THEORETICAL FORMULATION OF STRONG MOTION ATTENUATION BASED ON SEISMIC SOURCE MODELS

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SUMMARY

This paper focuses on the attenuation of rms acceleration in relation to distance from the source and the size of the earthquake represented by seismic moment. A theoretical attenuation relationship for rms ground acceleration is presented and compared with empirical attenuation formulas obtained using regression analysis. Based on this theoretical relation, we also derive closed form attenuation models for peak ground acceleration. The theoretical attenuation models for ground motion are verified using acceleration data from earthquakes recorded in Iceland during the period 1986 to 1997. The theoretical relation was shown to fit the recorded rms acceleration data adequately. The standard deviation of the data fit to the theoretical equations is comparable to that of the regression analysis models. The theoretical attenuation models presented here relate rms and peak acceleration to parameters of the extended Brune model (M_0, $\Delta\sigma$, β, ρ, κ) as well as the duration of the ground shaking, T_d. These models can be considered alternatives to empirical regression formulas, especially for regions where information about geological and geophysical properties exists, but few strong motion records are available.

INTRODUCTION

The attenuation of earthquake-induced ground motion has been dealt with thoroughly in the literature emphasising the use of regression-type models (see for instance Ambraseys et al., [1996]). In this paper we present a theoretical model of the attenuation of rms acceleration and its relation to the distance to source and the earthquake size represented by the seismic moment. The source is approximated using the modified Brune source model, and an exponential model is applied to describe the high frequency decay of the local earthquake spectra. Based on this model an attenuation relation for the peak ground acceleration is also derived. The attenuation relations are compared with available Icelandic data as well as regression formulas. The data used in this study were recorded by the Icelandic Strong Motion Network during the period 1986 to 1987 [Thórarinsson et al., 1999]. The theoretical foundation of this work is largely due to Hanks and McGuire [1981] and Boore [1983], which demonstrated that ground motions could be described by a stochastic model defined in terms of seismological parameters defining the source and the propagation process.

THEORETICAL ATTENUATION FORMULA

A suitable representation of the earthquake-induced ground acceleration, for engineering purposes, is furnished in Fourier spectral models. Different models of this type have been put forward and are available in the literature [Haskell, 1964; Brune, 1970; 1971]. In this study, a simplified model is adopted from a recently published paper by Ólafsson et al. [1998]. This model is based on a modified Brune spectrum [Brune, 1970; 1971] with an additional exponential term to account for anelastic attenuation. In the following, as an approximation, the high frequency part of the Fourier spectrum is assumed to be modelled by a simple exponential term where the decay is described by a frequency independent quality factor, Q [Anderson and Hough, 1984; Anderson, 1986]. It

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should be pointed out that more refined models for the shape of the high frequency part of earthquake spectra are available in the literature (see for example [Dainty, 1981], [Boore, 1986], [Baldvinsson et al., 1995] and [Boore, 1996]). The adopted simplified Fourier spectrum of intermediate and far-field ground acceleration of shear waves can be expressed as follows:

$$|A(\omega)| = \frac{2 M_0 R_{\theta\phi} C \omega^2}{4\pi\beta^3 \rho \mathcal{R} \,(1+(\omega/\omega_c)^2)} \mathrm{e}^{-\omega R/2Q\beta} \tag{1}$$

where ω is the circular frequency, the number 2 accounts for the free-surface effect, M_0 is the seismic moment, $R_{\theta\phi}$ is the radiation pattern, C is a reduction factor accounting for the partitioning of the energy into two horizontal components, $C = 2^{-1/2}$, Q is a path-averaged quality factor, $\mathcal{R}$ is a function representing geometrical spreading (defined below by Eq. (2)), R is the hypocentral distance given below (see Eq. (3)), β is the shear wave velocity, ω_c ($=2\pi f_c$) is the corner frequency, and ρ is the density of the crust.

