FUNDAMENTAL STUDY OF DYNAMIC RESPONSE OF POROUS MEDIA WITH RANDOM VARIABLES

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SUMMARY

Dynamic response of porous media where porosity and permeability coefficient are varied in space is discussed. Dynamic behaviour of porous media is expressed by Biot's theory. Porosity and permeability coefficient are defined by multivariable normal density function. Porosity and permeability coefficient are also connected by correlation coefficient. One-dimensional wave propagation of P wave is studied by Monte Carlo technique in which one calculation makes 100 runs. As a result, although the random permeability coefficient gives little effect to surface amplification of soil, the random porosity influences much more. In case that both porosity and permeability coefficient are random variables, the difference of the amplification ratio becomes greater in relatively high frequency compared to the homogeneous soil. Especially the effect of randomness of porosity is great. Correlation between porosity and permeability coefficient doesn't play an important role on wave propagation of P wave. Fluctuation of permeable coefficient doesn't affect on wave propagation of P wave, but fluctuation of porosity gives clear effect to it. Large fluctuation of porosity reduces wave amplification and makes the resonance frequencies move toward lower range.

INTRODUCTION

It is getting more and more important to understand dynamic behaviour of soil, and soil should be modelled more precisely for that purpose. Generally soil can be considered to be an porous medium consisting of solid phase (soil skeleton) and fluid phase (water and gas). When pore is filly occupied by water, soil is saturated. On the other hand, when gas is contained in pore water, soil is unsaturated. The effect of pore water can be observed especially when soil is subjected to strong ground motion and liquefaction occurs. Biot originally presents a theoretical formulation of wave propagation of porous media [Biot, 1956]. Based on Biot's theory, dynamic behaviours of soil skeleton and pore water are connected by interaction of viscous force through permeable coefficient that plays very important role along with porosity in porous media. Using Biot's theory, porosity and permeability coefficient should be introduced into filed equations. Those parameters are difficult to be determined as well as other physical properties as density, elastic wave velocities etc. It is pointed out that all soils have random variations in the values of parameters in space. There are many practical and theoretical researches on the fluctuation of soil properties and its effect to dynamic characteristics of soil. Hydrogeologic parameters like permeability coefficient, porosity and compressibility are also dealt with as random variables in water flow analysis. Many researches are done to analyse water flow in soil where hydrogeologic parameters are randomly fluctuated in space. But almost all the problems in water flow analysis are considered static or quasi-static. Study of dynamic response of soil considering hydrogeologic parameters to be random variables can be hardly found in earthquake engineering in spite of its importance. In this paper, we study the dynamic response of porous media considering the fluctuation of hydrogeologic parameters (porosity and permeability coefficient) in space. Basic equations are described by Biot's theory. Random variables are followed by normal distribution, and generated by multivariate normal density function [Freeze 1975]. Characteristics of one-dimensional wave propagation of P wave in saturated porous soil are studied by Monte Carlo technique. The effects of correlation of parameters and their fluctuation to the dynamic response are discussed.

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METHOD

Field equations

Porous media can be expressed as a mixture of the solid phase (soil skeleton) and the fluid phase (pore water and gas). Biot [Biot 1956] gives the equations governing the dynamic behaviour of porous media. Biot's equations are written as below [Simon et al. 1986].

$$(1-n)\mathbf{L^T}\sigma \ \ +(1-n)\rho_s b+k^{-1}n^2(\dot{U}-\dot{u})-(1-n)\rho \ \ \ddot{u}=0 \tag{1}$$

$$n\nabla\pi+n\rho_f b-k^{-1}n^2(\dot{U}-\dot{u})-n\rho_f\ddot{U}=0 \tag{2}$$

If total stress σ is expressed by $\sigma=(1-n)\sigma_s+n\pi m$, σ_s is the effective stress, n is porosity, and π is pore pressure, which is positive value for tension. The vector b is the body force per unit volume, and this is usually neglected in dynamic response analysis. If ρ_s is the density of soil particle and ρ_f is the density of pore water, the bulk density of mixture ρ can be written as $\rho=(1-n)\rho_s+n\rho_f$. k is the permeability coefficient and expressed by Darcy's permeability coefficient k_0 , $k=k_0/\rho_f g$. The formulation above is so-called $u-U$ formulation where a superposed dot means a time derivative, and $u^T=\{u_1,u_2,u_3\}$, $\sigma^T=\{\sigma_{11},\sigma_{22},\sigma_{33},\sigma_{12},\sigma_{23},\sigma_{31}\}$, $m^T=\{1,1,1,0.0,0\}$ has the same meanings of the Kronecker delta, $\nabla^T=\{\partial/\partial x_1,\partial/\partial x_2,\partial/\partial x_3\}$, and L^T is written as below [Simon 1986].

