SIMPLIFIED APPROACH TO THE NON-LINEAR BEHAVIOR OF RC MEMBERS

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SUMMARY

In this paper, a simplified one-dimensional analytical tool based on finite difference technique to analyze reinforced concrete members under cyclic deformations is presented along with the details of the algorithm. Relatively simple assumption of plain section remains plain before and after bending is used. Nonlinear material properties with cyclic stress-strain relationship have been adopted for both reinforcement and concrete to check the significance of implementation of the various characteristics of the stress-strain relationship. Finally this tool is used to successfully simulate a few experiment results to check the validity of this developed analytical tool.

INTRODUCTION

The numerical analysis of the behavior of the reinforced concrete members has been a subject of intensive research in the past two decades. Several methods for the analysis are already available. Different researchers are trying to formulate more sophisticated analytical tools for more accurate simulation of different structural phenomenon.

Though stiffness matrix based method is used for formulating the analytical tools, some researchers have also used flexibility based analytical methods(e.g. fiber model). These methods generally use theory of elasticity, plasticity or progressive damage for concrete using discrete crack or smeared crack approaches. These approaches generally attempt to simulate experimental results of reinforced concrete members under monotonic and cyclic loading using finite element based techniques. Some of these methods involve rigorous analysis, requiring considerably high amount of computing time and hardware requirements. However, many of these results can be simulated using relatively simpler methods so as to reduce the computational expenses.

In this research, a simplified analytical tool based on finite difference method is developed to analyze the reinforced concrete members under cyclic loading. Stiffness matrix based method is adopted in such way that displacement-controlled algorithm can be implemented.

Earlier, similar methods have also been used for simulating the behavior of reinforced concrete members. Those methods required the sectional property of the moment-curvature relationship as input. The moment curvature relationship is generally implemented using multi-linear models with varying number of transition points and is used to calculate the load displacement behavior of the member. For cyclic behavior of moment-curvature relationship, various models exist, e.g. Raufaiel and Meyer model, Takeda model, etc. However, all these models require the calculation of the envelope of moment-curvature relationships, which often depends on the level of axial force, which is not necessarily constant. The unloading rules of these models are not dependent on the actual material stress-strain relation.

In this proposed method, no such implementation of moment-curvature relationships is required as input. The non-linearity in the member behavior is taken care by the direct implementation of the non-linear material

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properties in the calculation of internal forces based on the current condition of stress, strain, axial force, moment, etc. Complete stress-strain relationship of both concrete and reinforcement is implemented with full unloading criteria.

One more positive aspects of this model is that, the effect of implementation of various characteristics of the stress-strain curves of both concrete and reinforcement can be check with lesser computational cost in comparison to the more sophisticated methods, e.g. 3D analysis etc.

In this research, the developed simplified method for the analysis of the reinforced concrete members is presented in brief. The material properties with hysteric stress-strain relationship for both reinforcement and concrete adopted in the analysis are also presented with due consideration to various phenomenon.

