INDICES OF EFFECTS OF TORSIONAL COUPLING ON EARTHQUAKE RESPONSE OF STRUCTURES

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SUMMARY

The indices of the effects of dynamic-torsion coupling on the peak response were investigated. A deductive method based on the calculation formula induced to the explicit form was adopted using a minimum model which is a single story linear system with an uniaxial eccentricity. Based on the calculation formula, two simple indices were proposed. These indices can uniformly show the dynamic-torsion effect in spite of the dimension of eccentricity. For example, they can evaluate the special torsion due to the small eccentricity which is peculiar to the dynamic torsion.

INTRODUCTION

In order to introduce the effect of the dynamic torsion into seismic design, it is necessary to find the structural index which accurately specifies its degree. Such as the index R_e of the current Japanese Seismic Design Code [1], various indices have been proposed by the present. Naturally the phenomenon of actual torsion is very complicated, and it is also abounding on the parameter which is related to this phenomenon. However, it can be not stated that the rational index was found at present, even if it tries to limit to a minimum model for this phenomenon which is a single story linear system with uniaxial eccentricity. For example, not only large eccentricity but also small eccentricity causes the torsion [3,4].

In this paper, by taking up a minimum model as an investigation object, the dynamic-torsion effect which appeared in the peak response to simplified ground motions is examined, and then the convenient indices which can uniformly show the dynamic-torsion effect in spite of the dimension of eccentricity are proposed. Although the investigation object is a minimum model, the deductive method based on the calculation formula induced to the explicit form is adopted.

ANALYTICAL MODEL

An analytical model is shown in Figure 1. The floor shape is arbitrary. As parameters expressing the torsional characteristics, *e* (the eccentricity factor) and *j* (the stiffness radius factor of gyration) are employed:

$$e = \frac{e_y}{i} \quad , \quad j = \frac{j_x}{i} = \frac{\omega_{o\theta}}{\omega_{ox}} \qquad (1),(2),(3)$$

where, e_y is the stiffness eccentricity and i is the distance of radius of gyration. j_x is the stiffness radius of gyration which is different from the usual definition (j'_x) and is defined by the torsional stiffness around the mass center. $\omega_{o\theta}$ and ω_{ox} are the natural circular frequencies of the torsional vibration around the mass center and the translational vibration, respectively, in uncoupled systems. j is related to the usual j' ($= j'_x / i$) by the following relation:

$$j = \sqrt{j'^2 + e^2} \quad \text{(therefore, } j \geq e\text{)} \qquad (4),(5)$$

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DYNAMIC CHARACTERISTICS [5,7]

The square of the natural circular frequency ω_s (s = 1,2) is given in the following equation.

$$\omega_{\substack{2\\1}}^{\,2} = (1 \pm \lambda)\omega_o^{\,2} \tag{6}$$

in which

$$\omega_o^{\,2} = 0.5\left(\omega_2^{\,2} + \omega_1^{\,2}\right) = 0.5\left(1 + j^2\right)\omega_{ox}^{\,2} \qquad (7),(8)$$

$$\lambda = \frac{\omega_2^{\,2} - \omega_1^{\,2}}{2\omega_o^{\,2}} = \frac{\sqrt{(1-j^2)^2 + (2e)^2}}{1+j^2} \qquad (9),(10)$$

$\omega_o^{\,2}$ is an average of the first and the second modes, and is determined regardless of e. λ is the deviation normalized by the average, and shows the degree of the difference between these circular frequencies. Figure 2 shows λ in the contour line on the e-j plane. Both circular frequencies are extremely close to each other in the region where $e \approx 0$ and $j \approx 1$.

The participation vectors of the torsional component (z) and the translational component (x) in case of $j \leq 1$ are expressed as follows, respectively:

$$\beta_{\substack{2\\1}} Z_{\substack{2\\1}} = \mp \Psi_z \,, \qquad \beta_{\substack{2\\1}} X_{\substack{2\\1}} = 0.5 \pm \Psi_x \qquad (11),(12)$$

in which

$$\Psi_z = \frac{0.5\Delta}{\sqrt{1+\Delta^2}} \;, \quad \Psi_x = \frac{0.5}{\sqrt{1+\Delta^2}} \;, \quad \Delta = \frac{2e}{1-j^2} \qquad (13),(14),(15)$$

and z is the input-direction component ($i\theta$) at the distance of i from the mass center. In case of $1 < j$, the first mode should be changed for the second mode. At a given point on the floor, the participation vector of the total component (u), which added the translational component (x) to the torsional component ($-\alpha_y \cdot z$), is given in the following equation.

