

## **DUCTILITY AND ENERGY DISSIPATION DEMANDS OF ASYMMETRIC BUILDINGS**

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### **SUMMARY**

In the present paper energy dissipation and hysteretic energy ductility demands of asymmetric-plan systems are discussed. The structural behaviour is studied by means of one-storey models, endowed with a rigid deck supported by resisting elements arranged along two orthogonal directions and having strength and stiffness in their plane only. Resisting members are firstly designed by the standard application of the multi-modal analysis then by its double application by means of a design eccentricity. The models are subjected to a set of thirty artificial accelerograms matching the elastic response spectrum proposed by EC8 for hard layer soil and having a peak ground acceleration equal to 0.35 g. A parametric analysis has been carried out so as to consider the seismic behaviour of most actual asymmetric buildings. Furthermore, the effectiveness of a formulation of the design eccentricity proposed in the past by Gherzi and Rossi (1999), aiming at reducing the normalised displacement ductility demand, is herein verified in decreasing the hysteretic energy ductility demand.

### **INTRODUCTION**

In the past several earthquakes have produced severe damages in asymmetric structures owing to the inadequate evaluation of the coupling of lateral and torsional motions in the phase of design. The seismic behaviour of in-plan irregular buildings and the influence of the design approach on the structural damage level has been widely studied by means of one-storey simplified models [Rutenberg et al., 1995]. The structural scheme has been quite always designed by static analysis and the damage evaluated by means of the maximum displacement ductility demand of the resisting elements. The results of such studies have led to important contributions to the assessment of a correct design procedure but, anyway, based on the control of the displacement ductility demand only. From a long time now structural damaging is widely conceived as dependent on displacement ductility and energy dissipation demands but, in spite of that, the study of the energy dissipation demands and that of the hysteretic energy ductility demands in the analysis of asymmetric structures has been rarely faced [Chandler et al., 1996; Goel, 1997]. Furthermore, the influence of the parameters which govern the elastic and inelastic response of asymmetric structures on such terms does not seem to be completely clear yet and its study is anyway limited to structures designed by static analysis only, according to the design procedures proposed by some seismic codes. The Author has from some years now faced the problem of the design of asymmetric structures by means of the multi-modal analysis as alternative to the static analysis [Rossi, 1998]. A planned numerical investigation has led to the definition of a design procedure which aims at limiting the displacement ductility demand with respect to that of the corresponding torsionally balanced systems [Gherzi and Rossi, 1999]. The formulation of the proposed design eccentricity has been firstly tested with reference to mass and stiffness eccentric systems subjected to uni-directional earthquakes and then verified with reference to asymmetric systems subjected to bi-directional ground motions [Gherzi and Rossi, 1998]. The study of the energy dissipation and that of the hysteretic energy ductility demands aims at providing a further contribution to the comprehension of the inelastic behaviour of asymmetric buildings by the analysis of the simplified one-storey model. Indeed, together with the analysis of the displacement ductility demand that of the hysteretic energy ductility demand allows to foresee more accurately the structural damaging level and to further validate the afore-mentioned proposed design procedure [Gherzi and Rossi, 1999].

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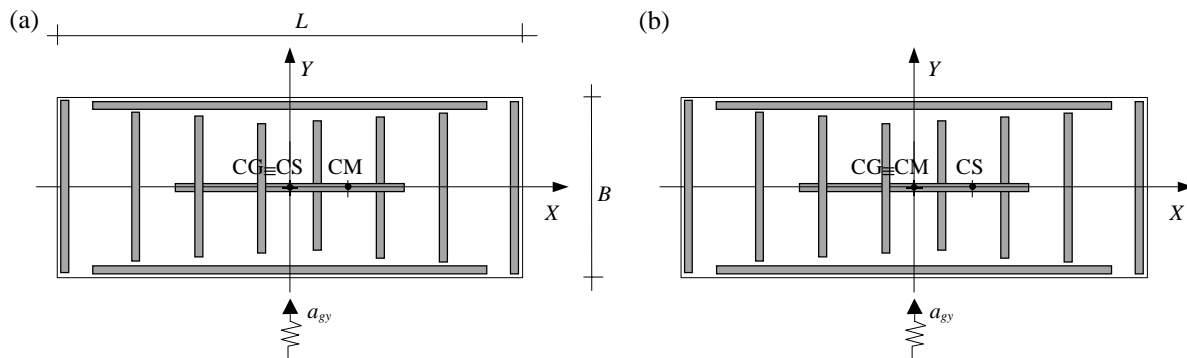


