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APPLICATIONS OF PROBABILITY THEORY IN THE EARTHQUAKE RISK ASSESSMENT AND ITS CONSEQUENT POSSIBLE REDUCTION

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SUMMARY

The paper analyzes available previous seismic data in terms of maximum annual recorded intensity of earthquakes on Richter scale at various locations of Pakistan with a view to know future earthquakes intensity and return periods required for making decision regarding the design of lifeline systems and other structures. Earthquakes are classified as natural hazards along with floods, landslides, cyclones and hurricanes etc. These natural hazards follow an almost fixed cycle and are ideally suited for analysis by using normal or Gaussian distribution and specially Pearson's type III distribution. This concept is also supported by seismic gap theory according to which big earthquakes are generated due to tectonic movements and follow a cycle. The results of analysis are plotted on log probability paper which contains very useful information required for making vital decisions for design of the life line supported systems and other structures thus bringing safety and economy in design.

INTRODUCTION

Richter instrumental scale is used for the separation of large, medium and small earthquake shocks. It takes into account maximum wave amplitude without specifying its type whether P, S or surface waves. According to tectonic plate theory, once the internal stress arising due to accumulated strain reaches the strength of rock, faulting takes places thus releasing accumulated strains and hence stresses. If this strain accumulation takes place on stronger rocks, bigger would be the corresponding earthquake upon its faulting. The largest recorded magnitude on Richter scale is 8.9. It is known that destructiveness of an earthquake, although dependant partly on its magnitude, is a function of some other equally important factors like focal depth, epicentral distance, local geology and structural characteristics and responses. Earthquakes are normally divided into four types i.e, great, major, moderate and small having corresponding magnitudes of greater than 8, between 7.0 to 7.9, 6.0 to 6.9 and smaller than 6.0 respectively.The relevant details are given elsewhere [Bullen.K.E] , [Hald, A].and [Longwell,C.R et al].

SEISMIC ZONES OF PAKISTAN

Pakistan has been divided into various earthquake zones depending upon the degree of seismicity. These include Makran Zone, Murray Ridge, Quetta transverse Zone, Suleman Range, Chamans Fault, Ornach – Nal Fault Zone, Pamir Karakoram Zone and Hazara Zone which are shown in figure 1. Work on irregular and special apllication structures is reported in various research papers [RIZWAN, S.A and Ahmad, A] and [RIZWAN, S.A and Ahmad, A] and [RIZWAN, S.A and Ahmad, A].