The spreading function is assumed to be modelled by

$$\mathcal{R} = \begin{cases} R_0^{1-n}\ R^n & R_1 < R \le R_2 \\ R_0^{1-n}\ R_2^{n-1}\ R & R_2 < R \le R_3 \\ R_0^{1-n}\ R_2^{n-1}\ R_3^{0.5}\ R^{0.5} & R_3 < R \end{cases} \tag{2}$$

where

$$R = \sqrt{\Delta^2 + d^2} \tag{3}$$

Here, Δ is the epicentral distance, d is the hypocentral depth, and n ($1 < n < 2$), R_0, R_1, R_2 and R_3 are model parameters presented in the following for the study area. This model is an approximation of the spreading function given by Aki and Richards [1980] and is found to be reasonable for the study area, provided that the epicentral distances are not too small. Rögnvaldsson and Slunga [1993] have proposed a similar model for the study area.

The corner frequency is assumed to be defined as $\omega_c = 2.34\beta/r$, where r is the fault radius [Brune, 1970; 1971]. The radius of a circular fault is given according to Brune [1970; 1971] as $r = (7M_0/16\Delta\sigma)^{1/3}$, where $\Delta\sigma$ denotes the stress drop. This is deemed a reasonable approximation for the study area, at least for moderate-sized earth-quakes [Ólafsson et al., 1998].

The rms ground acceleration, a_{rms}, can be defined formally as follows:

$$a_{\mathrm{rms}} = \sqrt{\frac{1}{T_\mathrm{d}} \int_0^{T_\mathrm{d}} |a(t)|^2\, dt} \tag{4}$$

where t denotes the time, $a(t)$ is the ground acceleration series and T_d is the duration of ground shaking; that is, it is assumed that the significant motion is confined to a closed time interval, $t \in [0, T_\mathrm{d}]$.

Various methods have been suggested and applied to assess the duration, T_d (see for instance [McCann, 1980]). Here only two methods will be mentioned. First, the duration T_d is chosen as the time window containing the direct S-wave arrivals [Ólafsson et al., 1998; Ólafsson, 1999]. Second, the duration T_d is obtained as a specified fraction of the cumulative energy [Ólafsson and Sigbjörnsson, 1995]. In both cases the significant part of the motion, at least for specific engineering purposes, is assumed to occur within a closed time interval defined by the duration, T_d. It then follows that the limits of the integral in Eq. (4) can be redefined as the infinite open interval, $t \in\ ]-\infty, \infty[$. Assuming further a finite rms value and using the Parseval theorem (see for instance [Oppenheimer and Willsky, 1983]), the rms value in Eq. (4) can be expressed in terms of the Fourier spectrum, $A(\omega)$, as follows:

$$a_{\mathrm{rms}} = \sqrt{\frac{1}{2\,\pi}\int_{-\infty}^{\infty}\frac{|A(\omega)|^2}{T_{\mathrm{d}}}\,d\omega} \tag{5}$$

It is seen that the integrand may be interpreted as reflecting the overall power spectral density of the shaking. It is worth pointing out that predictions based on Eq. (5), after substituting Eqs. (1), are found reasonable for the study area [Ólafsson et al., 1998; Ólafsson, 1999].

The rms ground acceleration can be expressed as follows after substituting the Fourier spectrum of ground acceleration given by Eq. (1) into Eq. (5) and performing some algebraic manipulations:

$$a_{\mathrm{rms}} = \frac{1}{\sqrt{\pi T_{\mathrm{d}}}}\,\frac{M_0\,R_{\theta\phi}\,C}{2\pi\,\beta^3\,\rho\,\mathrm{R}}\,\omega_{\mathrm{c}}^{5/2}\left\{\int_0^{\infty}\frac{\varpi^4}{(1+\varpi^2)^2}\,e^{-\lambda\varpi}\,d\varpi\right\}^{1/2} \tag{6}$$

Here $\varpi = \omega/\omega_c$ is a dimensionless frequency, $\lambda = \kappa\omega_c$ is a measure of the anelastic attenuation, and $\kappa = R/Q\beta$ is the spectral decay parameter. Frequency dependent Q has been used in several studies. Anderson and Hough [1984] have found that a frequency independent Q that increases with depth could fit acceleration data equally well in most cases.

