GENETIC ALGORITHM-BASED MULTI-OBJECTIVE STRUCTURAL CONTROL WITH ACCELERATION FEEDBACK

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SUMMARY

In this paper the genetic algorithm-based control methodology that have been proposed by authors is described. The results of three different controllers corresponding to different robustness criteria are presented and compared. Only the acceleration responses of scaled building model are used as feedback. To estimate the full state space of the system, the state space is reconstructed from the time delayed state feedback based on the embedding theorem.

INTRODUCTION

This paper presents the results and comparisons of three controllers developed by the genetic algorithm based control methodology (Kim and Ghaboussi 1999). As a model building three-story scale building suggested in the first benchmark problem (Spencer 1997) are used. Active mass driver system placed at the top of the scaled building has been modelled in the benchmark problem. As feedback only the acceleration responses of the building are used because the acceleration measurement is most reliable and least expensive compared with other measured responses such as velocity and displacement. However, just the acceleration measurements are not enough to represent full state responses of the system. To approach full state control performance, the proposed control method uses the state space reconstruction technique. The method is based on the Takens' embedding theorem that states the observed time series data can recover the full state space of the original system. This means the system can be estimated only by using measured time history responses instead of using uncertain system parameters. Accelerations of floors and an AMD are measured and used as feedback in this study.

To improve the controller's performance in real applications, the robustness of the controller to the unmodelled dynamics and signal noises should be considered in the design. However there are tradeoffs among the design objectives such as the response reduction, control efforts, and robust stability. Overall performance of the control system should be evaluated by those multiple objectives. Genetic algorithms are used to optimize the parameters of the proposed controllers. Nonlinear polynomial functions of the explicit design criteria are used as a multi-objective function in this study. The proposed method is applied to the vibration control of a seismically excited model building. Several controllers are developed with different design criteria. The results are compared and discussed.

A GENETIC ALGORITHM-BASED CONTROL METHODOLOGY

Genetic algorithms are optimization search methods that are inspired by natural selection and natural genetics. In GAs the parameters of the problem are coded as finite length strings which are composed of genes, and strings undergo evolution over several generations. GAs have three major operators designed to model the evolutionary forces such as selective reproduction, recombination and mutation. Compared with the traditional gradient based search methods, GAs are very simple but powerful search methods because GAs do not need to reformulate the problem to search a nonlinear and non-differentiable space. The flexibility in the formulation of the fitness function is also one of the advantages of using GAs. The fitness function can be formulated as a polynomial function of the output of the system to be optimized. Therefore multiple optimal design criteria can be considered easily.

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In the field of control design, GAs have been used successfully to obtain gains for optimal controller (Kundu and Kawata 1996), tune the weights of neuro-controllers (Lewis and Fagg 1992), and scale parameters of fuzzy controllers (Kim et al. 1995).

For the control of the civil structures, the GA based control method has been proposed (Kim and Ghaboussi 1999). The proposed control method estimates the system states from the observed time series data using the state space reconstruction technique that is based on the embedding theorem.

STATE SPACE RECONSTRUCTION FOR SYSTEM ESTIMATION

The *n*-dimensional reconstructed state space at time step t is defined as $\mathbf{W}^n(t)$ by the following equation in terms of the one-dimensional observed time series w(t) with time delay τ.

$$\mathbf{W}^{s\times n}(t) = \left[w(t) \quad w(t-\tau) \quad \ldots \quad w[(t-(n-1)\tau)] \right]^T \tag{1}$$

The reconstructed state space is not the same as the original state space, but it can characterize the dynamical properties of the original system for sufficiently large value of *n*. When the dimension of the original state space is *k*, a value of $n > 2k$ is recommended (Takens 1981).

CONTROLLER DESIGN

Evaluation Model and Control Constraints

Controllers are designed and tested on a benchmark problem. The structure considered in the benchmark problem is a scale model of a three-story building using an active mass driver as a control device. The state space parameters of this structural system, including the actuator and sensor dynamics, have been obtained from the experiment. More details on the benchmark problem can be found in the reference (Spencer et al. 1997).