$$L^T=\begin{bmatrix} \partial/\partial x_1 & 0 & 0 & \partial/\partial x_2 & 0 & \partial/\partial x_3 \\ 0 & \partial/\partial x_2 & 0 & \partial/\partial x_1 & \partial/\partial x_3 & 0 \\ 0 & 0 & \partial/\partial x_3 & 0 & \partial/\partial x_2 & \partial/\partial x_1 \end{bmatrix} \tag{3}$$

The total stress σ effective stress σ_s and pore water pressure π can be described as below.

$$\sigma=(D+\alpha^2Qmm^T)e+\alpha Qm\zeta \tag{4}$$

$$\sigma_s=\{D+(\alpha-n)^2Qmm^T\}e+n(\alpha-n)Q\varepsilon \tag{5}$$

$$-\pi=(\alpha-n)Qm^Te+nQ\varepsilon \tag{6}$$

where D is the drained material stiffness matrix, e is the strain of soil skeleton, ε is the stain of pore water, ζ is the strain of the relative displacement of water w defined as $w=n(U-u)$, α and Q are defined using the bulk modulus of soil particle K_s, pore water K_f and soil skeleton K_d as below.

$$\alpha=1-K_d/K_s \tag{7}$$

$$1/Q=n/K_f+(\alpha-n)/K_s \tag{8}$$

The value α and Q vary $n<\alpha<1$, $0<Q<\infty$. When soil particle is considered incompressible, K_s is assumed to be ∞. When pore water is completely incompressible, K_f is infinity. Contrarily if pore water is compressible, K_f has some value and unsaturated soil can be considered.

FEM formulation

Discretized equations are organised by applying Galerkin method to equation (1) and (2). Displacement of each element can be expressed by displacement of nodes by shape functions.

$$u = N_u \bar{u} \quad e = B_u \bar{u} \quad \dot{u} = N_u \dot{\bar{u}} \quad \ddot{u} = N_u \ddot{\bar{u}} \tag{9}$$

$$U = N_U \overline{U} \quad \zeta = B_U \overline{U} \quad \dot{U} = N_U \dot{\overline{U}} \quad \ddot{U} = N_U \ddot{\overline{U}} \tag{10}$$

where N_u and N_U are shape functions, and B_u and B_U are defined by $B_u = LN_u$ and $B_U = \nabla^T N_U$.
The discrete dynamic equations are obtained as below.

$$\begin{bmatrix} m_{uu} & 0 \\ 0 & m_{UU} \end{bmatrix} \begin{Bmatrix} \ddot{\bar{u}} \\ \ddot{\overline{U}} \end{Bmatrix} + \begin{bmatrix} c_{uu} & c_{uU} \\ c_{Uu} & c_{UU} \end{bmatrix} \begin{Bmatrix} \dot{\bar{u}} \\ \dot{\overline{U}} \end{Bmatrix} + \begin{bmatrix} k_{uu} & k_{uU} \\ k_{Uu} & k_{UU} \end{bmatrix} \begin{Bmatrix} \bar{u} \\ \overline{U} \end{Bmatrix} = \begin{Bmatrix} f_u \\ f_U \end{Bmatrix} \tag{11}$$

where each element is described as below.