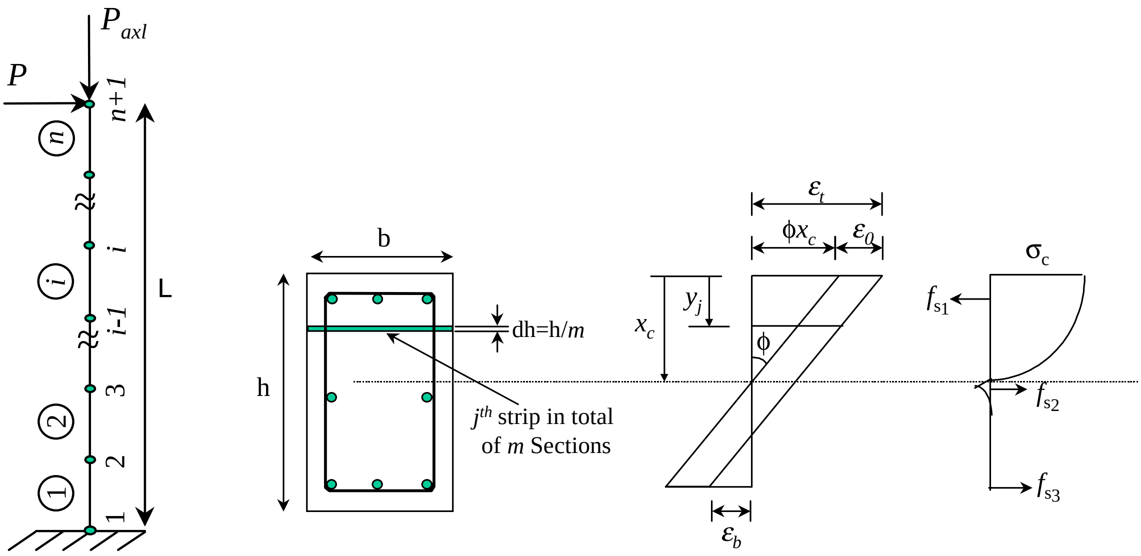
In this formulation, the structure is discritized into elements like parts. The applied load and the displacements, slopes and curvatures at the nodes are taken as global variables. Here, the assumption that a plain section remains plain before and after bending is used as the basis of calculation of internal forces. The stiffness matrix is calculated based on the present conditions of material stress, strain, etc.

In this paper experimental results of reinforced concrete specimen under cyclic loading have been simulated to find out the effectiveness of this model.

FORMULATION

Here a cantilever reinforce concrete column with horizontal cyclic load applied at the top is considered. The secondary moment in the column (P-Δ effect) is neglected. At the nodes, it is assumed that plain section remains plane during bending and the deformation due to shear force is negligible. The concrete is assumed to be a homogeneous isotropic material with perfect bond between concrete and reinforcement.

Based on these assumptions, sectional properties are considered at the nodes as shown in Figure 1b. In this paper, only uni-directional horizontal loading at the top of the column is considered. The column of height L is first divided into n element with $n+1$ node and the section is subdivided into m strips and non-linear material properties with appropriate stress-strain relationship are considered at these points to calculate the internal forces. For bi-directional horizontal loading, this method can be applied by dividing the section into square blocks, rather then strips.
Lateral load P and axial force P_{axl} are applied at the top of the column. Nodal variables of displacement y_i,



a) Column Discretization

b) Sectional Properties at Nodes

Figure 1: Analytical model

rotation θ_i, curvature ϕ_i and strain at the top of the section ε_{ti} at i^{th} node are considered. Strain at j^{th} strip can be defined as

$$\varepsilon_j = \varepsilon_{ti} - y_j \phi_i \tag{1}$$

Each strip of the section is further subdivided into parts of area A_{jm} according to the different material properties. These areas are core concrete, cover concrete and reinforcement. Stress σ_{jm} is calculated for each of these areas based on their material characteristics. The internal axial capacity P_{ax}, the external bending moment (M_{iEX}) and internal bending moment (M_{iIN}) at the nodes can be calculated as follows

$$P_{ax} = \sum_j \sigma_j A_j \tag{2}$$

$$M_{iEX} = l_i P \tag{3a}$$

$$M_{iIN} = \sum_j \sigma_j A_j z_j \tag{3b}$$

Here, the A_j represent the summations for all material parts A_{jm} and $z_j = h/2 - y_j$. Writing Eq.(2) and (3) in incremental form,

$$\delta P_{ax} = \sum_j \delta\sigma_j A_j = \sum_j \frac{\partial \sigma_j}{\partial \varepsilon_j} A_j (\delta\varepsilon_{t_i} - y_j \delta\phi_i) \tag{4}$$

$$\delta M_i = l_i \delta P = \sum_{j=1}^{m} \frac{\partial \sigma_j}{\partial \varepsilon_j} A_j Z_j \delta\varepsilon_j = \sum_j \frac{\partial \sigma_j}{\partial \varepsilon_j} A_j Z_j \delta\varepsilon_{t_i} - \sum_j \frac{\partial \sigma_j}{\partial \varepsilon_j} A_j Z_j y_j \delta\phi_i \tag{5}$$