$$\beta_{\substack{2\\1}} U_{\substack{2\\1}} = \beta_{\substack{2\\1}} X_{\substack{2\\1}} - \alpha_y \cdot \beta_{\substack{2\\1}} Z_{\substack{2\\1}} = 0.5 \pm \Psi_u \qquad (16),(17)$$

in which

$$\Psi_u = \Psi_x + \alpha_y \Psi_z = \frac{0.5\left(1+\alpha_y \Delta\right)}{\sqrt{1+\Delta^2}} \qquad (18),(19)$$

and α_y ("the normalized distance") is a distance normalized by i from the mass center to the point in the y-direction. The participation vector becomes a function of Δ and α_y.

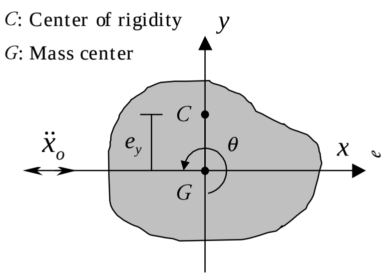

Figure 1: Analytical model

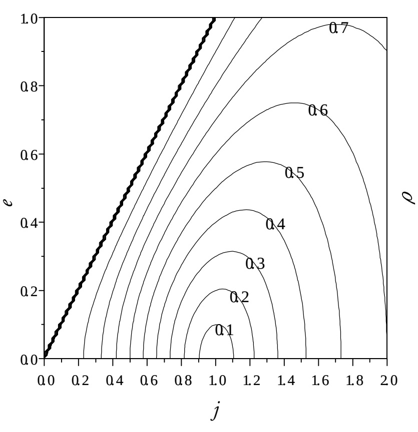

Figure 2: Coefficient λ

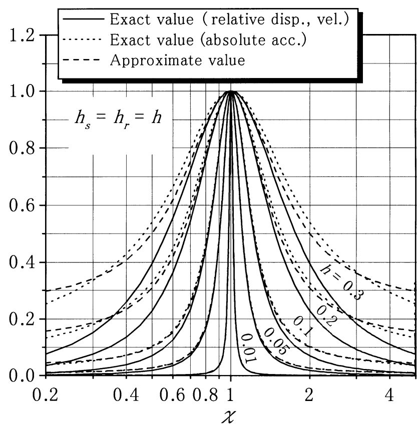

Figure 3: Coefficient ρ

PEAK RESPONSE [7]

Calculation Method and Assumptions

The peak response is formulated by a response spectrum method. The CQC method [8] are used, because the circular frequencies may be close to each other. For convenience, the various response quantities (the relative displacement w, the relative velocity $\dot{w}$ and the absolute acceleration $\ddot{w}$) and the various response components (the torsional component z, the translational component x and the total component u) are represented by w. w_{max} is given in the following equation.

$$w^2{}_{max} = \sum_{s=1}^{2} (\beta_s W_s \cdot S_s)^2 + 2\rho(\beta_1 W_1 \cdot S_1)(\beta_2 W_2 \cdot S_2) \qquad (20)$$

where, suffix s is the mode number and $\beta_s W_s$ represents the participation vector corresponding to the response components (z, x, u). S_s is the response spectrum value of ground motions, and ρ is the mode correlation coefficient between the first and the second modes corresponding to the response quantities (w, $\dot{w}$, $\ddot{w}$). The first term of the right-hand side is the peak response in disregarding the mode correlation ($\rho = 0$), and is equal to the calculation formula of the SRSS method. The second term gives the effect of the mode correlation between the first and the second modes.

