and  $c_{eff}$ , which were obtained using a harmonic linearization approach. These are:

$$k_{eff} = 2 \left( \left( \frac{\sqrt{U_o(\Delta - U_o)}}{\Delta} (2U_o - \Delta) + 0.5 \Delta \sin^{-1} \left( \frac{2U_o}{\Delta} - 1 \right) \right) K_{1_{full}} (1 - \alpha) + 0.25 \pi \Delta K_{1_{full}} (1 + \alpha) \right) / (\pi \Delta) \quad (10)$$

$$c_{eff} = 4U_0 K_{1,full} (1 - \alpha) \left( 1 - \frac{U_0}{\Delta} \right) / (\Omega \pi \Delta) \quad (11)$$

where  $\Omega$  is the value at which the spectral density function is maximal and  $\Delta$  is the maximal response displacement.

### Phase 2

Phase 2 consists of a two stage iterative scheme that produces the brace stiffnesses and the slip elongations as the design converges to the period of phase 1.

At first, the MDOF model is in motion without slip, and therefore the full stiffness of the structure is assembled from both the stiffnesses of the frame elements and the stiffnesses of the braces. Then as one of the braces slips all the friction dampers are assumed to be activated, and the stiffness of the structure reduces to the stiffness of the frame alone. At the completion of the design the dampers will have acted simultaneously.

The distribution of the non-linear *friction* forces,  $\mathbf{f}_y$ , in the dampers is assumed proportional to the equivalent forces,  $\mathbf{f}_e$ , obtained for the same MDOF with *viscoelastic* dampers, i.e.

$$f_{y_i} \propto f_{e_i} \quad (12)$$

Moreover, at maximum response, this viscoelastic MDOF system should exhibit a displaced shape of a straight line for the originally defined equal drifts criterion.

Following is the procedure whereby the equivalence is obtained. The equation of motion of the frame structure with *linear* devices can be presented as follows:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) + \mathbf{f}_e^0(t) = -\mathbf{a}(t)\mathbf{M}\mathbf{i} \quad (13)$$

Here  $\mathbf{f}_e^0(t)$  is the horizontal force vector of the equivalent viscoelastic dampers at floor levels and  $\mathbf{i}$  is a vector with entries of  $1.0$ . This force vector may be defined as:

$$\mathbf{f}_e^0(t) = \mathbf{K}_e \mathbf{y}(t) + \mathbf{C}_e \dot{\mathbf{y}}(t) \quad (14)$$

in which, the  $\mathbf{K}_e$  and  $\mathbf{C}_e$  are respectively the diagonal horizontal stiffness and damping matrices at floor levels.

### Evaluation of $\mathbf{K}_e$

The stiffnesses of the braces can be obtained by assuming a desired first mode behavior for the equivalent MDOF system having viscoelastic dampers:

$$[\mathbf{K} + \mathbf{K}_e] \Phi_1 = \omega_{eff}^2 \mathbf{M} \Phi_1 \quad (15)$$

If the stiffness matrix  $\mathbf{K}_e$  is approximated as a tri-diagonal matrix algebraic manipulations will transform Eq. (15) to:

$$\begin{aligned} \mathbf{\Phi} \mathbf{k}_e &= \mathbf{K} \Phi_1 + \omega_{eff}^2 \mathbf{M} \Phi_1 \\ \text{or} \\ \mathbf{A} \mathbf{k}_e &= \mathbf{b} \end{aligned} \quad (16)$$

Because the stiffnesses,  $\mathbf{k}_e$ , cannot be negative the solution of Eq. (16) is performed using standard least squares approximation with non negative variables, which is usually written as:

$$\min \|\mathbf{A} \mathbf{k}_e - \mathbf{b}\|_2^2 \quad \text{subject to} \quad \mathbf{k}_e \geq \mathbf{0}. \quad (17)$$