Rearranging Eq.(4),

$$\delta\varepsilon_{t_i} = (\delta P_{ax} - \sum_j \frac{\partial \sigma_j}{\partial \varepsilon_j} A_j \delta y_j \delta\phi_i) / \sum_j \frac{\partial \sigma_j}{\partial \varepsilon_j} A_j \tag{6}$$

Substituting Eq.(6) in Eq.(5), we will get,

$$\delta M_i = l_i P = (\partial M / \partial \phi)_i \delta\phi_i + (\partial M / \partial P_{ax})_i \delta P_{axi} \tag{7}$$

The nodal variables displacement y_i, rotation θ_i, curvature ϕ_i in the incremental form can be related to each other as follows

$$\delta\theta_{i+1} - \delta\theta_i - (\delta\phi_{i+1} + \delta\phi_i)\Delta l/2 = 0 \tag{8}$$

$$\delta y_{i+1} - \delta y_i - (\delta\theta_{i+1} + \delta\theta_i)\Delta l/2 = 0 \tag{9}$$

The stiffness matrix in the incremental form can be formed using Eq.(7), (8) and (9). There are n +1 variables of displacement δy_i, rotation $\delta\theta_i$, curvature $\delta\phi_i$ at each node and δP, the applied cyclic load is taken as global variable. There are n +1 equations from Eq.(7) and n equations each from Eq.(8) and (9). Since from boundary conditions $\delta\phi_{n+1}$= 0 and l_{n+1}=0 at the top, Eq.7 will not be relevant at the n+1th node. Therefore 3n equations have been taken with 3n +4 unknowns and 4 boundary conditions (at the support $\delta y_1 = 0$, $\delta\theta_1 = 0$ and at the top $\delta\phi_{n+1} = 0$, applied δy_{n+1}). Therefore, system of equations can be solved. Even though the stiffness matrix was not symmetric, it did not create any problem in the analysis.

After solving the nodal variables of displacement y_i, rotation θ_i and curvature ϕ_i are calculated. At each node, based on the calculated curvature ϕ_i, strain at the top of the section ε_{ti} is calculated in an iterative manner such that internal axial force balances with the external axial force. The gradient of the strain diagram ϕ is used in calculating the moment at the section. After the convergence of internal forces, the global convergence at each node is checked. The unbalanced moments due to internal and external forces is taken as the convergence criteria and iterated until the convergence criteria has been satisfied.

MATERIAL MODELS

Material model for concrete

The stress-strain curves for monotonic case serves as the envelope for the cyclic stress-strain relationship. In compression, the stress-strain envelope with parabolic behavior before the peak and linear softening after the peak is similar to the model proposed by Kent and Park [1] is adopted. Confinement effect due to ties or lateral reinforcement is considered by adopting more ductile post-peak softening curve.