Control constraints are placed on the system for a realistic numerical simulation. The RMS constraints and the peak response constraints are listed in Eq. (2), where σ represent the RMS value of its subscript.

$$\sigma_u \le 1 \text{ volt}, \ \sigma_{\ddot{x}_{am}} \le 2g, \text{ and } \sigma_{x_m} \le 3 \text{ cm} \tag{2}$$

$$|u|_{max} \le 3 \text{ volt}, \ |\ddot{x}_{am}|_{max} \le 6g, \text{ and } |x_m|_{max} \le 9 \text{ cm} \tag{3}$$

As additional constraints (control implementation constraints), sampling time is 0.001 seconds, computational time delay is 200 m seconds, A/D & D/A converter has 12 bit precision and a span of ±3 volts, and each of the measured responses contains an RMS noise of 0.01 volts.

Genetic Algorithm-Based Controller

We have chosen to use four sensors which measure the absolute accelerations of three floors, $\ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}$, and the absolute acceleration of the AMD mass, $\ddot{x}_{am}$. The feedback vector **y**(t) contains four sensor readings at time t. By using the reconstructed state feedback, we are using the current vector of sensor reading **y**(t) plus previous samples of sensor readings. Therefore, the dimension of the reconstructed state space of the four sensor feedback will be equal to 4×*n* with *n*-1 previous time histories. The controller also uses previous time histories of control signals as feedback.

$$\mathbf{Y}^{4\times n}(t) = \left[\mathbf{y}(t) \quad \mathbf{y}(t-\tau) \quad \ldots \quad \mathbf{y}[t-(n-1)\tau] \right]^T \tag{4}$$

In the remainder of this study, we have used the 23-dimensional reconstructed state space (m=3, n=5) which consists of 20-dimensional reconstructed state space vector $\mathbf{Y}^{4\times5}(t)$ and the 3-dimensional reconstructed state space vector $\mathbf{U}^3(t-\tau)$.

$$\mathbf{U}^3(t-\tau) = \begin{bmatrix} u(t-\tau) & u(t-2\tau) & u(t-3\tau) \end{bmatrix}^T \quad (5)$$

The control input is calculated from Eq. (5) with the additional constraint from the saturation of the actuator which requires that $|u| \le +3$ volts as a limit. The controller gain matrix $\mathbf{G}_R$ has 23 gains. The elements of the gain matrix $\mathbf{G}_R$ are optimised through evolution by using GAs.

$$u(t) = u(t-\tau) + \Delta u(t), \text{ where } \Delta u(t) = \mathbf{G}_R \begin{Bmatrix} \mathbf{Y}^{4\times5}(t) \\ \mathbf{U}^3(t-\tau) \end{Bmatrix} \quad (6)$$

Genetic Algorithm Parameters

The simple GA (Goldberg 1989) is used to optimise feedback gains. Ten bits are used to represent each gain as a real number by mapping, making the string length equal to 230 bits. The population size was 50 and the evolution was continued up to 1000 generations. Genetic operators used are fitness proportional (roulette wheel type) random reproduction, two point crossover at a rate of 0.8 and mutation at a rate of 0.003.

Fitness Function

The fitness function F is a nonlinear polynomial which consists of powered products of the normalized peak and RMS values of the responses of floors and the AMD. Each criterion in C_1 - C_3 has been designed to converge to 1.0 when the corresponding system response is reduced to zero.

$$C_1 = \prod_{i=1,3} \left(1+\frac{|\ddot{x}_{ai}|_{max}}{\beta_i}\right)^{\alpha_i} \cdot \left(1+\frac{|x_i|_{max}}{\delta_i}\right)^{\gamma_i} \cdot \left(1+\frac{\sigma_{\ddot{x}_{ai}}}{\zeta_i}\right)^{\varepsilon_i} \left(1+\frac{\sigma_{x_i}}{\theta_i}\right)^{\eta_i} \quad (7)$$

$$C_2 = \prod_{i=1,3} \left(1+\frac{P_1(|\ddot{x}_{am}|_{max})}{\beta_m}\right)^{\alpha_m} \cdot \left(1+\frac{P_2(|x_m|_{max})}{\delta_m}\right)^{\gamma_m} \cdot \left(1+\frac{\sigma_{\ddot{x}_{am}}}{\zeta_m}\right)^{\varepsilon_m} \cdot \left(1+\frac{\sigma_{x_m}}{\theta_m}\right)^{\eta_m} \quad (8)$$