### Evaluation of $\mathbf{C}_e$

The damping matrix of the model is assumed to be proportional to the stiffness and mass matrices by Rayleigh's proportionality factors  $a_0, a_1$  (see e.g. Clough and Penzien, 1993) and can be written as:

$$\mathbf{C}_e = a_0 \mathbf{M} + a_1 (\mathbf{K} + \mathbf{K}_e) \quad (18)$$

The proportionality factors  $a_0, a_1$  are given by:  $a_0 = \xi_{eff} \frac{2\omega_{eff j} \omega_{eff k}}{\omega_{eff j} + \omega_{eff k}}$ ;  $a_1 = \xi_{eff} \frac{2}{\omega_{eff j} + \omega_{eff k}}$ ; where  $\omega_{eff j}$  and  $\omega_{eff k}$  are two chosen natural frequencies of the frame without braces, which are determined by solving the

undamped eigenvalue equation:  $(\mathbf{K} + \mathbf{K}_e) - \omega_{eff}^2 \mathbf{M} \Phi = \mathbf{0}$ . This paper uses  $j=1$  and  $k=3$ . If the mass matrix,  $\mathbf{M}$ , is taken as a diagonal matrix and the frame stiffness matrix,  $\mathbf{K}$ , is also taken as tri-diagonal, the viscous matrix,  $\mathbf{C}_e$ , becomes tri-diagonal and the terms of this matrix are the viscous coefficients of the linear dampers in the braces. If the mass matrix,  $\mathbf{M}$ , and/or the stiffness matrix,  $\mathbf{K}$ , are taken as full matrices, the viscous matrix,  $\mathbf{C}_e$ , is also a full matrix. The evaluation of the viscous coefficients, in the latter case, can be obtained using story drift transformation in the following manner.

First a transformation matrix,  $\mathbf{T}$ , that transforms inter-story drifts to story displacements is defined  $\mathbf{y}(t) = \mathbf{T}\mathbf{d}(t)$  then the viscous damping matrix,  $\mathbf{C}_e$ , is transformed into  $\mathbf{C}_d$ , (in the inter-story formulation of the governing Equations (13)), using  $\mathbf{K}_d = \mathbf{T}^T \mathbf{K}_e \mathbf{T}$  and  $\mathbf{C}_d = \mathbf{T}^T \mathbf{C}_e \mathbf{T}$ . The viscous damping parameters of each brace can now be obtained using a fundamental (first) mode approach. This approach takes a full matrix  $\mathbf{C}_e^d$  and transforms it into an approximate diagonal matrix as follows: the difference in viscous components of forces squared over the time history (least square approximation) is first minimized. Then if drift velocities are approximated as fundamental modal drifts the following diagonal terms result (Reinhorn, 1993):

$$c_e^k = \frac{\sum_j c_{e,kj}^d \Delta \phi_{j1}}{\Delta \phi_{k1}} = \sum_j c_{e,kj} \Delta \phi_{j1}^k \quad (19)$$

Here  $\Delta \phi_{j1}^k = \Delta \phi_{j1} / \Delta \phi_{k1}$ ;  $\Delta \phi_{j1} = \phi_{j1} - \phi_{(j-1)1}$ ;  $\Delta \phi_{k1} = \phi_{k1} - \phi_{(k-1)1}$ ; and  $\phi_{0,1} = 0$ . For equal drifts the first mode of the brace is  $\Phi_1 = [1/N, 2/N, \dots, N/N]^T$ . These values of the stiffnesses (from solution of Eq. 17) and the viscous dampers (Eq. 19) now define the forces in the braces as:

$$f_{ei} = k_e^i (y_i - y_{i-1}) + c_e^i (\dot{y}_i - \dot{y}_{i-1}) \quad (20)$$

#### Equal drifts correction strategy

At this point a linear dynamic analysis is performed to yield the maximum top story displacement,  $\Delta$  and the drifts  $d_i$ . The force vector is then corrected according to  $f_{e,corrected,i} = f_{e_i} \times d_i / \Delta_{average}$  where  $\Delta_{average} = \Delta / N$ .

#### Stiffness of the brace

The stiffness of each brace is obtained using Eq. (12) as:

$$k_{y_i}^* = \frac{f_{y_i}}{u_{0_i}} \propto \frac{f_{e_i}}{u_{0_i}} \text{ or as: } k_{y_i}^* = q \frac{f_{e_i}}{u_{0_i}} = q k_{y_i}^{**} \quad (21)$$

where  $q$  is as yet an unknown proportionality coefficient.

The full stiffness matrix of the structure can be presented as the sum of the stiffness matrix of the frame and  $q$  times the tri-diagonal matrix composed of the elements  $k_{y_i}^{**}$ .