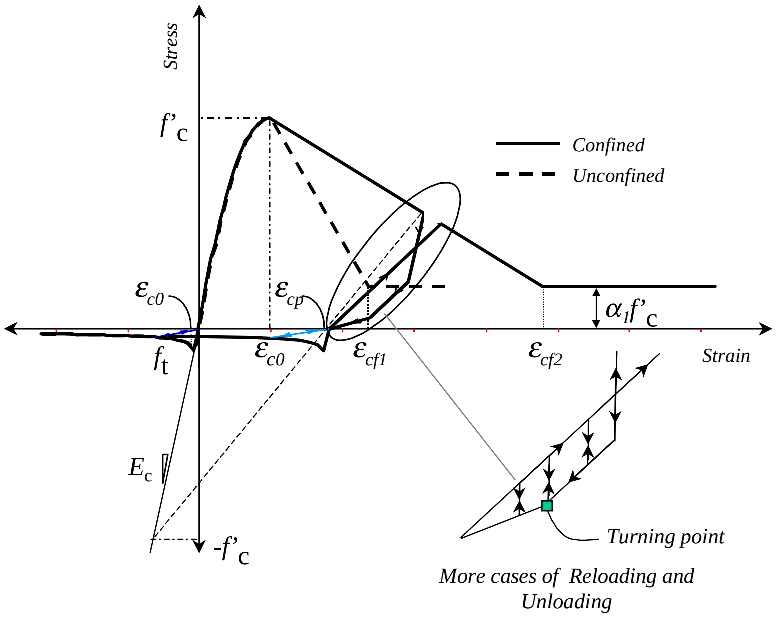


Figure 2: Concrete model

$$\sigma = f_c'\left(2\varepsilon/\varepsilon_{c0} - (\varepsilon/\varepsilon_{c0})^2\right) \quad \varepsilon \le \varepsilon_{c0}$$
$$\sigma = m_1(\varepsilon - \varepsilon_{c0}) + f_c' \quad \varepsilon_{c0} < \varepsilon \le \varepsilon_{cfi} \qquad (10)$$
$$\sigma = \beta\ m_1(\varepsilon - \varepsilon_{cfi}) + \alpha\ f_c' \quad \varepsilon > \varepsilon_{cfi}$$

Where ε_{c0} =0.002 is the strain at the peak stress, $m_1 = -0.8 f_c'/(\varepsilon_{cfi} - \varepsilon_{c0})$ is the post peak slope controlling the softening/ductility of the concrete and strain ε_{cfi} is the controlling parameter for this post peak slope where i=1 for unconfined concrete and i=2 for confined concrete. This factor ε_{cfi} is dependent on the amount and the spacing of transverse reinforcement and the strength of concrete and is calculated as mentioned in Kent and Park [1]. In tension, linear behavior until peak has been taken and gradual degradation is adopted in post peak behavior and is shown below:

$$\sigma = E_c\varepsilon \quad \varepsilon \ge \varepsilon_{t0}$$
$$\sigma = E_c\varepsilon_{t0}\left(\varepsilon_{t0}/(\varepsilon - \varepsilon_{cp})\right)^{0.4} \quad \varepsilon < \varepsilon_{t0} \qquad (11)$$

where f_t' and E_c are calculated on the basis of CEB and $\varepsilon_{t0} = f_t'/E_c$.

In various experimental works, it is shown that plastic strain is accumulated for cyclic cases in both tension and compression. However, the accumulation of plastic strain in tension is not taken in this analysis for simplicity. The plastic strain accumulated in compression ε_{cp} is taken as the origin of the stress-strain curve in tension as if the whole stress-strain curve has shifted to compression side. Linear unloading in the tension side is assumed.

For the cyclic behavior of concrete in compression, model proposed by Darwin and Pecknold [2] based on experimental results from Karson and Jirsa [3] is adopted. The strategy of unloading and reloading is shown in Figure 4. Unlike Darwin and Pecknold model, the unloading branch from the turning point is curtailed at 90% stress level and connected to the ε_{cp} point in order to avoid zero slope for better convergence.

The plastic strain ε_{cp} is calculated by the method adopted in focal point model [4] for better numerical convergence. Here ε_{cp} is the intersection of strain axis with line joining from the unloading point at the compression envelope curve to the point with stress $-f_c'$ and strain $-f_c'/E_c$.

Material model for reinforcement

For the reinforcement, the cyclic stress-strain model adopted in the numerical analysis is shown in Figure 3. This is a multi-linear model that deals with one-dimensional stress-strain relationship for reinforcement. This model takes care of reinforcement hardening and bilinear unloading is implemented. In tension after yielding at (f_{yt}, ε_{yt}), there is a yield plateau to the point ($f_{yt1}, \varepsilon_{yt1}$) with a nominal slope of E_{s1} (=0.001E_s). After this for simplicity, the hardening is assumed to start to reach the ultimate strength in a linear manner to ($f_{yt2}, \varepsilon_{yt2}$) with