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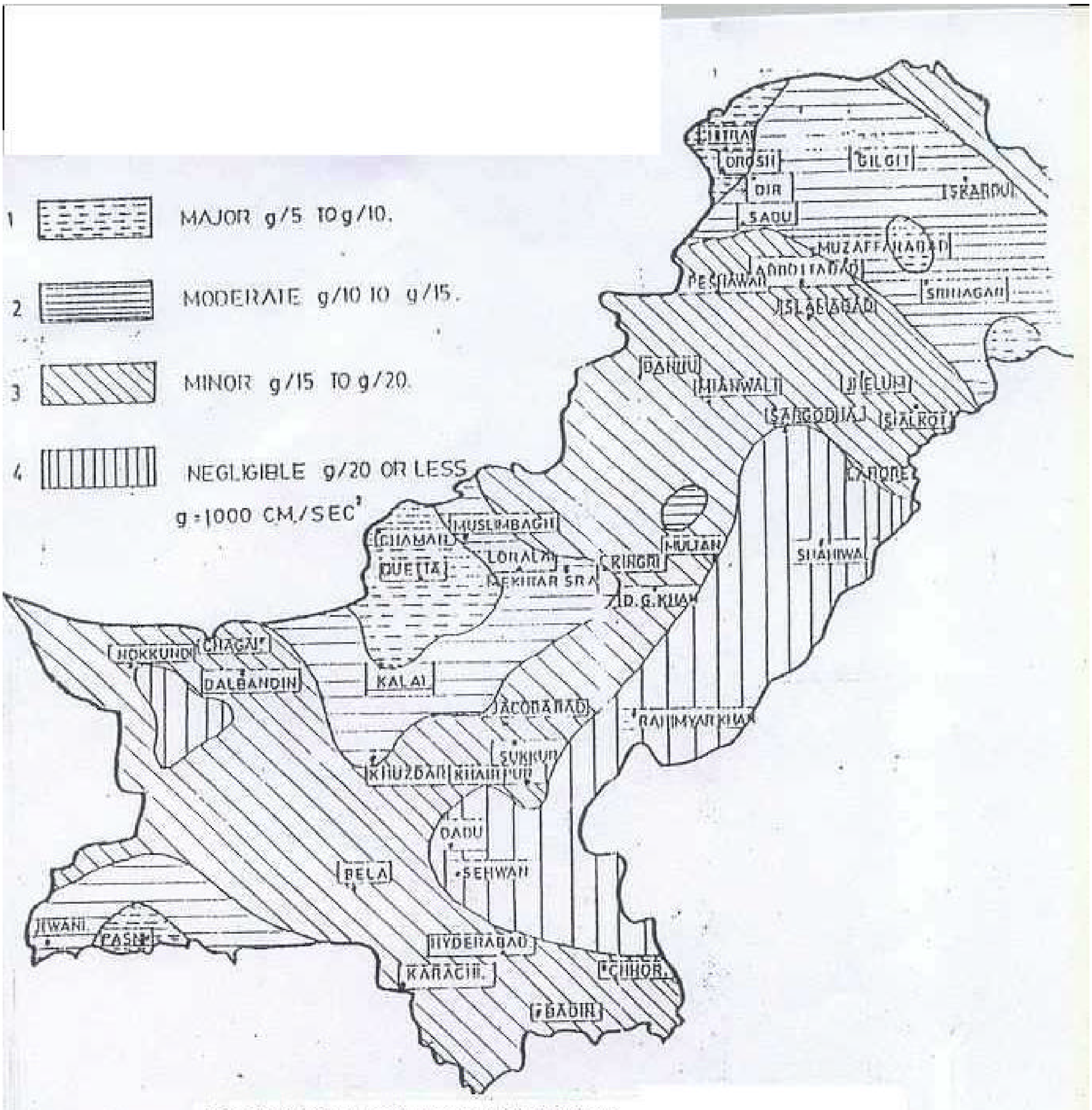

Fig 1: Seismic Zones of Pakistan

STATISTICAL AND PROBABILISTIC ANALYSIS

Statistical analysis when applied in earthquake engineering consists of estimating the future frequency or probability of seismic events based on information contained in previous seismic records. In probability analysis, statistical methods permit co ordination of observed data to yield a more accurate estimate of future frequencies than is indicated by the observed data and also provides criteria for judging the reliability of frequency estimates.The basic work has been done elsewhere [Rizwan, S.A and Azam, M, 1994].

FREQUENCY DISTRIBUTION

For developing probability estimates, in addition to identifying the possible events in the parent population, it is necessary to have some knowledge of the frequency of occurrence of each event, comprising the present/existing population. A complete description of the frequency of occurrence of each event in a parent population is called frequency distribution or more commonly a distribution.

NORMAL OR GAUSSIAN DISTRIBUTION

Through analysis of sampling experiments, statisticians and mathematicians have developed theoretical probability distributions that can be defined by general mathematical functions. A large number of natural phenomena can be closely approximated by these theoretical distributions. One of the most commonly used probability distribution is known as normal or Gaussian distribution which is completely defined by the population mean of random variable X’ and its standard deviation. When dealing with seismic data it is assumed that the sample mean and standard deviation is similar to that of parent population due to limited number of recorded events available in terms of maximum annual intensity of earthquakes.

PROBABILITY AND FREQUENCY

In many practical applications of probability analysis, it is often desirable to work with frequency of occurrence (Return period) rather than probability occurrence, although both are inter-related. In the analysis of seismic data, the population is defined as a set of all annual recorded earthquakes at a particular location. Even a 100 years record in terms of annual maxima is small sample from this population and should therefore be adjusted according to the assumptions of Gaussian distribution. In general, the accuracy of probability increases with an increase in size of sample. In case of normal distribution, the sample mean is represented by X’ is assumed to be equal to population mean. On the same lines, the sample standard deviation, represented by S, is assumed to be equal to population standard deviation. Table 1 and 2 give relevant data for the normal distribution.