#### Two stage iterative procedure for the design of the braces and slip elongations

Step 1 Choose  $u_{0_i} = u_{0_{max}}$  and  $q=1.0$

Step 2 Calculate  $k_{y_i}^{**}$  from Eq. 21. The values of  $f_{e_i}$  in Eq. 21 are obtained by solving the system of equations 6 with each  $f_{y_i}$  replaced by the expression of Eq. 20.

Step 3 Redesign using  $q^{n+1} = (T_{full} / T^n)^p q^n$  where  $T^n$  is the period of the full i.e. braced structure, for the current parameters obtained for the 1<sup>st</sup> eigenvalue with  $\mathbf{K}_{full}$  and  $\mathbf{M}$ .

Step 4. Calculate  $u_{0_i}$  from  $u_{0_i} = \delta_i / \delta_{max} u_{0_{max}}$  where  $\delta_i = \phi_{i1} - \phi_{(i-1)1}$  and  $\delta_{max} = (\phi_{i1} - \phi_{(i-1)1})_{max}$  where  $\phi_{i1}$  is the element  $i$  of the 1<sup>st</sup> mode shape with  $\mathbf{K}_{full}$  and  $\mathbf{M}$ .

Step 5. Go to step 2 until convergence.

## SELECTION OF EARTHQUAKE RECORDS

Strong motion earthquake records are basic data for the earthquake engineers. A study of such records can give insight into the nature of earthquake ground shaking. When used as inputs for experiments or analyses, they

permit a realistic simulation to study the nature of structural responses to seismic excitation. Earthquake ground motion records can be used to evaluate the performance of a particular structure, or assess the adequacy of building code requirements. One major parameter that has a significant effect on the structural response is the frequency content of the record. For estimation of the frequency content, a simple approach based on the peak ground acceleration to peak ground velocity ratio (A/V ratio) is often accepted (Heidebrecht and Lu, 1988, Naumoski et al., 1988). Motions with high A/V generate significant response in short period structures, whereas those with low A/V generate significant response in long period ones.

The Naumoski et al. (1993) report describes five ensembles of fifteen earthquake records, each having been assembled to defined ranges of A/V ratios. The ensembles are categorized as high (VH: A/V=2.63-3.52), new high (NH: A/V=1.60-2.43), new intermediate (NI: A/V=0.82-1.21), new low (NL: A/V=0.62-0.79), and new very low (NVL: A/V=0.36-0.59).

This work uses 35 (twelve from the NVL, eleven from NI and twelve from NH categories, respectively) out of the 75 records and are identified according to the record numbers that appear in the Naumoski et al. (1993) report.

### Normalization of the Earthquake Records

This section describes the parametric dynamic analyses that was performed and the rationale behind the scaling strategy of the earthquake records that was adopted in this work. Assuming that the seismic response is to be based on some average of the response results from the selected ensembles of records, the first question is how this averaging is to be carried out. The normalization rule used in this work is the one that minimizes the dispersion of the response obtained for the different sets of records.

The stiffness ratio coefficients  $\alpha=K_{frame}/K_{full}$ , the effective stiffness and the effective damping of the braces of the equivalent SDOF frame of a 10-story frame that is used in the example below, with an equivalent mass taken as unity, were calculated. The earthquake motion time histories were normalized according to peak ground acceleration (PGA or A) and peak ground velocity (PGV or V). The average results for the chosen A/V categories are shown in Table 1. Records falling into the A/V=1.0 category are, indifferent to which of the two modes of scaling is used. The results clearly indicate that scaling to PGA produces a wide dispersion of results whereas scaling to PGV leads to a significant reduction in scatter. The design which follows will, therefore, be constructed for records of all categories scaled to PGV=0.4 m/sec (Seismic Risk Zone 4).

TABLE 1. MEAN RESPONSE FOR 35 CHOSEN EARTHQUAKE RECORDS						
PGA=0.4g				PGV=0.4m/sec		
	$\alpha$	$c_{eff}$	$k_{eff}$	$\alpha$	$c_{eff}$	$k_{eff}$
A/V=0.5	0.077	0.92	1.67	0.17	0.8	4.22
A/V=1.0	0.18	0.79	4.11	0.18	0.8	4.07
A/V=2.0	0.67	0.23	1.89	0.24	0.72	3.92
Mean	0.30	0.66	2.0	0.19	0.78	4.08