The integral in Eq. (6) is readily obtained as follows:

$$\Lambda = \int_0^{\infty}\frac{\varpi^4}{(1+\varpi^2)^2}\,e^{-\lambda\varpi}\,d\varpi = \frac{1}{\lambda}\,\Psi \tag{7}$$

where

$$\Psi = 1 - \frac{1}{2}\lambda\,ci(\lambda)\big(\lambda\cos(\lambda + 3\sin(\lambda)\big) - \frac{1}{2}\lambda\,si(\lambda)\big(\lambda\sin(\lambda - 3\cos(\lambda)\big) \tag{8}$$

Here the sine- and the cosine-integrals are given as [Gradshteyn and Ryzhik, 1980]

$$si(x) = -\int_x^{\infty}\frac{\sin t}{t}dt = -\frac{\pi}{2} + \int_0^x \frac{\sin t}{t}\,dt \tag{9}$$

$$ci(x) = -\int_x^{\infty}\frac{\cos t}{t}dt = \gamma + \ln(x) + \int_0^x \frac{\cos t - 1}{t}\,dt \tag{10}$$

where γ is the Euler constant ($\gamma \approx 0.5772$).

Hence, the rms ground acceleration can be expressed as follows:

$$a_{\mathrm{rms}} = \frac{1}{\sqrt{\pi T_{\mathrm{d}}}}\,\frac{M_0\,R_{\theta\phi}\,C}{2\pi\,\beta^3\,\rho\,\mathrm{R}}\,\omega_{\mathrm{c}}^{5/2}\sqrt{\Lambda} \tag{11}$$

This equation can be restated in a more convenient way after substituting an equation relating corner frequency to the stress drop and moment magnitude [Ólafsson and Sigbjörnsson, 1999]. The result is as follows, using base 10 logarithm:

$$\log(a_{\mathrm{rms}}) = \log\left\{\frac{2.34^2\,(16/7)^{2/3}\,\Delta\sigma^{2/3}\,R_{\theta\phi}\,C}{2\pi^{3/2}\,\beta\,\rho\,\kappa^{1/2}}\right\} + \frac{1}{2}\log\left\{\frac{\Psi}{T_{\mathrm{d}}}\right\} + \frac{1}{3}\log(M_0) - \log(\mathrm{R}) \tag{12}$$

The second term on the right hand side includes the Ψ-function, which depends on the corner frequency and hence the seismic moment. Furthermore, Ψ is a function of the anelastic attenuation properties of the material carrying the seismic waves. For large earthquakes it is seen that $\Psi \to 1$ when $\lambda \to 0$. This term also includes the duration, T_d, which increases with increasing seismic moment. As a hypothesis, it is here suggested that $T_d \propto \Psi$ holds as a crude approximation for small- to moderate-sized earthquakes with seismic moments in the range, $M_0 = 10^{21}$-10^{25} dyn cm. In this case, the second term equals a constant, and the above model resembles some of the common attenuation formulas found in the literature (see for instance [Boore, 1983]).

The Λ-function given in Eq. (7) can be approximated fairly well in log-log scale by a multi-linear function [Ólafsson and Sigbjörnsson, 1999]. This yields the following approximation:

$$\Lambda = c_i\,\lambda^{n_i} \qquad \text{for} \qquad \lambda \in \left[\lambda_{i-1};\lambda_i\right[\tag{13}$$

where c_i and n_i are constants characterising the integral in the i-th interval, defined by λ_{i-1} and λ_i. The number of intervals should be defined such that a necessary accuracy is achieved. Furthermore, it should be pointed out that $c_i = 1$ and $n_i = -1$ for small values of λ, that is, if $\lambda << 1$. After carrying out this approximation Eq. (12) can be restated as follows:

$$\begin{aligned}\log(a_{\mathrm{rms}}) = \log&\left\{\frac{2.34^{(5+n_i)/2}(16/7)^{(5+n_i)/6}c_i^{\,1/2}\kappa^{n_i/2}\Delta\sigma^{(5+n_i)/6}\,R_{\theta\phi}\,C}{2\pi^{3/2}\,\beta^{(1-n_i)/2}\,\rho T_{\mathrm{d}}^{\,1/2}}\right\}\\ &+\frac{1-n_i}{6}\log(M_0)-\log(\mathcal{R})\end{aligned} \tag{14}$$

This attenuation relation is especially useful for a dataset of earthquakes with magnitudes in a rather narrow range. It is then possible to use a single equation that represents the data with reasonable accuracy.

