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A STUDY ON IMPROVEMENT OF PUSHOVER ANALYSIS

Pu YANG[1] And Yayong WANG[2]

SUMMARY

The static pushover analysis, POA, is becoming popular as a simplified computer method for seismic performance evaluation of structures. This method implies that the response of the structure is only controlled by the first mode, and the mode keeps constant during time history. Several examples illustrate that the structural maximum responses under-estimated the influence of higher modes compared to the results obtained from dynamic analysis. There will be a noticeable error especially for the structure with long period or when a local mechanism forms, then the dynamic properties of the structures changes accordingly. In this paper, a lateral load pattern is taken as the approximation of the distribution of the inertia force obtained from results of dynamic analysis of the story equivalent MDOF system of the structure, which is time dependent. Then the static pushover analysis is used to analyze the structure step by step until the top displacement reaches the target one.

It is indicated from the analysis of several frame structures that, story displacement and base shear force calculated by the presented method agree better with the results obtained from dynamic analysis than the general POA method, and it takes much less computer time than from the dynamic analysis.

INTRODUCTION

The actual behavior of a structure at each time subjected to strong motion can be realistically described by carrying out a non-linear dynamic analysis. You can also find order of crack and yield of members, and evaluate the behavior of structure. Therefore, the non-linear dynamic analysis is considered a rigor and reasonable tool for seismic design. At present time, many special, complex and important structures are analyzed by non-linear dynamic analysis. It has been accepted by many seismic codes. However, it is time-consuming, and there are many uncertainties to be completed (for example, time-histories of several ground motions and the hysteresis behavior of structural members, etc). Therefore, non-linear dynamic analysis is commonly used in theoretical study and not practical for everyday design use. It is worth to devise a simplified computer method for seismic performance evaluation of structures. POA is competent for this purpose. For many cases, we can obtain more important information from POA than dynamic analysis, and it is simple and economical. Lately, more and more studies are focused on it.

[1] *Faculty of Civil Engineering, Chongqing Jianzhu University, China, Email: yangpu@ynmail.com*
[2] *China Academy of Building Research, China*

BACKGROUND AND LIMITS

Background to POA

POA has no rigorous theoretical foundation. Its basic assumptions are:
(1) The response of the structure can be related to the response of an equivalent single degree-of-freedom (SDOF) system, which implies that the response is controlled by a single mode.
(2) The shape vector $\{\phi\}$ (see figure 1) remains constant throunghout the time history reponse.

Obviously, both assumptions are incorrect. But pilot studies carried out by several investigators have indicated that these assumptions lead to rather good predictions of the maximum seismic response of multi degree-of-freedom (MDOF) structures, provided their response is dominated by a single mode.

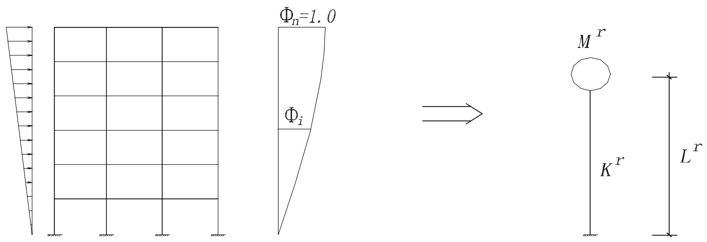


Figure 1 Sketch for Translation MDOF to SDOF system

The formulation of the equivalent SDOF system is not unique, but many methods are based on the shape vector and dynamic equilibrium equation. The governing differential equation of MDOF system is:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+\{Q\}=-[M]\{1\}\ddot{X}_g \qquad (1)$$

Where $[M]$ are the mass matrices, $[C]$ are damping matrices, $\{X\}$ is the relative displacement vector, $\{Q\}$ denotes the story force vector, $\ddot{X}_g$ is the ground acceleration history.