## DESIGN OF FRICTION DAMPERS FOR A 10-STORY STEEL FRAME

### Results

A 10-story steel frame is used as an example for which the braces (stiffnesses and slip elongations) are to be designed. Each story has a mass of 54 tons and the stiffness matrix  $\mathbf{K}$  is given below. Proceeding with phase 2 of the outlined procedure ( $\alpha=0.19$  in phase 1) obtain the stiffness and viscous coefficients of the equivalent linear as:

$c_e=[2.03 \ 1.99 \ 1.92 \ 1.81 \ 1.66 \ 1.47 \ 1.26 \ 1.00 \ 0.47 \ 0.36]$ ton sec/cm;  $k_e=[26.34 \ 16.71 \ 16.96 \ 15.60 \ 13.47 \ 12.69 \ 10.15 \ 5.96 \ 7.66 \ 0]$ ton/cm. The maximum viscous forces are:  $f_e=[62.33 \ 44.53 \ 40.52 \ 35.93 \ 29.96 \ 25.90 \ 20.25 \ 16.30 \ 10.90 \ 2.98]$ ton and the maximal interstory displacements (drifts) of the MDOF system with viscous dampers  $d=[1.79 \ 1.79 \ 1.74 \ 1.71 \ 1.63 \ 1.56 \ 1.42 \ 1.27 \ 1.02 \ 0.78]$  having a maximum top story displacement is  $\Delta=14.78$ cm. Figure 1 shows the mean story displacements plus one standard deviation (M+SD) for this case. An equal drift correction is now performed and the design process continued according to the two stage iterative algorithm to yield results that are summarized in Table 2. A full nonlinear analysis of the MDOF system with friction dampers is now performed and the mean story displacements plus one standard deviation (M+SD) are

displayed in Figure 2. Figure 3 shows the deviation from the equal drift requirement of the frame when designed for the mean plus one standard deviation (M+SD) and for the individual earthquake records.

**Table 2. Brace stiffnesses and slip elongations of the friction dampers.**

PGV=0.4 m/sec			
story #	$u_0$ (cm)	brace stiffness(ton/cm)	friction forces(ton)
1	0.19	156.40	30.06
2	0.26	95.42	25.04
3	0.31	76.32	24.29
4	0.36	62.38	22.74
5	0.42	45.66	19.44
6	0.47	34.87	16.84
7	0.54	23.06	12.67
8	0.60	8.37	5.13
9	0.53	8.85	4.75
10	0.40	2.38	0.94

$$\mathbf{K} = \begin{bmatrix}
 141.66 & -121.59 & 38.41 & -7.19 & 1.43 & -0.19 & 0.15 & -0.52 & 0.73 & -0.36 \\
 -121.59 & 182.01 & -121.25 & 37.57 & -7.01 & 1.35 & -0.45 & 0.52 & -0.72 & 0.41 \\
 38.41 & -121.25 & 171.99 & -112.91 & 33.85 & -5.92 & 1.02 & -0.26 & 0.34 & -0.23 \\
 -7.19 & 37.57 & -112.91 & 160.38 & -105.33 & 31.53 & -6.26 & 1.58 & -0.57 & 0.23 \\
 1.43 & -7.02 & 33.85 & -105.33 & 151.05 & -99.40 & 31.22 & -6.93 & 1.58 & -0.28 \\
 -0.19 & 1.35 & -5.92 & 31.53 & -99.40 & 143.54 & -94.79 & 28.53 & -5.41 & 0.83 \\
 0.15 & -0.45 & 1.02 & -6.26 & 31.22 & -94.79 & 136.79 & -89.03 & 24.92 & -3.60 \\
 -0.52 & 0.52 & -0.26 & 1.58 & -6.93 & 28.53 & -89.03 & 125.78 & -77.18 & 17.35 \\
 0.73 & -0.72 & 0.34 & -0.57 & 1.58 & -5.41 & 24.92 & -77.18 & 100.63 & -44.09 \\
 -0.36 & 0.41 & -0.23 & 0.23 & -0.28 & 0.83 & -3.60 & 17.35 & -44.09 & 29.61
 \end{bmatrix} \text{ ton/cm}$$

**CONCLUSIONS**

The main contribution of this work relates to the fact that friction dampers having optimally attained slip elongations,  $u_{0i}$ , and effective viscoelastic dampers which are characterized by  $c_e^k$  and  $k_e^k$  display practically equal top story displacements at the instant of maximum response. The two-phase iterative scheme provides a methodology that works well, and is close to the designer’s heart due to its iterative redesign nature. The design procedure presented in this paper uses an optimality criterion of equal drifts. This criterion is evidently satisfactory when limited to first mode behavior. The use of a large number of earthquake records of varying characteristics becomes efficient when scaled to PGV. The resulting design is autonomous if properly scaled. Figure 3 indicates that 100% of the records produce designs within 40% of the target design, 80% within 30% and 57% within 20%. This is a good practical result. Another reason for the deviation is approximated 1st mode behavior only of the design, whereas analysis was based on all of the 10 modes.