$$C_3 = \left(1+\frac{\sigma_u}{\omega}\right)^{\psi} \quad (9)$$

For the evaluation of the fitness, peak accelerations, peak displacements, RMS accelerations and RMS displacements of the three floors and active mass driver and RMS value of control signal are used as the parameters of the cost function as in Eqs. (7) - (9). The denominators b, d, z, q, and w are the normalisation factors, and powers a, g, e, h, and y are the exponential weight factors used to adjust the weight of responses which are to be reduced according to the control objective. In this study the factors are chosen by trial and error as follows: bi=2.0, bm=1.0, di=dm=1.0, zi=2.0, zm=1.0, qi=qm=1.0, and w=1.0 for normalisation, and ai=1.0, am=3.0, gi=1.0, gm=2.0, ei=em=1.5, hi=hm=1.0, and y=1.0 for the exponential weight factors.

$$F=\frac{C_{ref}}{C_T}, \qquad C_T = \prod_{i=1,5} C_i \quad (10)$$

C_T in Eq. (10) is the total cost, and the fitness F is the inverse of the total cost with a normalisation factor C_{ref} (=1.0 in this study).

Penalty Function

The penalty function has been successfully used for solving the constrained optimisation problem by several researchers (Homaifar et al. 1994; Gray et al. 1995). This penalty function is employed to impose the benchmark problem's hard constraints, i.e. maximum displacement and acceleration of AMD. Functions P1 and P2 in Eq. (11)-(12) are the penalty functions.

$$P_1\left(|x_m|_{max}\right) = \begin{cases} |x_m|_{max} & \text{for } |x_m|_{max} \le 9 \text{ cm} \\ 50. & \text{for } |x_m|_{max} > 9 \text{ cm} \end{cases} \tag{11}$$

$$P_2\left(|\ddot{x}_{am}|_{max}\right) = \begin{cases} |\ddot{x}_{am}|_{max} & \text{for } |\ddot{x}_{am}|_{max} \le 6g \\ 50. & \text{for } |\ddot{x}_{am}|_{max} > 6g \end{cases} \tag{12}$$

EVALUATION CRITERIA

Ten evaluation criteria are defined in the benchmark problem (Spencer et al. 1997). These criteria are summarised in Table 1. The first five performance measures are RMS responses and the latter five are based on peak responses.

Table 1. evaluation criteria of benchmark problem

J1	J2	J3	J4	J5
$\max\limits_{\substack{\omega_g,\zeta_g \\ i=1,3}} \left\{ \frac{\sigma_{d_i}}{\sigma_{x_{30}}} \right\}$	$\max\limits_{\substack{\omega_g,\zeta_g \\ i=1,3}} \left\{ \frac{\sigma_{\ddot{x}_{ai}}}{\sigma_{\ddot{x}_{a30}}} \right\}$	$\max\limits_{\omega_g,\zeta_g} \left\{ \frac{\sigma_{x_m}}{\sigma_{x_{30}}} \right\}$	$\max\limits_{\omega_g,\zeta_g} \left\{ \frac{\sigma_{\dot{x}_m}}{\sigma_{\dot{x}_{30}}} \right\}$	$\max\limits_{\omega_g,\zeta_g} \left\{ \frac{\sigma_{\ddot{x}_{am}}}{\sigma_{\ddot{x}_{a30}}} \right\}$
J6	**J7**	**J8**	**J9**	**J10**
$\max\limits_{\substack{t,\ i=1,3 \\ \text{El Centro} \\ \text{Hachinohe}}} \left\{ \frac{\lvert d_i(t) \rvert}{x_{30}} \right\}$	$\max\limits_{\substack{t,\ i=1,3 \\ \text{El Centro} \\ \text{Hachinohe}}} \left\{ \frac{\lvert \ddot{x}_{ai}(t) \rvert}{\ddot{x}_{a30}} \right\}$	$\max\limits_{\substack{t \\ \text{El Centro} \\ \text{Hachinohe}}} \left\{ \frac{\lvert x_m(t) \rvert}{x_{30}} \right\}$	$\max\limits_{\substack{t \\ \text{El Centro} \\ \text{Hachinohe}}} \left\{ \frac{\lvert \dot{x}_m(t) \rvert}{\dot{x}_{a30}} \right\}$	$\max\limits_{\substack{t \\ \text{El Centro} \\ \text{Hachinohe}}} \left\{ \frac{\lvert \ddot{x}_{am}(t) \rvert}{\ddot{x}_{a30}} \right\}$