$$m_{uu} = \int_l N_u{}^T (1-n)\rho_s N_u dl \quad m_{uU} = \int_l N_u{}^T n\rho_f N_u{}^T dl \tag{12}$$

$$c_{uu} = \int_l N_u{}^T (n^2 k^{-1}) N_u dl \quad c_{uU} = c_{Uu}{}^T = -\int_l N_u{}^T (n^2 k^{-1}) N_U{}^T dl \quad c_{UU} = \int_l N_U{}^T (n^2 k^{-1}) N_U{}^T dl \tag{13}$$

$$k_{uu} = \int_l B_u{}^T [D + (\alpha - n)^2 Q m m^T] B_u{}^T dl \; k_{uU} = k_{Uu}{}^T = \int_l B_u{}^T n(\alpha - n) Q B_U dl \; k_{UU} = \int_l B_U{}^T (n^2 Q) B_U dl \tag{14}$$

$$f_u = \int_l N_u{}^T \sigma_{zz} dl \quad \sigma_{zz}\text{: effective stress} \quad f_U = \int_l N_U{}^T n\pi dl \tag{15}$$

Random model of hydrogeologic parameters

Freeze studies one-dimensional water flow analysis using multivariate probable model where porosity, permeable coefficient and compressibility are assumed random variables [Freeze 1975]. In this study, two parameters (permeability coefficient k and porosity n) are assumed random variables that are dependent each other. Random variables are defined to follow multivariate normal density function [Freeze 1975]. Permeability coefficient k is expressed as log function of k as $y = log_{10} k$. The correlation between the parameters is considered by correlation coefficient ρ_{yn}. If correlation function $R(\xi)$ is defined, spectral density function $S(\xi)$ is obtained by Wiener-Khintchine relation.

$$S(\xi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-i\xi x) R(x) dx \tag{16}$$

Here we have multivariate normal density function f_j and normal distribution g_j [Kanda and Motosaka 1995]. ξ is wave number x is space and ϕ_{il} is random number generated between 0 and 1. N is number of divisions of ξ and m is space dimension.

$$f_j(x) = \prod_{k=1}^{m} (2\Delta\xi_k)^{1/2} \sum_{i=1}^{j} \sum_{l=1}^{N} H_{ji}(\xi_l) \cos[\xi_l x + \phi_{il}] \tag{17}$$

$$g_j(x) = \mu + \sigma f_j(x) \tag{18}$$

where H_{ij} is defined like,

$$H_{11} = \sqrt{S_{11}} \quad H_{ij} = (S_{ij} - \sum_{k=1}^{j-1} H_{ik} H_{jk}) / H_{jj} \quad (i > j) \quad H_{jj} = \sqrt{S_{ii} - \sum_{k=1}^{i-1} H_{ik}}^{\,2} \quad (i > 1) \tag{19}$$

If correlation function $R(\xi)$ is assumed gaussian type function,

$$R(\xi) = exp\{-(\frac{\xi d}{2})^2\} \tag{20}$$

If normal distribution $N[\mu_x, \sigma_x]$ is defined by average value μ_x and standard deviation σ_x, each random variable is connected by the relations described below [Freeze 1975].

$$N[\mu_n, \sigma_n] \quad N[\mu_y + a, b\sigma_y] \tag{21}$$

where a and b are defined in the form of.

$$a = \rho_{yn} \frac{\sigma_y}{\sigma_n} (g_n - \mu_n) \tag{22}$$

$$b = (1 - \rho_{yn}^{\ 2})^{1/2} \tag{23}$$

NUMERICAL EXAMPLES

One-dimensional wave propagation for P wave incident problems is calculated. The configuration of the soil model is shown in Fig.1. Actual fluctuation of soil parameters is not well known, so soil parameters are referred to some literature [Taga and Togashi 1976] [Kazama and Nogami 1992] [Tanaka et al 1995]. In spite of the difficulty of indicating the value ρ_{yn}, the sign of ρ_{yn} can be determined. It is known that clays have low permeability and high compressibility, and sands have high permeability and low compressibility. Thus ρ_{yn} can be taken negative from the similar analogy.