Figure 1. Mass (a) and stiffness (b) eccentric systems

## NUMERICAL MODEL AND GROUND MOTIONS

Asymmetric buildings have been schematised by means of idealised one-storey models having one symmetry axis. The deck, rectangular in shape ( $29.50 \times 12.50 \text{ m}^2$ ), is rigid in its plane and supported by resisting elements arranged along two orthogonal directions. The resisting elements have stiffness and strength in their plane only and are characterised by an elastic perfectly plastic behaviour. Both mass and stiffness eccentric systems (Fig.1) are considered in the numerical analyses, having eight resisting elements in the principal direction ( $y$ -axis) and three resisting elements in the secondary direction ( $x$ -axis). The mass centre and its radius of gyration  $r_m$  ( $0.312 L$ ) are assigned independently of the size and the shape of the deck supposing that the mass ( $m = 1 \text{ t/m}^2$  in mean) can be non uniformly distributed in plan. An automatic procedure [Gherzi and Rossi 1999] allows to define structural systems having established torsional to lateral frequency ratios and fixed global torsional and lateral stiffness.

The numerical models have been subjected to thirty artificial accelerograms [SIMQKE, 1976] matching the elastic response spectrum proposed by Eurocode 8 for hard layer soil (class A) and characterised by a 5% damping coefficient. The accelerograms, having a peak ground acceleration equal to  $0.35 \text{ g}$ , are enveloped by a trapezoidal intensity function characterised by a central part (stationary phase) of  $22.5 \text{ s}$  [Eurocode 8, 1996] and by starting and ending parts of  $3$  and  $5 \text{ s}$  respectively. According to the rules of Eurocode 8, no value of the mean spectrum is more than 10% below the corresponding value of the code elastic response spectrum and the mean value of the maximum elastic responses in the constant acceleration region of the code elastic response spectrum is not smaller than the value of the spectral acceleration proposed by Eurocode 8 for such region of the elastic response spectrum.

## STRENGTH DISTRIBUTIONS

The strength of the resisting elements has been designed by a double application of the multi-modal analysis, by means of the EC8 elastic response spectrum reduced by a constant value of the behaviour factor  $q$ : the first time the analysis is carried out with reference to the nominal values of the mass and stiffness centres, the second time the analysis is performed with reference to the mass centre displaced towards the stiffness centre of a quantity  $e_d$ , named *design eccentricity*. The modal contributions are superimposed by means of the CQC rule. The design displacements, whatever is the value of the design eccentricity, cover the elastic displacements of the model subjected to short return period earthquakes matching the design response spectrum. At the same time the application of the design eccentricity modifies the strength distribution of the first multi-modal analysis (Fig.2) aiming at achieving in asymmetric structures a more uniform ductility demand distribution and levels not greater than those of the corresponding torsional balanced systems. No accidental eccentricity has been taken into account in the phase of design.

### Formulation of the design eccentricity

In a previous study [Gherzi and Rossi, 1999] a formulation of the design eccentricity aiming at reducing the damaging level of asymmetric structures, in terms of displacement ductility demands only, to that of the corresponding balanced systems has been analytically proposed. The numerical analyses have been carried out by using mass and stiffness eccentric one-storey systems subjected to uni-directional ground motions. The structural eccentricity  $e_s$ , the torsional to lateral frequency ratio  $\Omega_\theta$ , the lateral uncoupled period of vibration  $T_y$ , the ratio  $\gamma_x$  of the torsional stiffness due to the resisting elements along the  $x$ -axis to the total torsional stiffness and the behaviour factor  $q$  have been varied in a wide range of values so as to highlight their influence on the