Let the assumed shape vetor $\{\phi\}$ be normalized with respect to the roof displacement, x_t; that is:

$$\{X\}=\{\phi\}x_t \qquad (2)$$

Substituting this expression for $\{X\}$ in equation (1) :

$$[M]\{\phi\}\ddot{x}_t+[C]\{\phi\}\dot{x}_t+\{Q\}=-[M]\{1\}\ddot{X}_g \qquad (3)$$

Define the SDOF refrence displacement x^r as:

$$x^r=\frac{\{\phi\}^T[M]\{\phi\}}{\{\phi\}^T[M]\{1\}}x_t \qquad (4)$$

Pre-multiplying equation (3) by $\{\phi\}^T$ and substituting for x_t using equation (4) results in the governing differential equation for response of the equivalent SDOF system:

$$M^r\ddot{x}^r+C^r\dot{x}^r+Q^r=-M^r\ddot{x}_g \qquad (5)$$

where(FEMA[1998]):

$$M^r=\{\phi\}^T[M]\{1\} \qquad (6)$$

$$Q^r=\{\phi\}^T[P_y] \qquad (7)$$

$$C^r=\{\phi\}^T[C]\{\phi\}\frac{\{\phi\}^T[M]\{1\}}{\{\phi\}^T[M]\{\phi\}} \qquad (8)$$

$[P_y]$ is the story force vector at yield.

The initial period of the equivalent SDOF system (T_{eq}) can be computed as:

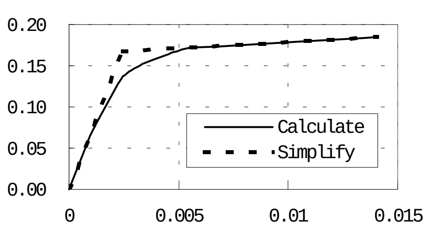
$$T_{eq}=2\pi\sqrt{\frac{M^r}{K^r}} \qquad (9)$$

Where K^r is elastic stiffness of the equivalent SDOF system (see figure 1 and 3).

Limits of POA

The basic steps of POA are: (1) Assume the nonlinear force-displacement relationship of individual elements of structure (including yield strength, post yield stiffness and stiffness degradation, etc); (2) Calculate the target displacement of structure; and (3) Select a reasonable lateral load pattern, and pushing the structure under this load pattern which is monotonically increasing step by step, when a structural member yields, then its stiffness is modified, until the roof displacement of structure is up to the target displacement or the structure collapses. At this time, the evaluation of seismic performance of structure is obtained.

When a structure is translated into equivalent SDOF system, the target displacement can be computed by the inelastic displacement spectra, or non-linear dynamic analysis method. Because the inelastic displacement spectrum is not presented by Chinese seismic code [1989], in this paper, the target displacement is calculated by the second method. The hysteresis model is bilinear (figure 3), no stiffness degradation, and it is simplified likely to figure 2 [Kilar, 1997; Fajfar, 1996]. Three natural earthquake records and one artificial wave which are match the design spectrum near the fundamental period of structure (see figure 4) are selected to calculate the target displacement.



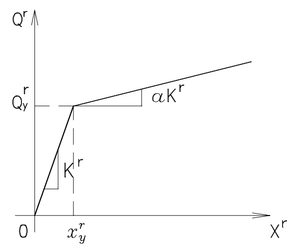
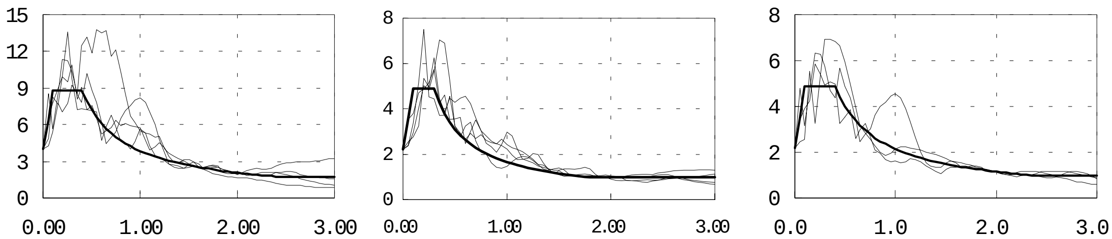


Figure 2 Normalized base shear Vs roof displacement Figure 3 The hysteresis model of equivalent SDOF



a) JG1 b) JG2 c) JG3

Figure 4 Spectrum of earthquake records

From the above steps, we can find that the target displacement and lateral load pattern is very important for POA to evaluate the seismic performance of structures. The target displacement is intended to represent the maximum displacement likely to be experienced during the design earthquake. The load patterns are intended to represent and bound the distribution of inertia forces in a design earthquake. The limits of POA at present time are also focusing on these two aspects.