TABLE 1: Pearsons type III Co ordinates

Skew	k = Magnitude in standard deviations from mean for excedence percentage of:												
	0.01	0.1	1.0	5	10	30	50	70	90	95	99	99.9	99.99
1.0	5.92	4.54	3.03	1.87	1.34	0.38	-0.16	-0.61	-1.12	-1.31	-1.59	-1.80	-1.88
0.8	5.25	4.25	2.90	1.83	1.34	0.42	-0.60	-0.13	-1.16	-1.38	-1.74	-2.03	-2.18
0.6	5.96	3.96	2.77	1.79	1.33	0.45	-0.58	-0.09	-1.19	-1.45	-1.88	-2.28	-2.53
0.4	4.60	3.67	2.62	1.74	1.32	0.48	-0.57	-0.06	-1.22	-1.51	-2.03	-2.54	-2.92
0.2	4.16	3.38	2.18	1.69	1.30	0.51	-0.55	-0.03	-1.25	-1.58	-2.18	-2.81	-3.32
0.0	3.73	3.09	2.33	1.64	1.28	0.52	-0.52	-0.0	-1.28	-1.64	-2.33	-3.00	-3.73
-0.2	3.32	2.81	2.18	1.58	1.25	-0.55	-0.51	-0.03	-1.30	-1.69	-2.18	-3.38	-4.16
-0.4	2.92	2.54	2.03	1.51	1.22	0.57	-0.48	-0.06	-1.32	-1.74	-2.62	-3.67	-4.60
-0.6	2.53	2.28	1.88	1.45	1.18	0.58	-0.45	-0.09	-1.33	-1.79	-2.77	-3.96	-5.04
-0.8	2.18	2.06	1.74	1.38	1.16	0.60	-0.42	-0.13	-1.34	-1.83	-2.90	-4.25	-5.18
-1.0	1.88	1.80	1.59	1.31	1.12	0.61	-0.38	-0.16	-1.34	-1.87	-3.03	-4.54	-5.92
SKEW CO-EFFICIENTS COMMONLY USED													
0.00	3.73	3.09	2.33	1.64	1.28	0.52	0.00	-0.52	-1.28	-1.64	-2.33	-3.09	-3.73
-0.04	3.65	3.03	2.30	1.63	1.27	0.53	0.01	-0.52	-1.28	-1.65	-2.36	-3.15	-3.82
-0.12	3.48	2.92	2.24	1.60	1.26	0.54	0.02	-0.51	-1.29	-1.67	-2.42	-3.26	-3.99
-0.23	3.26	2.77	2.16	1.57	1.25	0.55	0.03	-0.50	-1.30	-1.70	-2.50	-3.42	-4.23
-0.32	3.08	2.68	2.09	1.54	1.23	0.56	0.05	-0.49	-1.31	-1.72	-2.56	-3.55	-4.42
-0.37	2.98	2.58	2.05	1.52	1.22	0.57	0.06	-0.48	-1.32	-1.73	-2.60	-3.63	-4.53
-0.40	2.92	2.54	2.03	1.51	1.22	0.57	0.06	-0.48	-1.32	-1.74	-2.62	-3.67	-4.60

Note: k can be accomplished with the following equation:

$K = 2/g\ \{\{g/6(x - g/6) + 1\}^3 - 1\}$

TABLE 2: Values of PN ~ Pμ in Percent (For use in samples drawn from normal distribution)