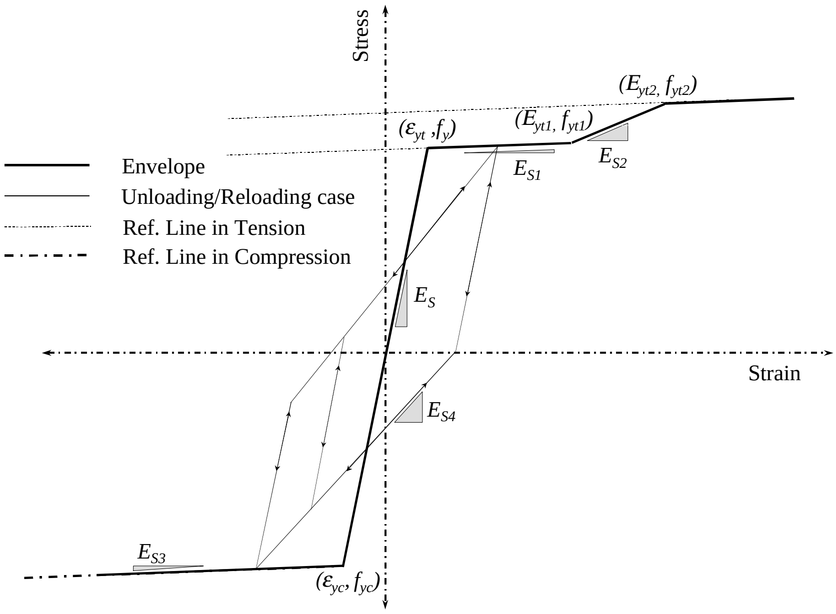


Figure 3: Reinforcing reinforcement stress-strain model

a slope of E_{s2} in monotonic stress-strain case. This model is quite similar to multiple surface models that are used for reinforcement in multi-dimension. Always, one line in tension and one in compression are used as reference lines with a reference point on each of them. When the strain is between these reference points, it is considered to be unloading. Unloading is assumed to occur elastically (E_s) initially and assumed to follow slope E_{s4} after reaching the surface midway between the two reference surfaces (or lines). This is assumed to take care of various different phenomena including Bauschinger effect, etc.

Typical envelope line and a typical case of unloading and reloading are presented in Figure 3. There are two reference lines in tension part and one in the compression part and they are shown clearly. When the stress-strain curve travels along the tensile part, the plastic strain is assumed to accumulate until it reaches the strain of $\varepsilon_{yt} - \varepsilon_{yt1}$. After that, the stress-strain behavior is assumed to harden with a slope E_{s2}. The reference line in tension moves up as the stress hardens. The hardening continues, till the stress-strain reaches another reference line in tension. A typical case of unloading-reloading case is also shown in Figure 3.

The reference line in compression does not shift like the reference line in tension. It passes through compression yield point (ε_{yc}, f_{yc}) and then takes the slope of E_{s3} (=0.001E_s) to avoid the zero slope problem in analysis. The different parameters adopted here in the analyses are E_s = 2.1x10^5 MPa, ε_{yt1} = 0.007, ε_{yt2} =0.05, E_{s4}=0.1E_s, $f_{yc} = -f_{yt}$.