For simplification, the following assumptions are adopted for the damping factor (h_s) and the response spectrum value of ground motions (S_s).

$$h_1 = h_2 = h, \quad h << 1 \qquad (21), (22)$$

$$S_1 = S_2 = S \qquad (23)$$

Moreover, the following approximation [6] is used with respect to ρ.

$$\rho = \frac{1}{1+\Lambda^2} \quad , \text{ in which } \quad \Lambda = \frac{\lambda}{2h} \qquad (24), (25)$$

This approximation is obtained by applying equation (22) and the condition, in which these two circular frequencies are close to each other, to the exact solution for the white noise input. The broken lines in Figure 3 are the approximate values. The abscissa χ is the natural circular frequency ratios ω_2/ω_1 and ω_1/ω_2. From equation (9), χ and λ are in the following relationship:

$$\lambda = \frac{|1-\chi^2|}{1+\chi^2} \qquad (26)$$

The solid lines (the relative displacement and the relative velocity) and the dotted lines (the absolute acceleration) in Figure 3 are the exact values shown for comparison. If h is under 0.1 at least, for either response quantity, the accuracy of this approximation will be sufficiently satisfied in the practical use, for not only $\chi \approx 1$ but also very wide range of χ.

Calculation Formula

The peak response of the pure translation in case of no eccentricity ($e = 0$) is equal to the response spectrum value S, if the response spectrum is constant. Then, w_{max} is normalized by S to evaluate the torsional effect. This normalized peak response w_{max}/S is shown by the product of $\overline{\Psi}_w$ (the normalized peak response in disregarding the mode correlation, which is equal to the calculated value by the SRSS method) and $\overline{\lambda}_w$ (the coefficient which gives the effect of the mode correlation).

$$w_{max}/S = \overline{\Psi}_w \cdot \overline{\lambda}_w , \qquad (27)$$

in which

$$\overline{\Psi}_w = \sqrt{(\beta_1 W_1)^2 + (\beta_2 W_2)^2}, \qquad \overline{\lambda}_w = \sqrt{1 + \left[\frac{2(\beta_1 W_1)(\beta_2 W_2)}{\overline{\Psi}_w{}^2}\right]\rho} \qquad (28), (29)$$

Both these terms $\overline{\Psi}_w$ and $\overline{\lambda}_w$ are unrelated to the response spectrum, and are given only by the characteristic of the system. Therefore, w_{max}/S becomes common for either response quantity.

The calculation formula for the torsional component ($w = z$) becomes the following equation.

$$z_{max}/S = \overline{\Psi}_z \cdot \overline{\lambda}_z \qquad (30)$$

in which

$$\overline{\Psi}_z = \sqrt{2}\left|\Psi_z\right| = \frac{\sqrt{2}}{2}\cdot\frac{|\Delta|}{\sqrt{1+\Delta^2}} \qquad (31),(32)$$

$$\overline{\lambda}_z = \sqrt{1-\rho} = \frac{\Lambda}{\sqrt{1+\Lambda^2}} \qquad (33),(34)$$

In the same way, the calculation formula for the total component ($w = u$) becomes the following equation.

$$u_{max}/S \;=\; \overline{\Psi}_u \cdot \overline{\lambda}_u \qquad (35)$$

in which

$$\overline{\Psi}_u = \sqrt{0.5 + 2\Psi_u{}^2} = \sqrt{\frac{1}{2}\left(1+\frac{(1+\alpha_y\Delta)^2}{1+\Delta^2}\right)} \qquad (36),(37)$$

$$\overline{\lambda}_u = \sqrt{1-\left[1-\left(1/\overline{\Psi}_u\right)^2\right]\rho} = \sqrt{\frac{\left(1/\overline{\Psi}_u\right)^2+\Lambda^2}{1+\Lambda^2}} \qquad (38),(39)$$

The calculation formula for the translational component ($w = x$) is obtained, if $\alpha_y = 0$ in equation (37).

PROPOSED INDICES OF TORSIONAL EFFECT

The normalized peak response z_{max}/S of the torsional component is given in the product of $\overline{\Psi}_z$ and $\overline{\lambda}_z$, as mentioned above. From equation (32), $\overline{\Psi}_z$ is a function only of Δ. From equation (34), $\overline{\lambda}_z$ is a function only of Λ. What are proposed as an index to evaluate the torsional effect are these Δ and Λ. It calls, for convenience, these "the torsion basic index" and "the mode independent index".