IMPROVEMENT OF POA

The Gibson model is adopted in this paper, so yields of members occur only at the ends of elements. The column is different from the beam. For column, the axial force and moment are all considered, but for beam, only the moment is considered. The yield strength is calculated by standard value of concrete and actual reinforcing steel. Let the ratio (*a*, see figure 3) of post-yield stiffness to effective stiffness be 1%.

Three different lateral load patterns (A, B and C) is used to evaluate the structures. Comparing with the results of dynamic analysis, a better pattern is selected.

Pattern A:

$$P_j = \frac{W_j h_j}{\sum_{i=1}^{n} W_i h_i} V_{base} \tag{10}$$

Pattern B: (FEMA 274[1998])

$$P_j = \frac{W_j h_j^k}{\sum_{i=1}^{n} W_i h_i^k} V_{base} \tag{11}$$

Where: n is number of stories, h_i is height from the base of a building to floor level i, W_i is weight of floor i, k can be computed by:

$$k = \begin{cases} 1.0 & T \le 0.5 \\ 1.0 + \dfrac{2.5 - 0.5}{T - 0.5} & 0.5 < T < 2.5 \\ 2.0 & T \ge 2.5 \end{cases} \tag{12}$$

Where T is the fundamental period of the building. Obviously, when T is smaller than 0.5 second, the Pattern A is the same as the pattern B.

Pattern C:

When the periods and modes of a structure at provious step is known, according to modal analysis, the story shear forces can be calculated using SRSS(Square Root of the Sum of the Square), then the equivalent lateral load can also be obtained, and it can be regarded as the lateral load pattern for the next step.

Define F_{ij}, Q_{ij} as lateral load, story shear of floor i corresponding to jth mode, Q_i as story shear of floor i using SRSS of N modes, P_i as equivalent lateral load of floor i, then Pattern C is described as following:

$$F_{ij} = \alpha_j \gamma_j X_{ij} W_i \tag{13}$$

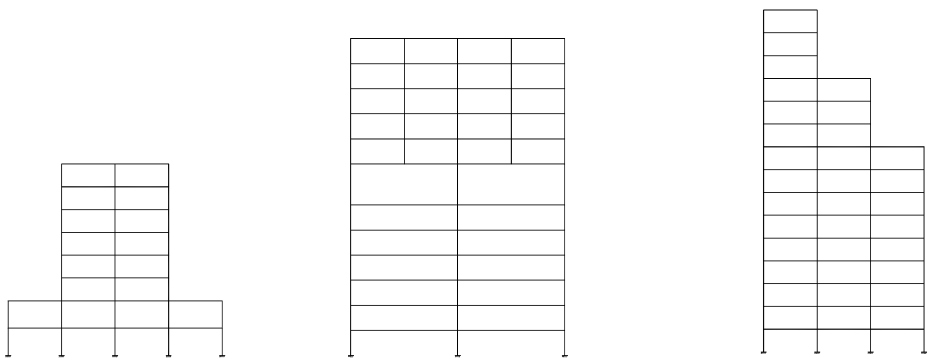
$$Q_{ij} = \sum_{m=i}^{n} F_{mj} \tag{14}$$

$$Q_i = \sqrt{\sum_{j=1}^{N} Q_{ij}} \tag{15}$$

$$P_i = Q_i - Q_{i+1} \tag{16}$$

Where α_j is horizon seismic effect coefficient corresponding to the natural period of jth mode of structure at previous step, Its value refers to Chinese Seismic Code[1989] under the action of rarely occurred earthquake. X_{ij} is relative horizon displacement of floor i corresponding to jth mode, γ_j is mode participation factor of jth mode, N is number of modes (in this paper, N=3), n is number of stories, W_i is weight of floor i.