P∝	50.0	30.0	10.0	5.0	1.0	0.1	0.01
1	50	37.2	24.3	20.4	15.4	12.1	10.2
2	50	34.7	19.3	14.6	9.0	5.7	4.3
3	50	33.6	16.9	11.9	6.4	3.5	2.3
4	50	33.0	15.4	10.4	5.0	2.4	1.37
5	50	32.5	14.6	9.4	4.2	1.79	0.92
6	50	32.2	13.8	8.8	3.6	1.38	0.66
7	50	31.9	13.8	8.3	3.2	1.13	0.50
8	50	31.7	13.1	7.9	2.9	0.94	0.39
9	50	31.6	12.7	7.6	2.7	0.82	0.31
10	50	31.5	12.5	7.3	2.5	0.72	0.25
11	50	31.4	12.3	7.1	2.3	0.64	0.21
12	50	31.3	12.1	6.9	2.2	0.58	0.18
13	50	31.2	11.9	6.8	2.1	0.52	0.16
14	50	31.1	11.8	6.7	2.0	0.48	0.14
15	50	31.1	11.7	6.6	1.96	0.45	0.13
16	50	31.0	11.6	6.5	1.90	0.42	0.12
17	50	31.0	11.5	6.4	1.84	0.40	0.11
18	50	30.9	11.4	6.3	1.79	0.38	0.10
19	50	30.9	11.3	6.2	1.74	0.36	0.091
20	50	30.8	11.3	6.2	1.70	0.34	0.084
21	50	30.8	11.2	6.1	1.67	0.33	0.078
22	50	30.8	11.1	6.1	1.63	0.31	0.073
23	50	30.7	11.1	6.0	1.61	0.30	0.068
24	50	30.7	11.0	6.0	1.58	0.29	0.064
25	50	30.7	11.0	5.9	1.55	0.28	0.06
26	50	30.6	10.9	5.9	1.53	0.278	0.057
27	50	30.6	10.9	5.9	1.51	0.26	0.054
28	50	30.6	10.9	5.8	1.49	0.26	0.051
29	50	30.6	10.8	5.8	1.47	0.25	0.049
30	50	30.6	10.8	5.8	1.45	0.24	0.046
40	50	30.4	10.6	5.6	1.33	0.20	0.034
60	50	30.3	10.4	5.4	1.22	0.16	0.025
120	50	30.2	10.2	5.2	1.11	0.13	0.017
∝	50	30.0	10.0	5.0	1.0	0.10	0.010

COMPUTATION OF PROBABILITY CURVES

Pearson’s type III distribution which is completely defined by sample mean X’ sample variance S^2 has been used in the analysis of samples of seismic record at various locations in Pakistan. For the purpose of demonstration only one typical analysis is presented while results for more locations are also available. The above three sample parameters are calculated as follows.

$$X' = \sum X / N \qquad (1)$$

$$S^2 = \sum(x^2 / (N-1)) \qquad (2)$$

$$g = N\,(\sum x^3)/((N-1)(N-2)(S^3)) \qquad (3)$$

where,

X = logrithm of magnitude of an event

X’ = the mean of logrithm

$x = X - X'$, the deviation of single event from the mean.

N = number of events in sample

S^2 = estimate of variance

g = skew co efficient

S = standard deviation.

Computation process involves calculating the values of logarithm of magnitudes corresponding to selected points (exceeding frequencies). The subscripted $P\propto$ indicates infinite sample size asking for adjustment as stated earlier. The logarithms of event magnitudes corresponding to each of the selected $P\propto$ points are computed for the following equation:

$$\text{Log } M@P\propto = X' + \underline{k@P\propto S} \qquad (4)$$

Where

$\text{Log } M@P\propto$ = Logarithm of a magnitude corresponding to an exceedence frequency value of $P\propto$.

$k@P\propto$ = Deviation from mean at selected exceedence frequency value of $P\propto$.

Values of k for each selected $P\propto$ value are obtained from table 1. If seismic data is less than 100 events, then value of g may be taken as zero.

PROCEDURE FOR PLOTTING FREQUENCY CURVES

Frequency curves can be computed as follows and plotted on log-probability paper later.

1. For selected values of $P\propto$, tabulate values of k obtained from table 1 corresponding to adopted skew-coefficient.

2. Multiply each of them by standard deviation and add the product to mean logarithm X'.

3. Tabulate values of P_N from table 2 corresponding to selected $P\propto$ values against value of $N-1$.

4. Plot on a log-probability paper, the anti-logarithm of each of the sums obtained at 2 above against P_N value obtained at 3 above.

A typical site Cholen whose seismic data is given in table 3 is analyzed and record is shown in fig 2.

I = Return period in years.