A single approximation can also be used for a dataset of earthquakes with a broad magnitude range. It then seems preferable to approximate the Ψ-function. One of the simplest functions is the following:

$$\Psi = \exp\left(-\xi\,\lambda^{\eta}\right) \tag{15}$$

The parameters $\xi = 1.4565$ and $\eta = 0.8369$ give a fair approximation for $\lambda < 1$.

VERIFICATION

The attenuation relation of Eq. (12) was applied to the available Icelandic data, assuming $T_d = 4\Psi$ for the small and moderate-sized earthquakes. The resulting equation is

$$\log(a_{\mathrm{rms}}) = \log\left\{\frac{2.34^2(16/7)^{2/3}\Delta\sigma^{2/3}\,R_{\theta\phi}\,C}{4\pi^{3/2}\,\beta\rho\kappa^{1/2}}\right\}+\frac{1}{3}\log(M_0)-\log(\mathcal{R}) \tag{16}$$

Here, the following values are applied for the study area [Ólafsson, 1999]: $\beta = 3.3\times10^5$ cm/s for shear wave velocity, crustal density of $\rho = 2.7$ g/cm^3, stress drop of $\Delta\sigma = 50\times10^6$ dyn/cm^2, $\kappa = 0.04$ s for the spectral decay parameter, $R_{\theta\phi} = 0.63$ for the mean radiation pattern, and a factor $C = 2^{-1/2}$ to account for the partitioning of the energy into two horizontal components. The function representing geometrical spreading, $\mathcal{R}$, given in Eq. (2) is used, taking $n = 1.41$, $R_0 = 14\times10^5$ cm, $R_1 = 5\times10^5$ cm and $R_2 = 15\times10^5$ cm. The results are plotted in Figure 1.

A theoretical attenuation relation for peak acceleration was obtained by inserting $a_{\max} = p a_{\mathrm{rms}}$ into Eq. (16), which gives the following equation:

$$\log(a_{\max}) = \log\left\{\frac{p2.34^2(16/7)^{2/3}\Delta\sigma^{2/3}\,R_{\theta\phi}\,C}{4\pi^{3/2}\,\beta\rho\kappa^{1/2}}\right\}+\frac{1}{3}\log(M_0)-\log(\mathcal{R}) \tag{17}$$

The peak factor, p (Vanmarke and Lai [1980]), was calculated for the dataset. The predominant period, T_0, was calculated by counting zero crossings of the acceleration series within the time window, T_d. The average predominant period for the dataset was $\langle T_0 \rangle = 0.17$ s, and the average time window was $\langle T_d \rangle = 2.0$ s. The peak factors obtained were $p = 2.82$ for 0.37 probability of non-exceedance and $p = 2.94$ for a 0.5 probability of non-exceedance. The results are plotted in Figure 2.