Three frame structures of 8, 12 and 15 stories (named JG1, JG2 and JG3 in turn) are studied in the paper. They have been designed according to Chinese Seismic Code. But when the seismic design of JG1 was finished, the weight of 3rd floor increased, and it has not been designed again. Their elevations and more information as following:



a) JG1 b) JG2 c) JG3

Figure 5 Elevation of frame structure

The height of JG1 is 3.0m except for the first and second level, which is 3.6m. The height from the base of structure is 25.2m. The span between columns is 6.0m. The cross-section of columns is 450*450 mm (side column at 1~6th floor, middle column at 3~6th floor), 500*500mm (middle column at 1~2nd floor), 400*400 mm (7~8th floor), the cross-section of beams is 200*500 mm. The grade of concrete is C20. The structure is in region of VIII. The characteristic period value is 0.40 second. The fundamental period of the structure is 1.376 second.

The height of 7th floor of JG2 is 5.4m and the others are 3.3m. The height from the base of structure is 41.7m. The span between columns is 12.0m (1~7th floor), 6.0m (8~12th floors). The cross-section of columns is 700*700 mm (side column at 1~7th floor), 900*900 mm (middle column at 1~7th floor), 700*700 mm (8~12th floors). The cross-section of beams is 300*900 mm (1~6th floor), 600*1800 mm (7th floor), 300*600mm (8~12th floor). The grade of concrete is C35. The structure is in region of VII. The characteristic period value is 0.30 second. The fundamental period of structure is 1.514 second.

The height of every floor of JG3 is 3.0m. The height from the base of structure is 45.0m. The span between columns is 6.0m. The cross-section of columns is 450*450mm (side column at 1~5th floor), 600*600mm (middle column at 1~5th floor), 400*450mm (side column at 6~15th floor), 500*500mm (middle column at 6~15th floor), the cross-section of beams is 250*550mm. The grade of concrete is C40. The structure is in region of VII. The characteristic period value is 0.40 second. The fundamental period of structure is 1.808 second.

Information of equivalent SDOF is listed in table 1.

Table 1 Equivalent SDOF Information

Parameter / *No. of structure*	M^{r} (kg)	T_{eq} (sec) [T_1]*	x_y (m)	e	a
JG1	250229.50	1.394[1.376]	0.0625	0.721	0.033
JG2	797351.44	1.544[1.514]	0.0672	0.805	0.037
JG3	536080.88	1.893[1.808]	0.0693	0.697	0.030

*T_1 is the fundamental period of structure.

Using the time history analysis for plan structure (the program PFEP was worked by China Academy of Building Research, P. R. of China), the maximum roof displacements (x_t) of structures subjected to selected strong ground motions (see table 2) are calculated. Translating the structure into SDOF system, the maximum roof displacements (x_t^*) of equivalent SDOF subjected to the same strong ground motions are also calculated. The comparison between results is given in table 2 (unit: meter):

Table 2 The Roof Displacement (x_t) andEquivalent Roof Displacement (x_t^*)