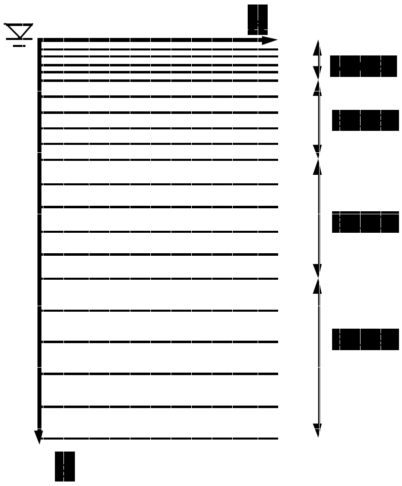

Figure 1: Configuration of the soil model

Shown in Fig.1, for 29.5m-depth soil divided into 30-layer lying on the half space. One calculation takes 100 Monte Carlo runs. One example of random porosity and permeability coefficient is shown in Fig.2 and Fig.3. The parameters needed in this study are set; weight density of soil ρ_s; 2.23 tf/m3, weight density of water ρ_f; 1.0 tf/m3, S wave velocity V_S; 100m/s2, P wave velocity V_P; 300 m/s2 , bulk modulus of soil K_s; 37.00×104 t/m2 , bulk modulus of pore water K_f ; 21.37×103 t/m2, average of porosity μ_n ; 0.3, standard deviation of porosity; 0.05, 0.10, average of permeability coefficient μ_y ; -1 (μ_k =10-1cm/s), standard deviation of permeability coefficient σ_y; 1, 2 (σ_k =101 cm/s , 102 cm/s), correlation coefficient ρ_{yn}; -0.3, -0.9, correlation distances of porosity and permeability coefficient d_n , d_y ; 20 m .

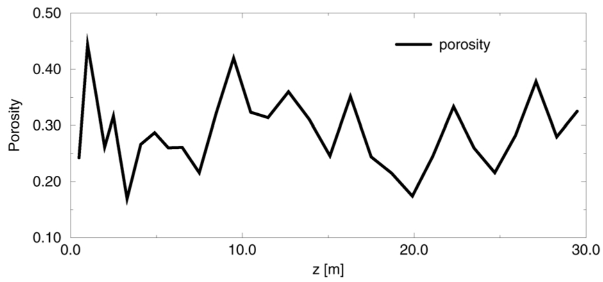

Figure 2:A sample of porosity distribution

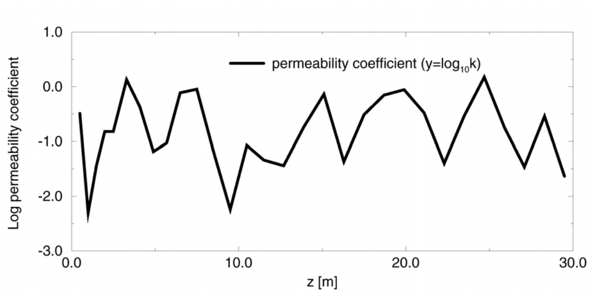

Figure 3:A sample of permeability coefficient distribution

RESULTS AND DISCUSSIONS

In this study, the influence of the fluctuation of porosity and permeability coefficient, and their correlation to one-dimensional wave propagation of P wave are discussed. Figure 4 shows the amplification ratios of the soil considering both porosity and permeability coefficient random variables with respect to different correlation coefficients. The difference of correlation coefficient doesn't give significant effect on the amplification ratio of wave. Figure 5 shows the amplification ratios of the soil with the random porosity compared to the soil with the random permeability coefficient. A significant difference can not be found in the amplification ratio of the soil with the random permeability coefficient. In case of the random porosity, the amplification ratio is affected greater than the random permeability coefficient in high frequency. Although the fluctuation of the permeability coefficient doesn't give large influence to the amplification of wave in soil, the fluctuation of porosity can give some influence. Figure 6 shows the amplification ratios of the soil in case that both porosity and permeability coefficient are random, compared to the homogeneous soil with respect to different standard deviations of permeability coefficient. The difference of the standard deviation of permeability coefficient doesn't seem to play an important role on wave propagation. The amplification ratios of the soil with respect to different standard deviations of porosity are shown in Figure 7. There can be seen some effect of the different standard deviation. The amplification ratio of wave in saturated soil becomes less and the resonance frequency is shifting toward slightly lower in case of large standard deviation. That means large fluctuation seems to make bulk modulus of porous soil less.