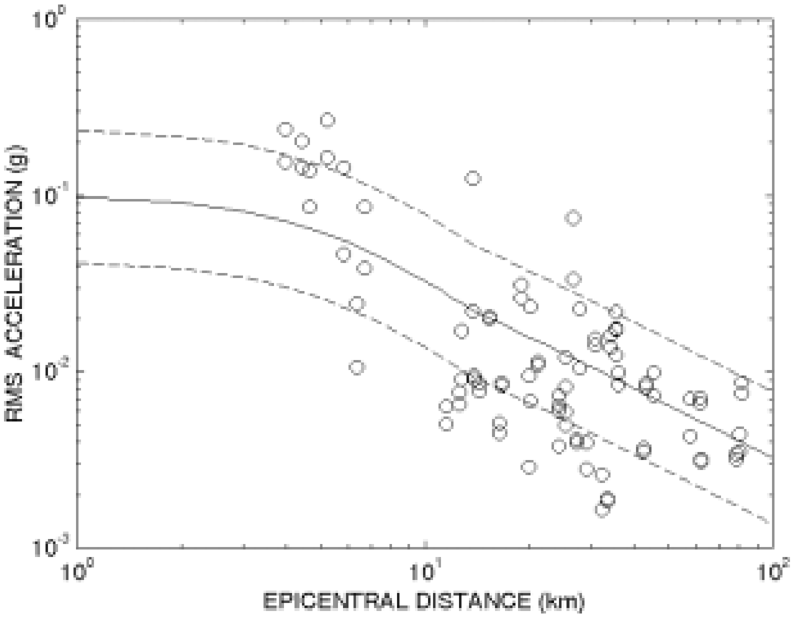

Figure 1: Rms acceleration data (circles) and the theoretical model of Eq. (16) (solid curve), plotted here for an earthquake of magnitude $M_w = 6.0$. The dashed curves represent theoretical values +/– one standard deviation ($\sigma = 0.375$).

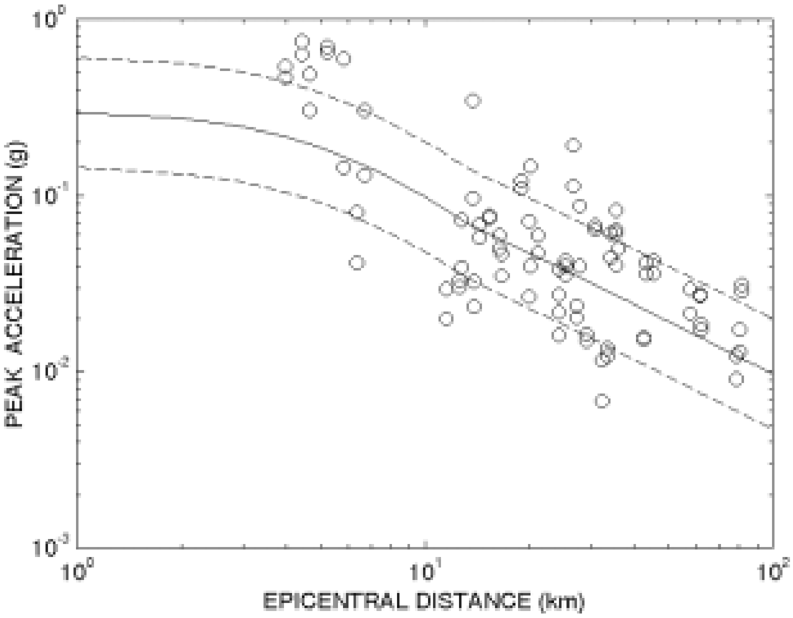

Figure 2: Peak acceleration data (circles) and the theoretical model of Eq. (17) (solid curve), plotted here for an earthquake of magnitude $M_w = 6.0$, and peak factor $p = 2.94$. The dashed curves represent theoretical values +/– one standard deviation ($\sigma = 0.317$).

In Figure 3, the attenuation of peak ground acceleration for an earthquake of magnitude M_w = 6.0 (corresponding to M_S = 5.85 [Ambraseys and Free, 1997]) obtained using the theoretical relation of Eq. (17) is compared with the attenuation obtained using other formulas. In Figure 3, the attenuation obtained from the Icelandic dataset using linear regression is represented with a dashed curve. Furthermore, attenuation obtained for rock sites in Europe by Ambraseys et al. [1996] is represented by a dotted curve. The theoretical relation of Eq. (17) was adjusted with a factor 1.29 (obtained from the dataset), to compensate for including both rather than the larger of two components.