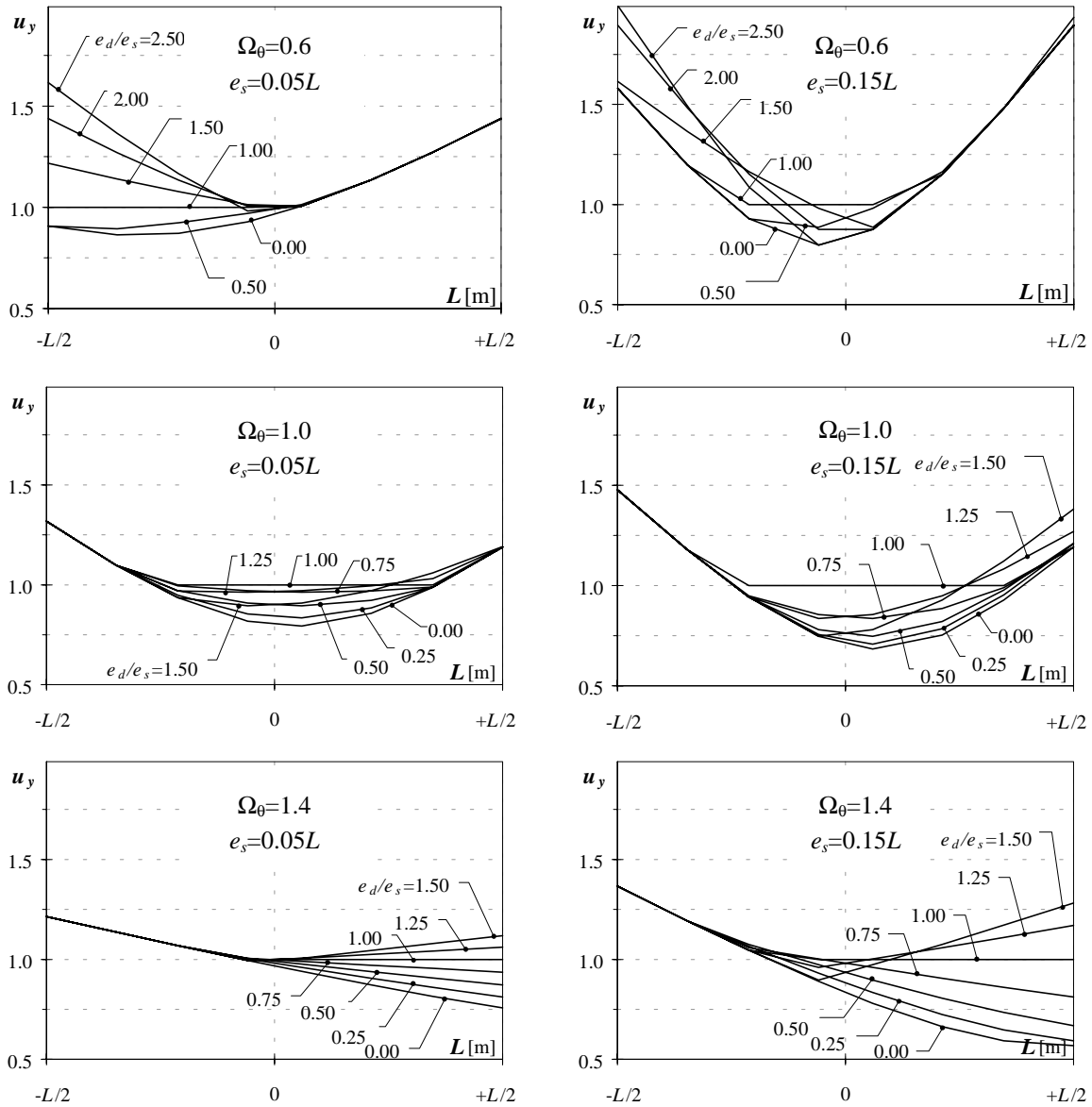


Figure 2. Design displacements of mass eccentric systems resulting from a double application of the multi-modal analysis with different values of the design eccentricity  $e_d$  (design parameters:  $\Omega_\theta = \text{variable}$ ,  $T_y = 1$  s,  $\gamma_x = 0.2$ ,  $q = 5$ ).

seismic response of asymmetric systems. Each model has been further designed by different values of the design eccentricity  $e_d$  so as to examine its influence on the damage level. The maximum responses to a set of thirty accelerograms in terms of displacement ductility demand, normalised to the corresponding values of the torsionally balanced systems, have been concisely taken into account by means of a gaussian density probability function of the normalised displacement ductility demand having the same mean value and standard deviation of the experimental data. The optimum value of the design eccentricity has been obtained as the value of the design eccentricity which reduces to 1.3 the characteristic value of the gaussian density probability function of the normalised displacement ductility demand. The proposed formulation of the design eccentricity has been presented in [Ghera and Rossi, (1999)] depending on the torsional to lateral frequency ratio  $\Omega_\theta$ , the behaviour factor  $q$  and the structural eccentricity  $e_s$ :

$$e_d = k(e_s - e_r) \quad (1)$$

where:

$$\begin{aligned}
k &= \max \begin{cases} 3.3 - 2.5 \Omega_{\theta} + 0.04 q \\ 1 \end{cases} \\
e_r &= \max \begin{cases} 0.1 (0.5 \Omega_{\theta} - 0.4) L \\ 0.01 L \end{cases}
\end{aligned} \tag{2}$$