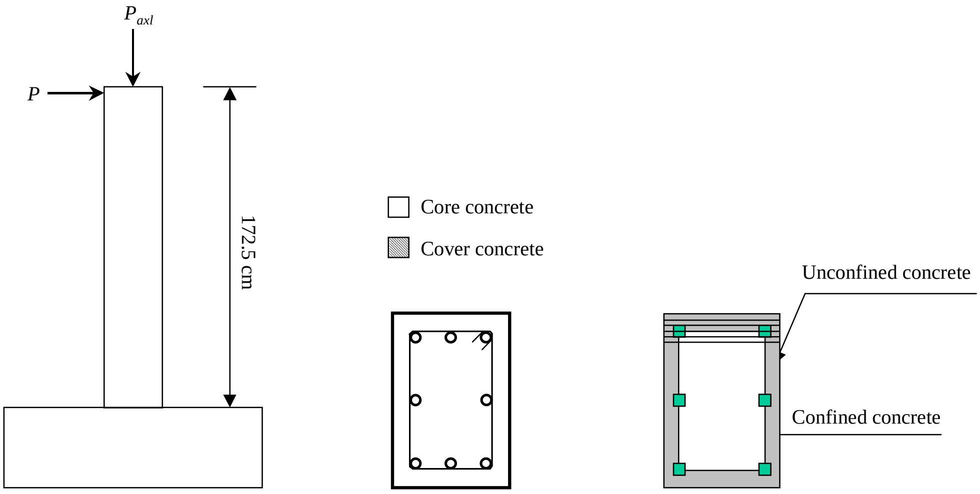
APPLICATION TO EXPERIMENTAL RESULTS

Table 2: Details of Specimen and Material Properties of Concrete

Case No	f'_c (Mpa)	Specimen Cross-section	Rings spacing	ε_{cf1} (Unconfined)	ε_{cf2} (Confined)	Axial Load (N/mm^2)
T1	20.0	300x150	D10@150 c/c	3.16ε_0	6.65ε_0	0.00
T2	20.0	-do-	-do-	3.16ε_0	15.0ε_0	3.00

Table 1: Material Properties of Reinforcements

Case	Type	f_{yc} (MPa)	f_{yt} (MPa)	f_{yt1}(MPa)	E_S(MPa)
T1 & T2	D10	-420.0	420.0	500.0	21.0E5



a) Specimen T1 & T2 b) Section of T1 & T2 c) Analytical model of Section

Figure 4: Specimen details

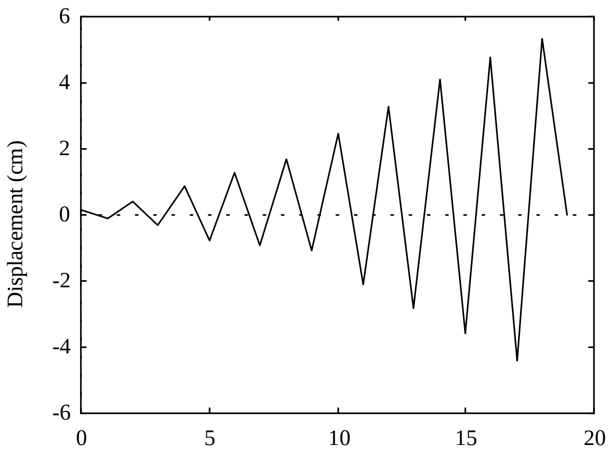


Figure 5: Applied cyclic displacement

In this paper two experiments are considered to understand the capabilities of this developed analytical method. Experiments were carried out on a reinforced concrete cantilever column under displacement controlled cyclic loading[5]. The sectional details and dimensions of the specimens are shown in Figure 4. Both the specimens T1 and T2 are of same dimensions. T1 is the specimen without axial load whereas T2 is the specimen with axial load. The axial load of 3.0 N/mm^2 was applied at the top of the column as shown in Figure 4. Each cycle of a particular magnitude was applied once and shown in Figure 5. The section details, material properties of concrete and values of different parameters of the concrete material model used in the analysis are presented in **Table 1**. The material properties of the reinforcement bars used are mentioned in **Table 2**.

The descretisation of the specimen into elements and the cross-section into layer is shown in Figure 4. The specimen is divided into n=5 parts and the cross-section of the specimen is divided into the m=300 layers as shown in Figure 1. In order to simulate member behavior and to see the capabilities of this analytical tool, the material models are also studied. It is well-known fact that the analytical results are the reflection of the material models adopted hence some parameters, which are very important in describing the structural behavior under cyclic loading will be discussed and applied in this section.

GENERAL DISCUSSION

In this section, the RC cantilever column specimen described in the previous section is analyzed and discussed in detail to check the validity of the analytical method.