NUMERICAL RESULTS

Numerical simulations of the GA based controllers have been performed on the benchmark problem. Three controllers have been developed in this study. The one has been developed without a sensor noise (Case A), and other two controllers have been designed by adding noises (Case B & C) to the input and output of the controller to consider a sensor noise and the model uncertainty (Figure 1).

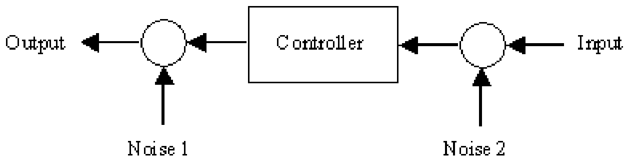

Figure 1. inclusion of noise for robustness consideration

The noise added to each of the input and output of the controller is a white noise span of ±0.1 volts in Case B and ±0.5 volts in Case C, which are 3.3 percent and 16.7 percent of the total span respectively. To develop each controller, only the El Centro earthquake excitation data provided by the benchmark problem have been used upto 5 seconds. The time delay t=0.001 seconds has been used for the state space reconstruction.

Table 2. comparisons of results using evaluation criteria

	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10
Case A	0.127	0.203	0.721	0.684	0.677	0.357	0.627	2.277	2.062	1.597
Case B	0.130	0.201	0.726	0.684	0.643	0.356	0.632	2.361	2.067	1.575
Case C	0.153	0.245	0.617	0.578	0.520	0.371	0.673	1.712	1.535	1.030
Uncont rolled	0.589	0.999	0.072	0.082	1.068	0.620	0.718	0.077	0.083	1.142

Table 2 shows the list of the evaluation results. Case A and Case B do not have many differences in the evaluation criteria, but there is a lot of gain reduction in the loop gain transfer function in high frequencies. In Case C there are some deterioration in inter-story drifts and floor accelerations as shown in criteria J1, J2, J6, and J7. However, control efforts are reduced very much instead. The loop gain transfer function of Case C has the lowest gains in high frequencies in this study. By increasing the noise levels at the input and output node of the controller while optimising the controller by GAs, more robust controllers have been developed as expected.

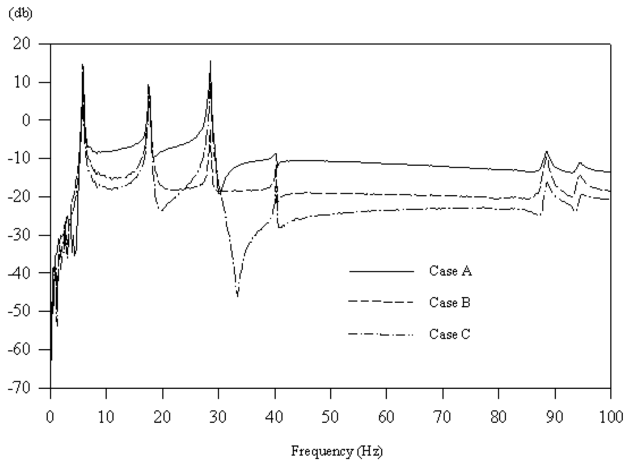

Figure 2. loop gain transfer function

CONCLUSIONS

In this paper three genetic algorithm based controllers have been designed and the results have been compared. Genetic algorithms are used to optimize the parameters of the proposed controllers with respect to the multiple design objectives. Only the accelerations are measured and used as feedback. Sensor noises and model uncertainties are considered (Case B & C) in the process of developing controllers to improve the controllers' robustness. Results shows the controllers' robustness is improved with some tradeoffs in the response reduction by increasing the noise levels at the input and output node of the controller while optimizing the controller.

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