The optimum value of the design eccentricity reduces also the mean normalised displacement ductility demand to values close to unity for all the examined values of the structural eccentricity  $e_s$  and torsional to lateral frequency ratio  $\Omega_{\theta}$  if the following formula is taken into account together with Eq.(1):

$$e_d \geq 0.6 e_s \tag{3}$$

The proposed formulation of the design eccentricity has been further verified with reference to generalised eccentric systems [Rossi, 1998], having both mass and stiffness locations different from the geometric centre, subjected to mono-directional ground motions and to mass eccentric systems subjected to bi-directional ground motions [Gherzi and Rossi, 1998].

## NUMERICAL ANALYSES

In the present study, involving the analysis of the seismic response of asymmetric systems in terms of energy characteristics, the numerical analyses have been carried out with reference to the following range of values of the above-mentioned parameters:

$$0.6 \leq \Omega_{\theta} \leq 1.6 \quad ; \quad 0 \leq e_s \leq 0.2L \tag{4}$$

while others have been held constant owing to their negligible influence on the inelastic response of asymmetric structures [Gherzi and Rossi, 1999]

$$T_x = T_y = 1.0 \text{ s} \quad ; \quad \gamma_x = 0.2 \quad ; \quad q = 5 \tag{5}$$

The design eccentricity  $e_d$  have been varied in the range 0-1.5  $e_s$  for torsionally rigid structures ( $\Omega_{\theta} \geq 1.0$ ) and in the ranges 0-2.0  $e_s$  and 0-2.5  $e_s$  for asymmetric models having  $\Omega_{\theta} = 0.8$  and 0.6 respectively.

## HYSTERETIC ENERGY DISSIPATION

The *hysteretic dissipation ratio*  $H$ , defined as the ratio of the total hysteretic energy to the total energy dissipated by the system during an earthquake has shown quite constant values in the range of  $e_s$  from 0 to 0.2  $L$ . This aspect has been already pointed out by other authors [Chandler et al. (1996)] but, while these ones refer to values of the parameter  $H$  ranging about from 0.60 to 0.75, the numerical analyses carried out in this study show values of the hysteretic dissipation ratio in a more narrow range of values from 0.70 to 0.75. Furthermore, the non negligible dependence of the hysteretic dissipation ratio, shown by Chandler et al. (1996), on the design method and structural eccentricity is not confirmed by the present numerical analyses. The slight disagreement might be caused by the different values of overstrength produced by the application of different design methods (e.g. no accidental eccentricity has been considered in this study) or by the different choice of the accelerograms which, in the present work, are artificially generated so as to match the elastic response spectrum proposed by EC8 and to show a quite equal input energy spectrum.

The distribution of the hysteretic energy dissipation between the resisting elements may be analyzed by means of the *proportional hysteretic energy ratio*  $H_i$ , defined as the ratio of the hysteretic energy dissipation demand of the single resisting element to the hysteretic energy dissipation demand of the system. The results of the numerical analyses, in terms of proportional hysteretic energy demand, are shown in Figure 3 with reference to mass eccentric systems only because the diagrams relative to stiffness eccentric systems have analogous aspect and lead to the same conclusions. In torsionally flexible systems ( $\Omega_{\theta} \leq 1.0$ ) the proportional hysteretic energy demand  $H_i$  reaches its greatest values in the central elements because of the great stiffness and strength of such members. It is generally quite independent of the value of the design eccentricity  $e_d$ , particularly for low values of the structural eccentricity; just slight increases of  $H_i$  are noticeable for high values of the structural eccentricity on the rigid side of the system, on increasing the value of the design eccentricity. Differently, in torsionally rigid structures the proportional hysteretic energy demand is greater in the outermost elements where strength and the stiffness have their highest values. The value of the proportional hysteretic energy demand