The analytical results of both the specimens showed stiffer behavior in the beginning in comparison to the experimental results. This is the draw back and generally noticed in most of the analytical tools and is outside the scope of this paper. The area bounded by the hysteric curve is actually the representation of the amount of the hysteric energy released. This hysteric energy is taken here to understand the cyclic behavior of the structural members. Since the analytical results are the reflection of the material models adopted hence special emphasis is given to the different parameters of the material models.

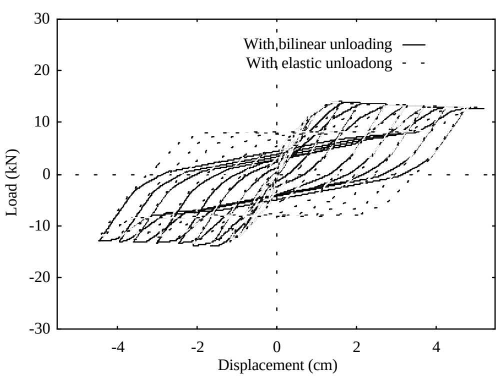


Figure 6: Reloading/unloading, T1

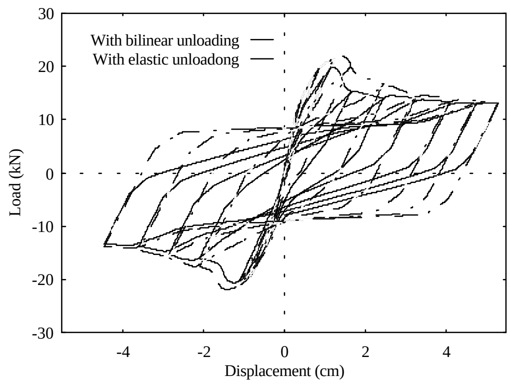
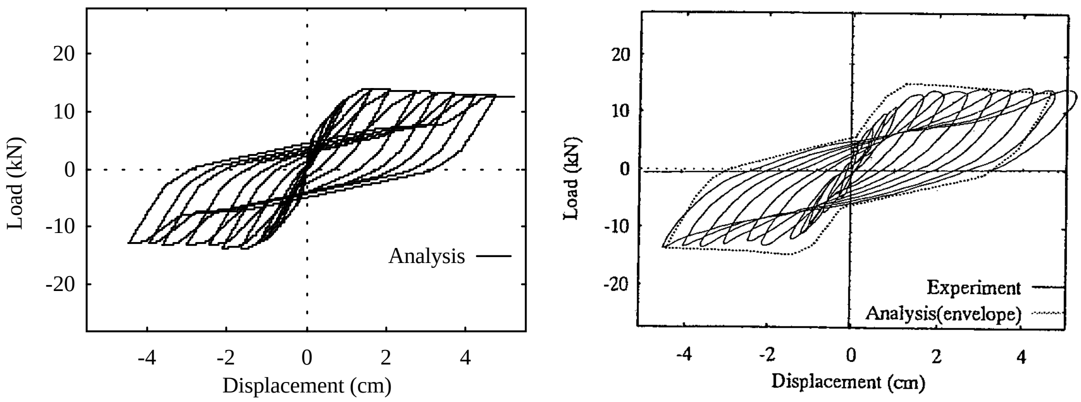
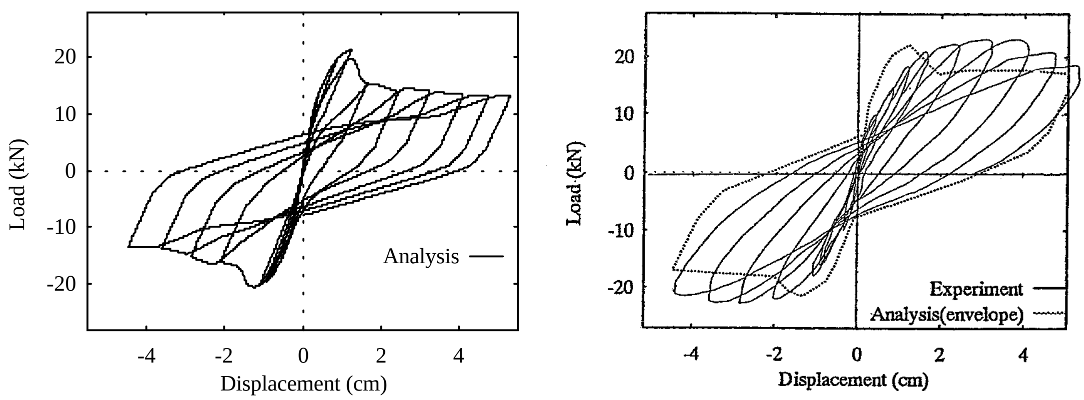