Figures 4,5 and 6 show the examples of the normalized peak response in the contour line on the Δ - Λ plane. Figure 4 is the torsional component z_{max}/S at $\alpha_y = \pm 1$. Figures 5 and 6 show the total components u_{max}/S at $\alpha_y = \sqrt{3}$ and $\sqrt{3/2}$. These values of α_y are respectively the upper limit corresponding to the very slender floor and the lower limit corresponding to the square floor at the stiff edge of the floor, if the rectangular floor of the

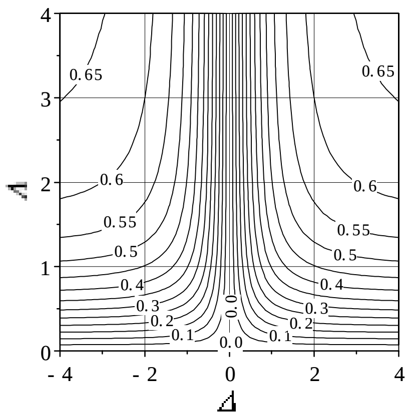

Figure 4: z_{max}/S at $\alpha_y = \pm 1$

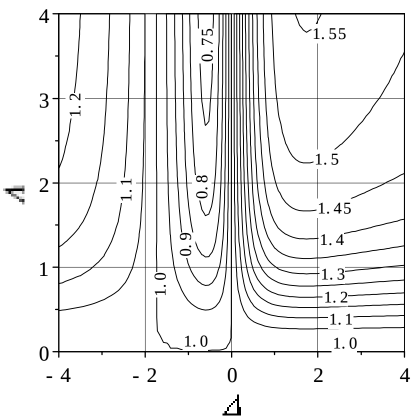

Figure 5: u_{max}/S at $\alpha_y = \sqrt{3}$

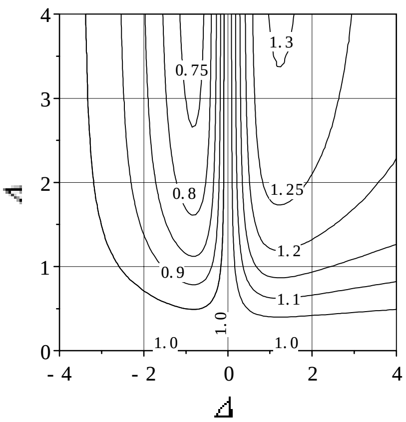

Figure 6: u_{max}/S at $\alpha_y = \sqrt{3/2}$

uniform distribution mass is assumed to have the eccentricity in the transverse direction. Here, if the sign of the abscissa Δ is reversed, the total component at the flexible edge $\alpha_y = -\sqrt{3}, -\sqrt{3/2}$ is obtained.

Index for Case in Disregarding Mode Correlation

As described in next 5.2, the mode correlation never occur, when index Λ is ∞. From Figure 4 of z_{max}/S, the normalized peak response in this condition ($\overline{\Psi}_z$) is monotonically increased with the increase of the absolute value of Δ. From this fact, it is possible to confirm that Δ becomes an index of the torsional effect. The normalized peak response is 0 when Δ is 0, and reaches the upper limit ($\sqrt{2}/2$) when $|\Delta|$ is ∞. However, even if $|\Delta|$ is about 2, it considerably approaches the upper limit.

In case of the total component of Figures 5 and 6, the swinging edge of the floor is decided by the sign of the index Δ, at which the peak response become larger: being the stiff edge ($\alpha_y > 0$) if positive and the flexible edge ($\alpha_y < 0$) if negative. This point is different from the static torsion in which the swinging edge is always the flexible edge.

For the absolute value of the index Δ, $\overline{\Psi}_u$ is not monotonically increased, because $\overline{\Psi}_x$ reversely decreases, if $\overline{\Psi}_z$ increases. However, it monotonically increases until the upper limit at the swinging edged. It becomes more than 1.3 when $|\Delta|$ reaches about 1, even in the square floor (Figure 6) where the proportion of the increase is the lowest among the above-mentioned rectangular floors. From equation (37), the value of Δ at the upper limit is α_y, and its limit value is given in the following equation.

$$u_{max}/S \quad = \quad \sqrt{1+0.5\,\alpha_y^{\;2}} \tag{40}$$

Index for Effect of Mode Correlation

From Figure 4, z_{max}/S is monotonically increased with the increase of index Λ. From equation (24), Λ increases with the decrease of ρ which gives the intensity of the mode correlation. From this fact, Λ shows the independent degree of the mode (the weakness of the mode correlation). Therefore, modes have no correlation, when Λ is ∞. Then, it can be confirmed that Λ becomes an index showing to what degree the normalized peak response in disregarding the mode correlation decreases by the mode correlation. The reason why the torsional effect decreases by the mode correlation is that the torsional response of the first and the second modes