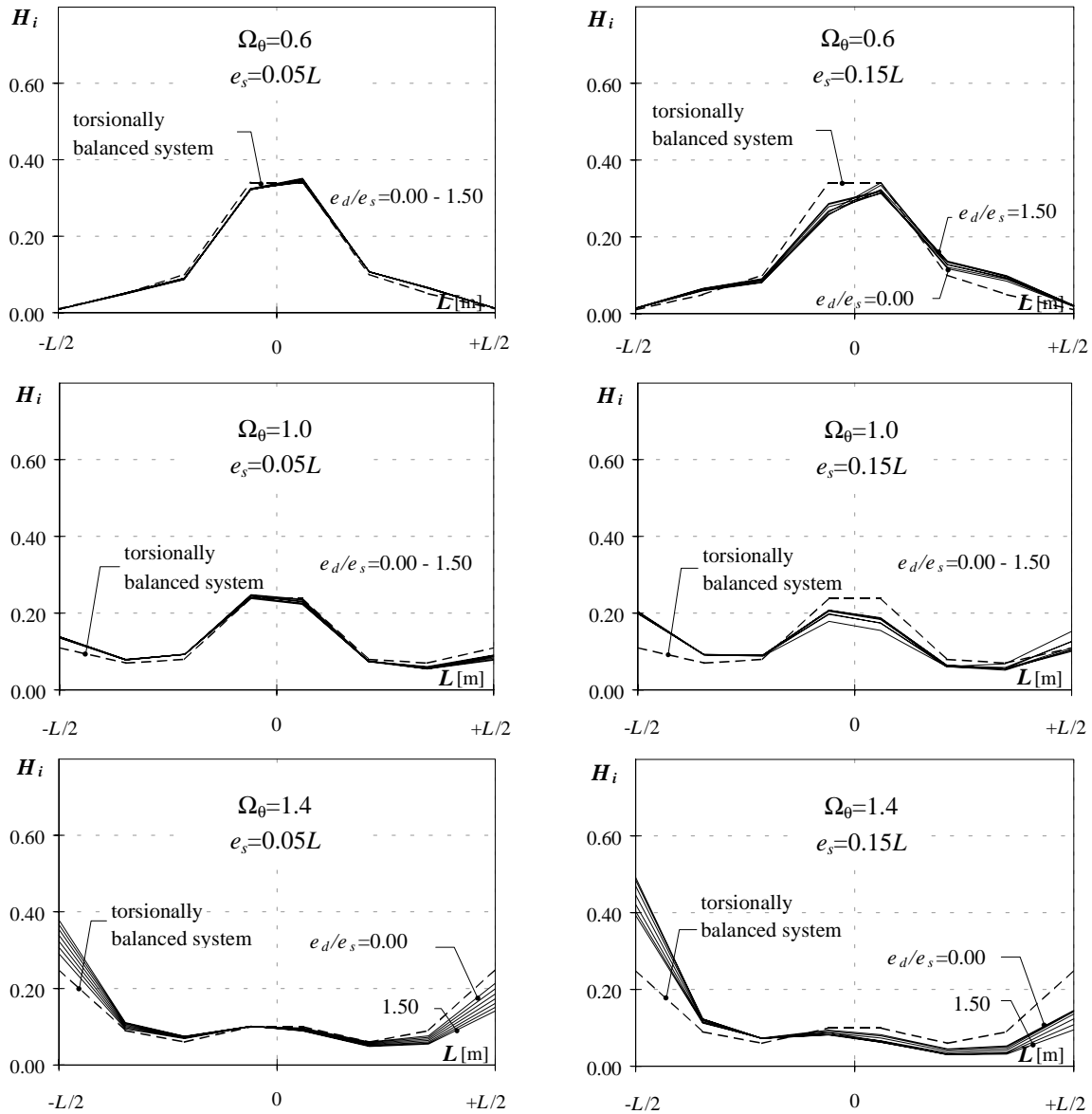


Figure 3. Hysteretic energy dissipation of mass eccentric systems designed by means of multi-modal analysis with different values of the design eccentricity  $e_d$  (design parameters:  $\Omega_\theta =$  variable,  $T_y = 1$  s,  $\gamma_x = 0.2$ ,  $q = 5$ ).

decreases at the stiff side and increases at the flexible side on increasing the design eccentricity  $e_d$ .

### HYSTERETIC ENERGY DUCTILITY

The capacity of the single member to resist repeated inelastic deformations has been herein analysed by means of the *hysteretic energy ductility*  $\mu_h$ , so as proposed by Mahin and Bertero, (1981):

$$\mu_h = \frac{E_h}{f_y u_y} + 1 \quad (6)$$

where  $E_h$  is the total hysteretic energy,  $f_y$  the strength of the resisting element and  $u_y$  the yield displacement. It is to be noticed that the expression of the hysteretic energy ductility adopted in this study agrees with that used by Goel (1997) but differs from that reported by Chandler et al. (1996) which does not consider the unity in the formula. In order to compare the results of asymmetric-plan structures with those of the corresponding torsionally balanced systems, the values of the hysteretic energy ductility demand obtained for the asymmetric systems have been divided by those of the corresponding torsionally balanced systems. The resulting index  $d_h$  is later on called as *normalised hysteretic energy ductility*.