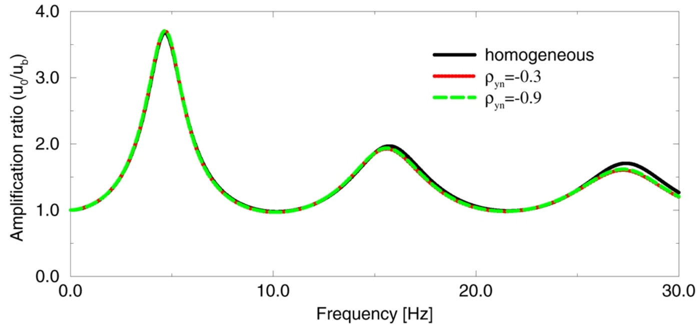

Figure 4: Amplification ratio with different correlation coefficient

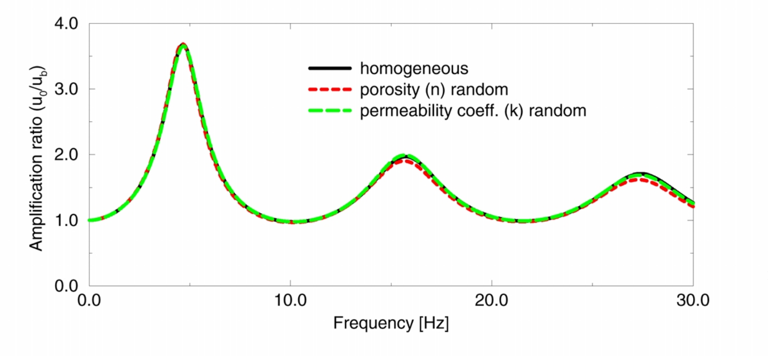

Figure 5: Amplification ratio with random variables

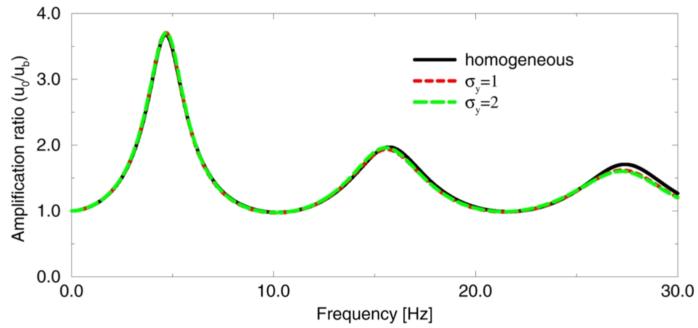

Figure 6: Amplification ratio with fluctuation of permeability coefficient

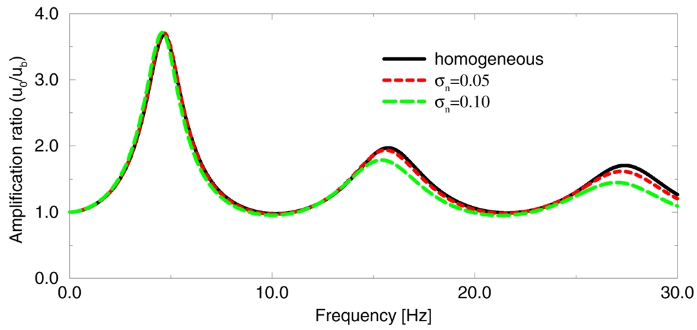

Figure 7: Amplification ratio with fluctuation of porosity

CONCLUSIONS

In this study, one-dimensional wave propagation of porous soil is studied. Soil is assumed random media in which porosity and permeability coefficient are varied in space. Random model is based on the multivariate normal distribution [Freeze 1975]. To focus the vertical wave amplification, only P wave incident problem is discussed. Calculations are carried out by Monte Carlo technique. Surface amplifications of waves are shown to compare the effect of the fluctuation of parameters and the correlation between parameters. The results obtained here are summarised as below.

1) Although the random permeability coefficient gives little effect to amplification of wave in porous soil, the random porosity influences much more. In case of both porosity and permeability coefficient are random variables, the difference of the amplification ratio becomes greater especially in relatively high frequency.

2) Correlation between porosity and permeability coefficient doesn't play an important role on wave propagation of P wave.

3) Fluctuation of permeable coefficient doesn't affect on wave propagation of P wave, but fluctuation of porosity gives some effect to it. Large fluctuation of porosity reduces wave amplification and makes the resonance frequencies move toward lower range.

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