M_b = Magnitude on Richter scale, (anti-log of sums obtained at step 2 of procedure for plotting frequency curves).

P_N = Adjusted value of an event corresponding to selected $P\propto$ value. (Probability of Excedence in percent)

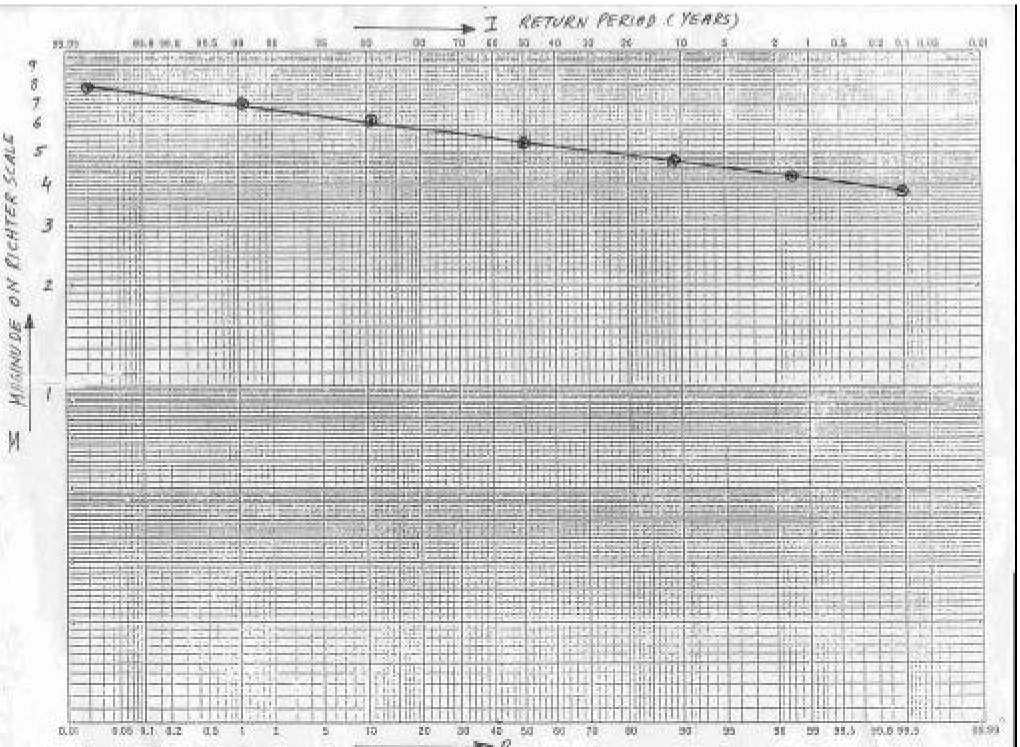

FIG 2 FREQUENCY CURVE FOR ANNUL MAXINUM INTENSITY EARTHQUAKES AT GOLEN

TABLE 3: Analysis of Seismic Data at location CHOLEN

Year	Magnitude	Log			Deviation		
		X	X^2	X^3	x=X – X’	x^2	x^3
1963	4.9	0.69020	0.47637	0.32879	-0.05254	0.00276	-0.00015
1964	5.4	0.73239	0.53640	0.39286	-0.01034	0.00011	-0.00000
1965	5.9	0.77085	0.59421	0.45805	0.02812	0.00079	0.00002
1966	4.9	0.69020	0.47637	0.32879	-0.05254	0.00276	-0.00015
1967	5.4	0.73239	0.53640	0.39286	-0.01034	0.00011	-0.00000
1968	5.4	0.73239	0.53640	0.39286	-0.01034	0.00011	-0.00000
1969	5.9	0.77085	0.59421	0.45805	0.02812	0.00079	0.00002
1970	5.4	0.73239	0.53640	0.39286	-0.01034	0.00011	-0.00000
1971	5.4	0.73239	0.53640	0.39286	-0.01034	0.00011	-0.00000
1972	6.9	0.83885	0.70367	0.59027	0.09611	0.00924	0.00089
1973	5.9	0.77085	0.59421	0.45805	0.02812	0.00079	0.00002
1974	6.4	0.80618	0.64993	0.52396	0.06345	0.00403	0.00026
1975	5.4	0.73239	0.53640	0.39286	-0.01034	0.00011	-0.00000
1976	4.7	0.67210	0.45172	0.30360	-0.07064	0.00499	-0.00035
1977	5.4	0.73239	0.53640	0.39286	-0.01034	0.00011	-0.00000
1978	5.2	0.71600	0.51266	0.36707	-0.02673	0.00071	-0.00002
1979	4.6	0.66276	0.43925	0.29112	-0.07998	0.00640	-0.00051
1980	4.8	0.68124	0.46409	0.31616	-0.06149	0.00378	-0.00023
1981	6.4	0.80618	0.64993	0.52396	0.06345	0.00403	0.00026
1982	5.2	0.71600	0.51266	0.36707	-0.02673	0.00071	-0.00002
1983	5.4	0.73239	0.53640	0.39286	-0.01034	0.00011	-0.00000
1984	5.9	0.77085	0.59421	0.45805	0.02812	0.00079	0.00002
1985	4.7	0.67210	0.45172	0.30360	-0.07064	0.00499	-0.00035
1986	5.3	0.72428	0.52458	0.37994	-0.01846	0.00034	-0.00001
1987	4.9	0.69020	0.47637	0.32879	-0.05254	0.00276	-0.00015
1988	5.7	0.75587	0.57135	0.43187	0.01314	0.00017	0.00000
1989	4.9	0.69020	0.47637	0.32879	-0.05254	0.00276	-0.00015
1990	6.3	0.79934	0.63895	0.51073	0.05661	0.00320	0.00018