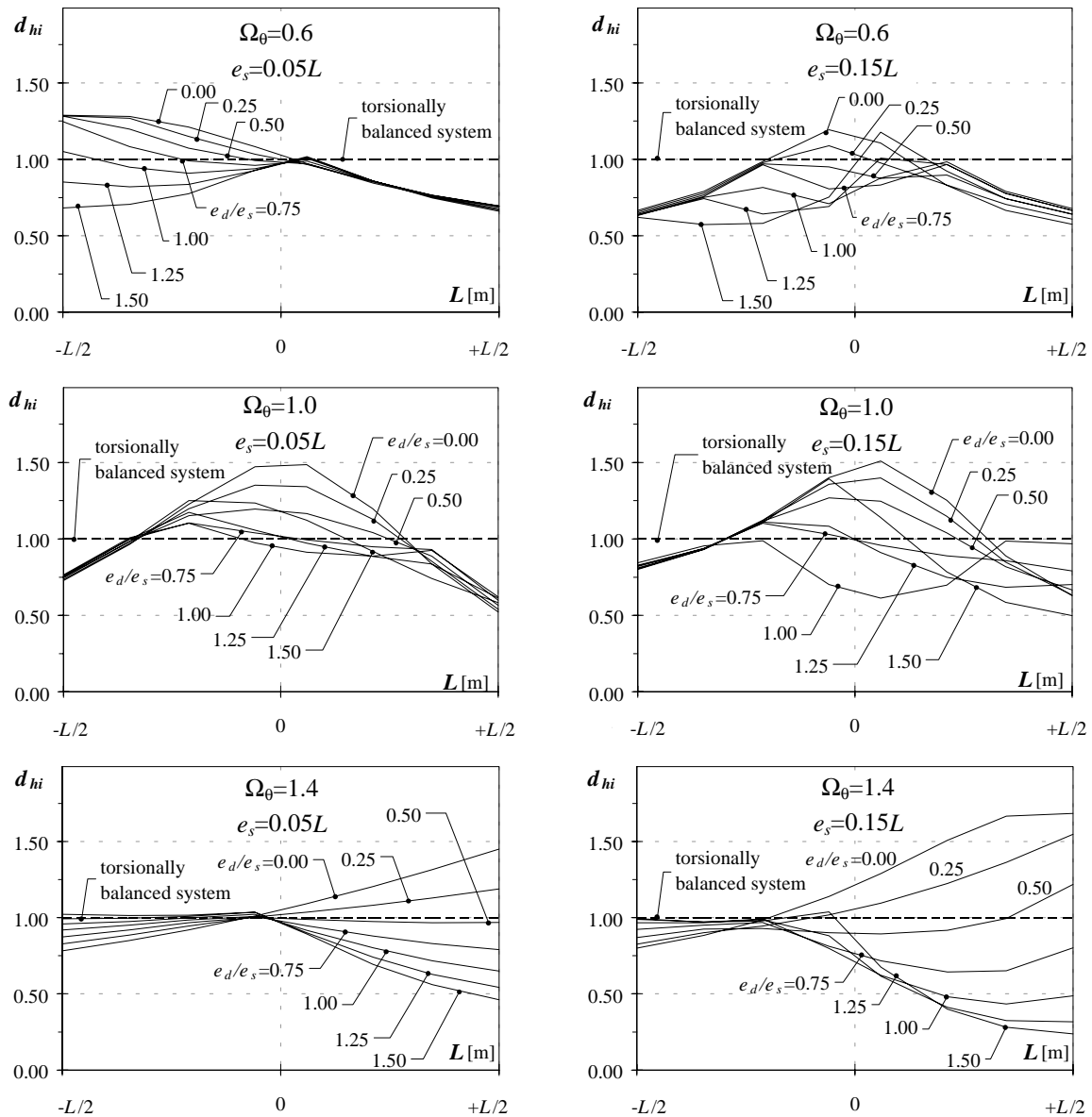


Figure 4. Normalised hysteretic energy ductility demands of mass eccentric systems designed by means of multi-modal analysis with different values of the design eccentricity  $e_d$  (design parameters:  $\Omega_\theta = \text{variable}$ ,  $T_y = 1$  s,  $\gamma_x = 0.2$ ,  $q = 5$ ).