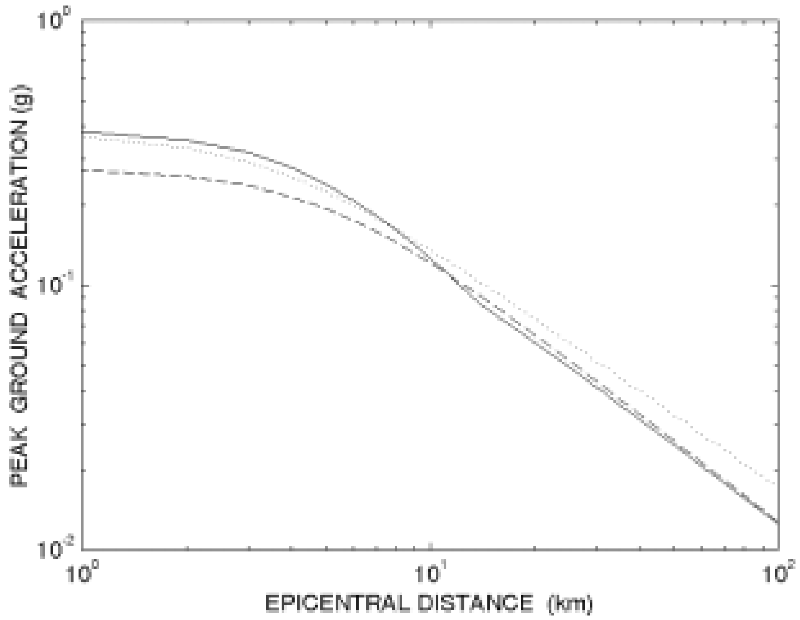

Figure 3: Comparison of attenuation relation, peak ground acceleration vs. epicentral distance, for an earthquake of magnitude M_w = 6.0. Results obtained using the theoretical attenuation relation of Eq. (17) (solid curve) and formula obtained from the Icelandic dataset using linear regression analysis (dashed curve). Attenuation obtained for rock sites in Europe by Ambraseys et al. [1996] (represented by dotted curve).

The relation from Ambraseys et al. [1996] is derived using the larger component of horizontal acceleration and are estimated using a linear regression method. The theoretical relation gives similar results as the curve from Ambraseys et al. [1996] for shorter distances, but gives a somewhat lower acceleration in the interval 15 to 100 km (by a factor 1.33 at 50 km). The difference in attenuation in the distance range of 15 to 100 km was found to be even greater for smaller magnitude earthquakes.

DISCUSSION AND CONCLUSIONS

A theoretical model has been derived to describe the attenuation of ground motion by using the Parseval theorem and the modified Brune spectrum with an additional exponential term to account for anelastic attenuation. By using source parameters obtained from the analysis of acceleration records from 17 moderate-sized earthquakes in Iceland ($M_0 = 1.4\times10^{21}$ to 7.1×10^{24} dyn cm), the theoretical model is shown to fit the recorded rms acceleration data adequately. The standard deviation of the data fit to the theoretical equations was found to be comparable to that of the regression analysis models.

A theoretical model for peak acceleration was arrived at by inserting $a_{max} = pa_{rms}$ into the relation for rms acceleration. Using the estimated value for probability of non-exceedance equal to 0.5, a good fit to the measured peak acceleration was obtained.

The seismic moment is used in the attenuation models instead of magnitude because we feel it is a measure consistent through the range of earthquake sizes and it is a parameter directly related to the physics of the earthquake process. A geometric spreading function is used to approximate the higher rate of attenuation than $1/R$ at distances closer to the source. An even more detailed model would also take into account the size of the earthquake where the parameters R_0, R_1, R_2 and R_3 could be taken as a function of M_0. However, the most notable effect of using the simplified geometric spreading function is that it gives an apparently more reliable estimate of the ground motion for sites close to the earthquake source.

Due to limited data, the relations presented here can only be used with sufficient confidence for a certain range of distances and magnitudes. More information is needed concerning how duration and stress drop are related for larger magnitudes. The assumption of $T_d = 4\Psi$ is a fair approximation for the small- and medium-sized earthquakes considered here. For larger magnitude earthquakes, $\Psi \rightarrow 1$ and the approximation are no longer valid.

A comparison of the attenuation of peak acceleration obtained in this study with results from Europe and western North America seems to indicate a somewhat greater attenuation in Iceland than in other areas for the distance range of 15 to 100 km. This can at least partly be attributed to the geological structure of our study area.

The theoretical attenuation models presented here relate rms and peak acceleration to parameters of the extended Brune model (M_0, $\Delta\sigma$, β, ρ, κ) as well as the duration, T_d. These models have been found to provide a good fit to rms and peak acceleration data from medium-sized earthquakes recorded in Iceland, based on source parameters estimated for the region. The models can be considered an alternative to empirical regression formulas, especially for regions where information about geological and geophysical properties exists, but few strong motion records are available.

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