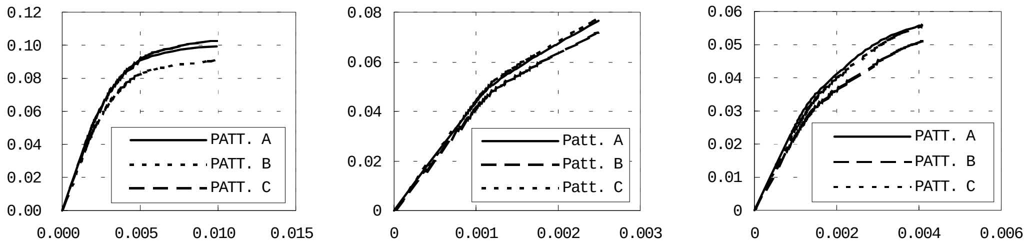
Struc.	Wave	x_t	x_t^*	Struc.	wave	x_t	x_t^*	Struc.	wave	x_t	x_t^*
JG1	USA00161	0.188	0.254	JG2	USA00118	0.098	0.100	JG3	USA00115	0.143	0.176
	USA00863	0.258	0.212		USA00193	0.105	0.088		USA00868	0.148	0.176
	USA00868	0.152	0.155		USA00293	0.082	0.076		USA01183	0.145	0.179
	AW002	0.194	0.232		AW027	0.108	0.106		AW010	0.147	0.143
	Average	0.198	0.213		Average	0.098	0.093		Average	0.146	0.168

Note: AW002, AW027, AW010 are artificial motions.

The results in table 2 shown that error between x_t and x_t^* subjected some ground motions is great, the possible reasons are: (1) the frequency of ground motions is different. The elastic spectrum of special earthquake record is not so smooth as the design spectra. There are apices or vales near the fundamental period (see figure 4). So roof displacement is sensitive to difference between the fundamental period of structure (T_1) and period of equivalent SDOF system (T_{eq}); (2) in order to decide yield displacement of equivalent SDOF system, there is error when taking the base shear Vs roof displacement curve into bilinear (shown in figure 2).

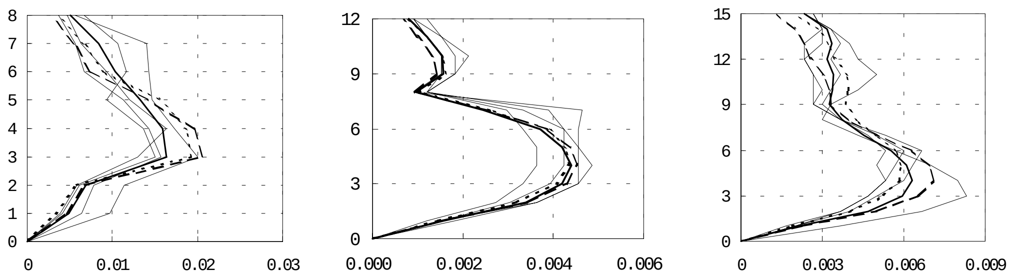
The target displacement of structure in the paper is defined as mean value plus one standard deviation of displacement x_t^* [1998]. Therefore, target displacement of JG1, JG2 and JG3 is 0.250, 0.104 and 0.185, respectively.

The curves of normalized base shear Vs roof displacement are illustrated in figure 6. Figure 7 gives the distribution of inter-story elasto-plastic displacement rotation of structures. The distribution of the maximum shear force subjected to different lateral load pattern is drawn in figure 8.



a) JG1 b) JG2 c) JG3

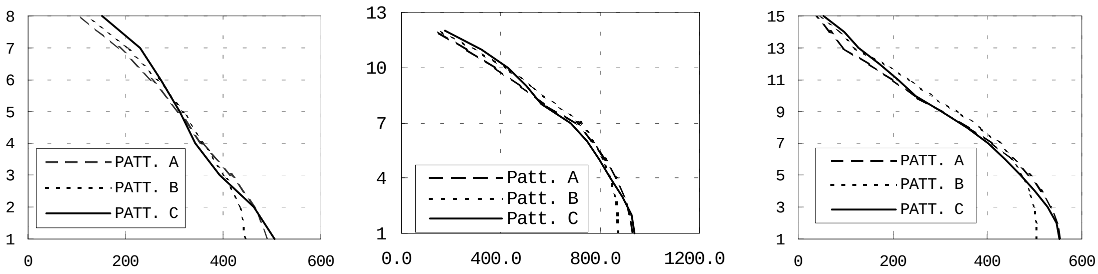
Figure 6 Normalized base shear Vs roof displacement