The resisting elements characterised by the maximum normalised hysteretic energy ductility demand are generally located at the flexible side in torsionally flexible systems and at the rigid side in torsionally rigid systems (Fig. 4). Greater values of the structural eccentricity extend the part the of the system where the mean of the maximum normalised hysteretic energy ductility demands exceed unity toward the stiff edge in torsionally flexible systems and toward the flexible edge in torsionally rigid systems. The trend of the maximum normalised hysteretic energy ductility demand, as already noticed for the normalised displacement ductility [Rossi, 1998], seems to be generally determined by the more translational aspect of the envelope of the maximum inelastic displacements respect to that of the elastic response.

Structural parameters representative of the global inelastic behaviour have been assumed as the mean and characteristic values of the maximum normalised hysteretic energy ductility demands evaluated between all the resisting elements with reference to each accelerogram (Fig. 5). These values have been evaluated for mass and stiffness eccentric systems designed by means of multi-modal analysis either without design eccentricity or with the design eccentricity (Eq. 1) proposed by Ghersi and Rossi (1998-1999).

Differently from displacement ductility demands [Ghersi and Rossi, 1998-1999; Rossi, 1998], hysteretic energy ductility demands present in all the systems small differences between mean and characteristic values and

therefore slight dispersion of the numerical data. This result is surely influenced by the choice of the artificially generated accelerograms which have similar pseudo-acceleration and input energy spectra but obviously different time-histories. In systems designed without design eccentricity the mean values of the maximum normalised hysteretic energy ductility demands are generally high, greater in torsionally rigid structures than in torsionally flexible systems and quite constant in the range of moderate and high structural eccentricity. The application of the proposed design procedure (Eq.1) generally cuts down the mean value of the maximum normalised hysteretic energy ductility demands; its effectiveness is quite perfect in torsionally rigid structures, good in systems having about a unity value of the torsional to lateral frequency ratio  $\Omega_\theta$  but poor in torsionally flexible systems having high structural eccentricity. As regard as the response of such last structures it has to be noticed that the maximum normalised hysteretic energy ductility demand is anyway quite low, even in absence of design eccentricity, and that the adoption of whatever value of the design eccentricity does not manage to reduce the mean value of the maximum normalised hysteretic energy ductility demands to unity. Some other values of the design eccentricity minor than those proposed by the author may lead to slightly minor values of the hysteretic energy ductility demand but they produce greater values of the displacement ductility demand; because the structural damaging, according to most recent damage indexes, is quite always dependent on the displacement ductility demand much more than on the hysteretic energy ductility demand (e.g. Park and Ang index [Park and Ang, 1985] ), the reduction of the displacement ductility demand appears to be more important than that of the hysteretic energy ductility demand.

The application of a design eccentricity which satisfies Eqs.1 and 3 leads to the reduction of the displacement and hysteretic energy ductility demands also in systems (particularly torsionally rigid structures) characterised by low and moderate values of the structural eccentricity, for which the design eccentricity evaluated according to Eq.1 would have led to quite high values of the maximum displacement and hysteretic energy ductility demands.

### NORMALISED DISPLACEMENT DUCTILITY

Diagrams analogous to those shown in Figure 4 but referred to the displacement ductility demand are shown in other papers [Gherzi and Rossi, 1998-1999; Rossi, 1998]. The results of the numerical analyses in terms of displacement ductility demand highlight remarkable analogies with those in terms of hysteretic energy ductility demands; the trend of the diagrams of the two damaging parameters is the same but the values of the displacement ductility demand, caused by different design eccentricities, are quite always closed in a more narrow range of values. No deficiency appears in the application of the proposed design procedure if the additional Eq.3 is taken into account for both torsionally flexible and rigid systems.

### CONCLUSIONS

A wide parametric analysis regarding one-storey mono-symmetric models designed by multi-modal analysis has allowed to study the influence of the parameters which govern the seismic response of asymmetric-plan systems on the energy dissipation and hysteretic energy ductility demands. A close analysis of such results has led to the following conclusions:

1. The energy dissipation reaches its greater values in the resisting elements where strength and stiffness are greater. Its values are quite independent of the value of the design eccentricity in torsionally flexible systems while show not negligible differences in torsionally rigid structures.
2. The trend of the hysteretic energy ductility demands within the structure quite resembles that of the displacement ductility demands. Their values, in absence of design eccentricity, are greater at the flexible side of torsionally flexible systems and at the rigid side of torsionally rigid systems. Greater values of the structural eccentricity extend the part the of the system where the resisting elements present high values of the hysteretic energy ductility demand.
3. The design procedure proposed by Gherzi and Rossi (1999), aiming at reducing the displacement ductility demand of asymmetric-plan systems to that of the corresponding torsionally balanced systems, has produced good results in decreasing also the values of the normalised hysteretic energy ductility demands for most values of the structural parameters examined in the paper and influencing the seismic response of asymmetric-plan systems.