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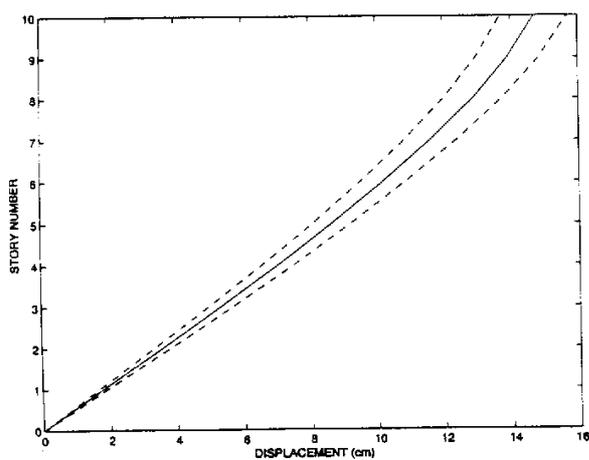
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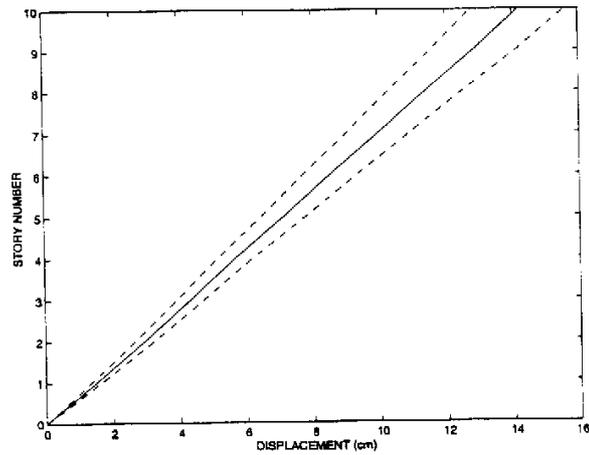
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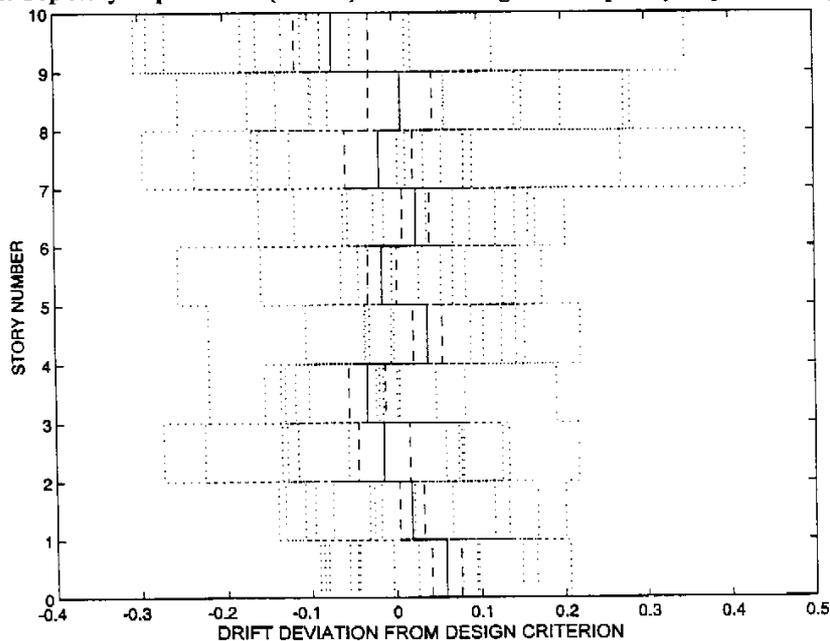
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**Figure 1: Top story displacement (viscous)**



**Figure 2: Top story displacement (friction)**



**Figure 3: Drift deviation from target design**