Figure 7: Reloading/unloading, T2



(a) **(b)**

Figure 8: Specimen T1



(a) **(b)**

Figure 9: Specimen T2

For the case of cyclic loading, reinforcement unloading/reloading branches are having major effect on the shape and size of the area bounded by the hysteric curve. The amount of hysteric energy released can be controlled by the application of the proper slopes of those branches of reinforcement material model and is studied here. Firstly elastic unloading/reloading has been implemented for the simulation of specimen T1 and T2. It was found that analyses showed much higher hysteric energy as compare to experiments. Various parametric studies were conducted using different characteristic of the envelope curve and with different slopes of the unloading branches of the material model. It was realized that characteristic stress-strain curve of concrete has lesser effect. It was also realized that adopting bilinear unloading/reloading in the reinforcement material model in such a way that the slope of unloading or reloading decreased from E_s to $0.1E_s$ from midway between the tension and compression envelope shows nice simulation of the experimental results. This is basically to take care of the Bauchinger effect and other phenomenon.

Figure 6 and Figure 7 show the analytical results of the implementation of bilinear unloading curve ($E_{s4} = 0.1E_s$) in comparison to the elastic unloading ($E_{s4} = E_s$). Figure 8 and Figure 9 show the analytical results adopting the bilinear model ($E_{s4} = 0.1E_s$) in comparison to the experimental results. Figure 8(a) and Figure 9(a) show the analytical results whereas Figure 8(b) and Figure 9(b) show the experimental results along with the outside envelope of the analytical results. It is noticed that the adoption of modified bilinear model ($E_{s4} = 0.1E_s$), the shape of the unloading branches and the hysterics energy matched with the experimental results. The shape of the load displacement diagram envelope after the yielding of reinforcement and the peak load is matching with that of the experimental results for specimen T1 whereas in case of T2 only the peak load matched. This implies that further attention is necessary for the T2 where axial load is present and higher confinement is expected for the concrete stress-strain curve.

CONCLUSIONS

A simple method for analysis of reinforced concrete members has been proposed based on finite difference technique. In this paper, the algorithm of this analytical tool is presented. Nonlinear material properties are adopted and the effect of few parameters of the material models on the analytical results is studied. Finally using the appropriate material parameter values, the analysis is done. The following conclusions can be made:

1. This analytical method can predict the peak strength quite appropriately
2. The characteristic of unloading/reloading branch of reinforcement plays prominent role in member hysteric behavior. Nice matching between the reloading/unloading branches was observed when bilinear unloading ($E_{s4} = 0.1E_s$) is adopted. Hence it is realized that adopting a lower slope from some where in the middle is reasonable. This phenomenon is also observed in various other cases that are not reported here.

Finally, this method is very fast and efficient method of analysis of reinforced concrete members. This method is numerically very efficient and can be used to simulate experimental results for the better understanding of the phenomenon occurring in the damage process. This analytical tool can help to study the effect of implementation of various parameters of the material model. Hence, this analytical tool can also be used in a supporting role to the finite element analysis with three-dimensional implementation, as it is difficult to check the effect of various characteristics of the material stress-strain curve using the sophisticated methods.

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