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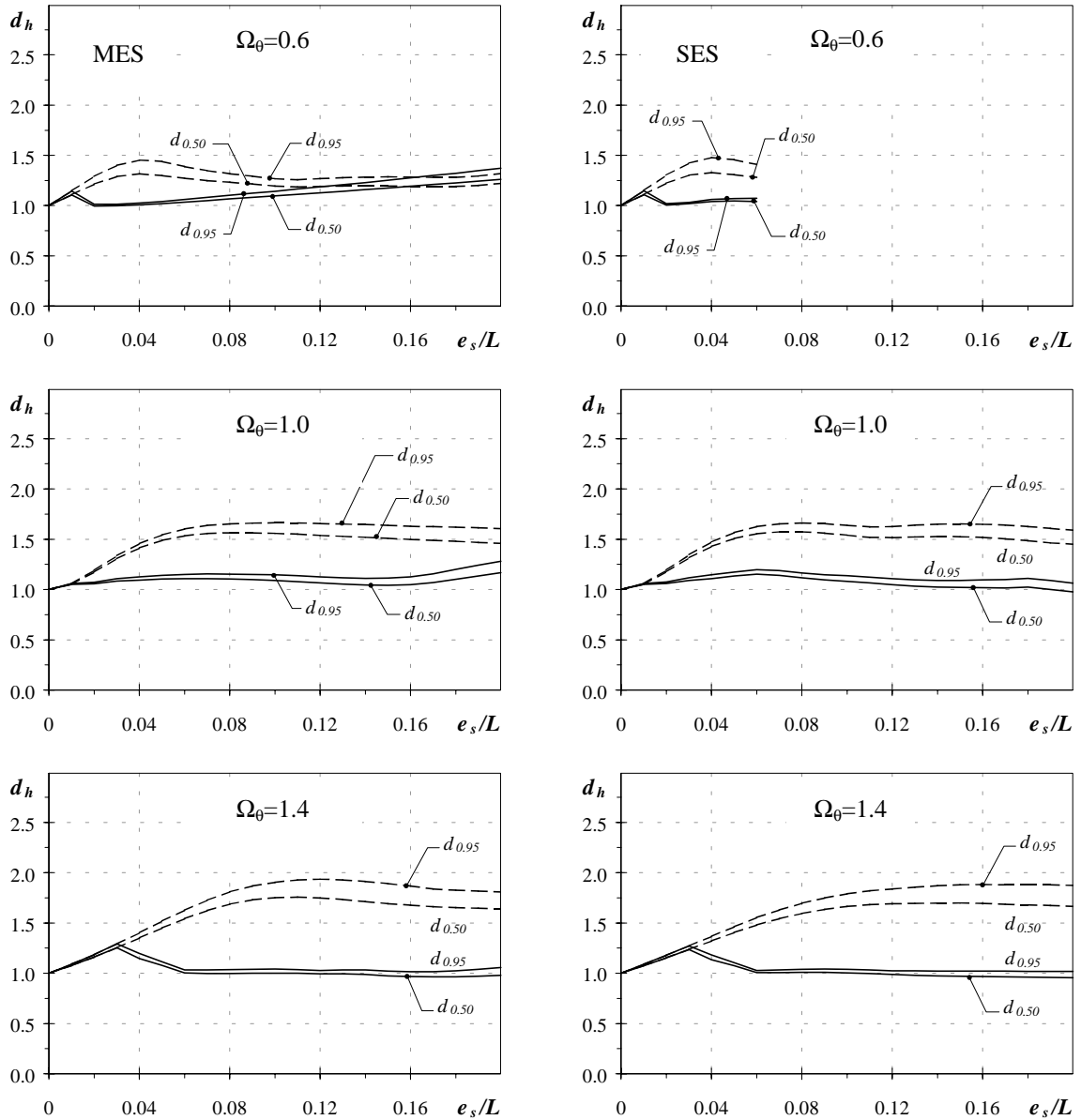


Figure 5. Normalised hysteretic energy ductility demands of asymmetric models designed by means of multi-modal analysis either without or with the proposed design eccentricity (design parameters:  $\Omega_\theta = \text{variable}$ ,  $T_v = 1 \text{ s}$ ,  $\gamma_x = 0.2$ ,  $q = 5$ )