1991	6.4	0.80618	0.64993	0.52396	0.06345	0.00403	0.00026
1992	6	0.77815	0.60552	0.47119	0.03542	0.00125	0.00004
1993	6.4	0.80618	0.64993	0.52396	0.06345	0.00403	0.00026
1994	6.3	0.79934	0.63895	0.51073	0.05661	0.00320	0.00018
1995	4.5	0.65321	0.42669	0.27872	-0.08952	0.00801	-0.00072
1996	6.2	0.79239	0.62788	0.49753	0.04966	0.00247	0.00012
1997	6.1	0.78533	0.61674	0.48435	0.04259	0.00181	0.00008
1998	5.8	0.76343	0.58282	0.44494	0.02069	0.00043	0.00001
		26.73846	19.94247	14.93491			
		0.74273			-0.000002	0.08288	-0.00018

$N = 36$

$\sum X = 26.73846$

$\sum x = -2.0 \text{ E} -06$

$X' = \sum X/N = 0.742735$

$\sum (x^2) = 0.082883$

$S^2 = \sum (x^2 / (N-1)) = 0.00236808$

$\sum x^3 = -0.00018$

$g = N (\sum x^3) / (N-1)(N-2)(S^3) = -0.0472532$

$S = 0.04866298$

TABLE 4: Results to be Plotted on Log Probability Paper

$P_{\propto}$	0.1	1.0	10	50	90	99	99.9
K(Tab 1)	3.09	2.33	1.28	0	-1.28	-2.33	-3.09
Log m@$p_{\propto}$	0.8931	0.8561	0.805	0.7427	0.6804	0.6243	0.592
M@P$_{\mu}$	7.81	7.18	6.38	5.53	4.79	4.26	3.91
P_N (Tab 2)	0.022	1.38	10.68	50	89.32	98.62	99.98

DISCUSSION:

It is very difficult to exactly predict the occurrence of a major earthquake at a given site due to complexity and a number of variables involved. However depending upon the length of availability of seismic data for particular location and considering the limitations of probability techniques. A reason estimate on the recurrence of a given magnitude of earthquake along with its return period can be determined easily.

CONCLUSIONS

Although the results of analysis of seismic data by statistical and probabilities approach are not 100% accurate, yet it is useful in assuming a reasonable intensity of future earthquake for the design of lifeline systems and other structures. If return period so calculated is many times the useful anticipated service life of a structure then the intensity of seismic input to the structure can be reduced resulting in economical structures. This approach is also useful in assessment of seismic risk at a given location which can be consequently reduced.

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