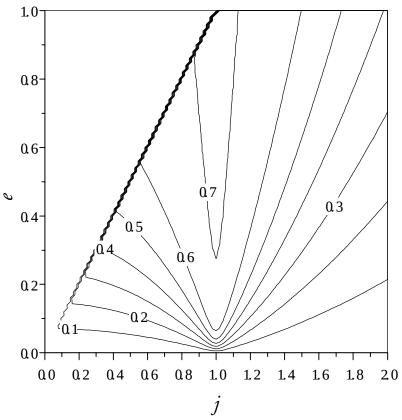

Figure 7: z_{max}/S ($h = 0.02$)

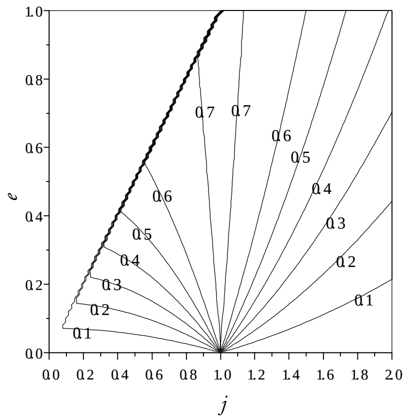

Figure 9: $\overline{\Psi}_z$, i.e. z_{max}/S ($h = 0$)

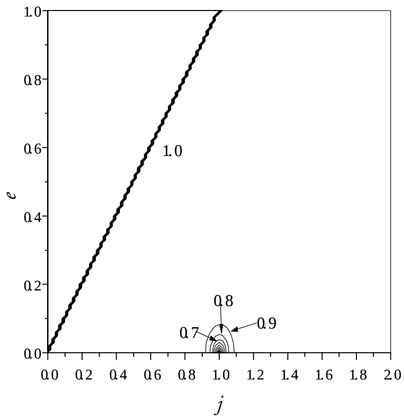

Figure 10: $\overline{\lambda}_u$ ($h = 0.02$)

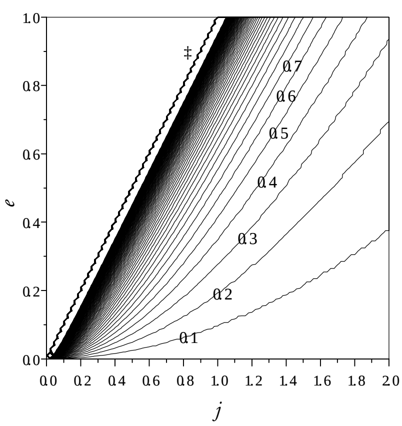

Figure 8: z_s/x_{os} (static torsion)

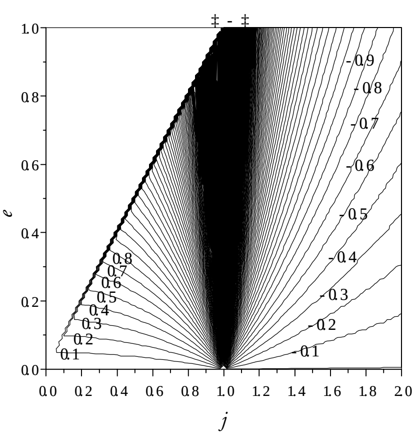

Figure 11: Index Δ

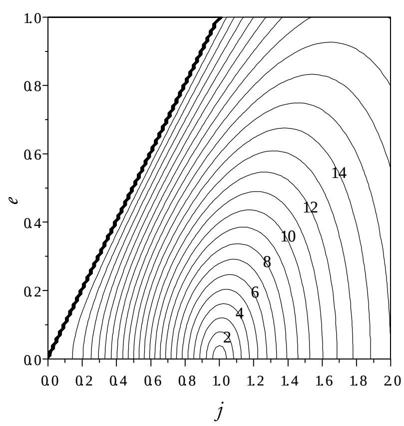

Figure 12: Index Λ

cancels out each other, because the sign of participation vector of these modes is reversed (equation (11)).
On the other hand, from equation (39), the coefficient $\bar{\lambda}_u$ of the total component is related to not only the index Λ but also the coefficient $\bar{\Psi}_u$. However, from Figures 5 and 6, the property which monotonically increases with the increase of the index Λ is similar to the coefficient $\bar{\lambda}_z$ of the torsional component, as far as the swinging edge is concerned.