a) JG1 b) JG2 c) JG3

Figure 7 Distribution of inter-story elasto-plastic displacement rotation

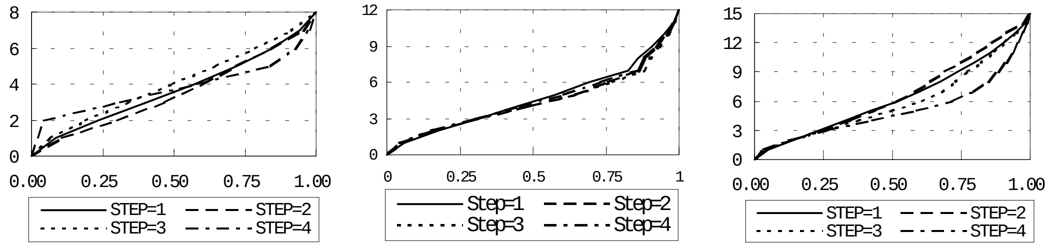
(Annotate: Pattern A····· Pattern B Pattern C, filaments: results of time history analysis)



a) JG1 b) JG2 c) JG3

Figure 8 The distribution of the maximum story shear force

Figure 7 and 8 imply that the elasto-plastic performance of structures (JG1, JG2 and JG3) can be well judged using three lateral load patterns (A, B and C). In compare with the results of dynamic analyses, pattern C is a better one, especially for the upper of structures, because it reflects the effect of higher modes and the change of the modes at each step. The change of first mode is illustrated in figure 9.

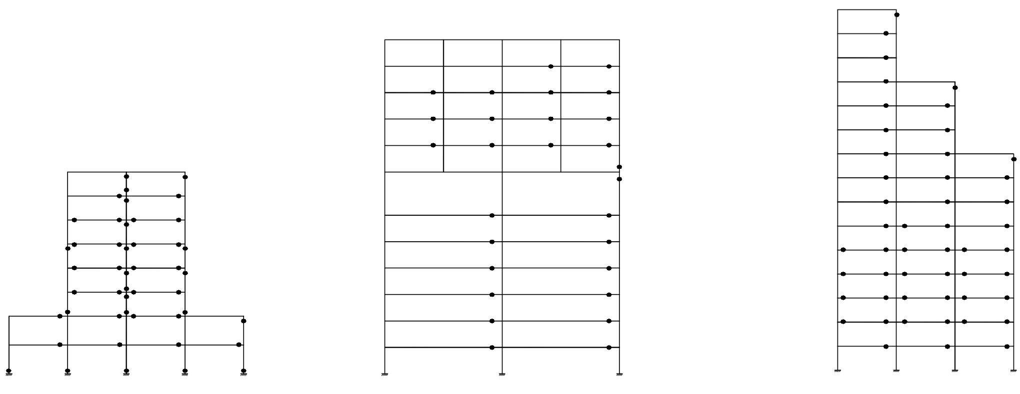


a) JG1 b) JG2 c) JG3

Figure 9 Change of first mode

Four representative curves of change of first mode are shown in figure 9. Step=1 is original mode (elastic mode), Step=4 is the mode at last. Moreover, Step=3 and Step=4 are shown that when structural members yields, and the mode of structure will change. Pattern A and B cannot describe this change.

The distribution of plastic hinges under pattern C is shown in figure 10.



a) JG1 b) JG2 c) JG3

Figure 10 Distribution of plastic hinges (Pattern C in positive direction)

Figure 10 illustrates that many plastic hinges of columns of JG1 are found, because it was not designed according to Seismic Code. However, there are only a few plastic hinges of columns are found in JG2 and JG3, even the huge beam of JG2 has not yielded, for they are designed according to Seismic Code.

CONCLUSIONS

The paper evaluated the performance of structures using three lateral load patterns, and compare with the results of dynamic analysis. For structure which fundamental period is less than 2 seconds, the elasto-plastic seismic performance of structures (in the paper JG1, JG2 and JG3) can be well evaluated by the lateral load pattern (A, B and C), but the Pattern C is more reasonable. It is advised in paper that it is better to evaluate the seismic behavior of structure using pattern C.

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