Either $\bar{\Psi}_z$ and $\bar{\Psi}_u$ is monotonically increased with the index Λ, as mentioned above. But, the value even when Λ is about 2 approximately reaches the value when Λ is ∞ (the case of no mode correlation). Therefore, the effect of the mode correlation hardly appears, if Λ is over about 2.

Condition in which Torsional Effect Increases

Figure 7 shows z_{max}/S $(h = 0.02)$ in the contour line on the *e-j* plane. If *e* is large, the normalized peak response is large in the wide range of *j*. Therefore, the torsional effect is surely large in this case. In addition, when *j* is near to 1, the torsional effect is also large, even if *e* is considerably small. This is the torsional vibration due to the small eccentricity. This special torsion is peculiar to the dynamic torsion, and is never observed in the static torsion. Figure 8 is this example, which shows the static torsional displacement (z_s) normalized by the pure translational displacement (x_{os}) in case of no eccentricity. The calculation is based on the following equation.

$$z_s \big/ x_{os} = \frac{e}{j^2 - e^2} \tag{41}$$

From the figure, the static torsional effect increases, as *j* is smaller and *e* is larger.

Figures 9 and 10 show $\bar{\Psi}_z$ (the normalized peak response in disregarding the mode correlation) and $\bar{\lambda}_z$ (the coefficient which gives the effect of the mode correlation), respectively. Figure 7 is obtained by these two figures.

The torsion due to the small eccentricity is remarkable in Figure 9, which disregarded the mode correlation ($\Lambda = \infty$), further than Figure 7 considering the mode correlation. For example, the normalized peak response has reached the upper limit even in case of almost no eccentricity ($e \approx 0$). From equation (25), the case of no damping ($h = 0$) is concretely correspondent to this condition. Figure 11 shows the index-value of Δ. It can be confirmed that the general tendency of the absolute value of the index resembles Figure 9, including the torsion due to the small eccentricity.

From Figure 10, $\bar{\lambda}_z$ is almost 1 in the major region of the *e-j* plane, while it becomes under 0.9 only in the vicinity of the point where *e* is 0 and *j* is 1. Therefore, the effect of the mode correlation is remarkable for the torsion due to the small eccentricity. Figure 12 shows the index-value of Λ in case of $h = 0.02$. It is possible to confirm that the region in which the index-value is smaller than 2 corresponds to the above-mentioned region. From Figure 2, the circular frequencies of both modes are very close to each other in this region. From equation (25), this region becomes narrow, and therefore the torsion due to the small eccentricity is easy to be generated, as h is smaller than 0.02.

5.4 Application of Indices

As shown in the above, the condition in which the torsional effect increases is explicitly explained by two indices proposed in this paper, including the special torsion due to the small eccentricity which is peculiar to the dynamic torsion. This fact shows that these indices are possible to uniformly express the dynamic-torsion effect in spite of the dimension of eccentricity. However, only the torsion due to the small eccentricity actually requires the mode independent index Λ, as shown in Table 1. The torsional effect will be overestimated, if the index Λ is disregarded for simplification. In the usual case, the evaluation of the torsional effect is approximately possible only by the torsion basic index Δ.

CONCLUSIONS

1. Based on the induced calculation formula (30)-(39) of the normalized peak response, two indices Δ and Λ were proposed as a convenient index for the torsional effect.
2. Under the condition of the elastic range, these indices can uniformly show the dynamic-torsion effect in spite of the dimension of eccentricity, including the special torsion due to the small eccentricity which is peculiar to the dynamic torsion.
3. It is possible that the torsion basic index Δ, defined in equation (15), shows the torsional effect in disregarding the mode correlation (that is, the case in which it was calculated by the SRSS method) in terms of its absolute value. In addition, by the sign of this index, it is possible to show the swinging edge of the floor (the edge at which the peak response become larger). It is the stiff edge, if positive.
4. It is possible that the mode independent index Λ, defined in equations (10) and (25), shows to what degree the torsional effect in disregarding the mode correlation decreases by the mode correlation.
5. Of these two indices, only Δ is sufficient in the usual case. Only for the case of the special torsion due to the small eccentricity, Λ should be required in addition to Δ.

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Table 1: Application of indices

	Torsion Basic Index Δ	Mode Independent Index Λ
Usual Case ($\Lambda \geq 2$)	Required	Not Required
Special Case ($\Lambda < 2$)	Required	Required (Overestimating if Disregarding)