EQUIVALENT LINEAR ANALYSIS CONSIDERING LARGE STRAINS AND FREQUENCY DEPENDENT CHARACTERISTICS

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SUMMARY

Equivalent linear dynamic response analysis has at least two shortages in evaluating the response of the surface ground. The one is overestimation of the peak acceleration and shear strength under large earthquake and the other is underestimation of peak acceleration that comes underestimation of amplification ratio in high frequency region. Since both shortages comes from the same cause, i.e., computing the effective strain from maximum strain, one cannot improve both of them in one time. The authors introduce the concept that effective strain is expressed in terms of frequency in order to improve both shortages at the same time. The accuracy and advantage of the proposed method is proved through the simulation of vertical array record at Industrial Institute of Science of the University of Tokyo during the 1987 Chiba-ken Toho-oki earthquake, in which proposed method shows much better agreement than other equivalent linear methods.

INTRODUCTION

Theoretically speaking, truly nonlinear dynamic response analysis, in which change of mechanical property is revised in each time increment, has more potential to simulate the dynamic response of the surface ground during earthquake than equivalent linear method, because, unlike the name of "equivalent", equivalent linear analysis is just an approximate method. Equivalent linear method has been used, however, especially in the field of engineering practice because of several reasons. One of them is simplicity in preparing the input data and stability of numerical analysis. These advantages are, however, not important in the point of view of the accuracy. In addition, for example, constitutive model that requires exactly the same information that equivalent linear analysis requires is proposed [Yoshida and Tsujino, 1993] and stable numerical integral scheme is proposed [Sakai et al., 1995]. In some textbook, it is written that computing time in equivalent linear analysis is shorter than nonlinear analysis because equivalent linear analysis is linear analysis and can reduce the computing time by FFT technique. This is, however, not true. According to author's experience and other researcher's experience, nonlinear analysis is much faster than equivalent linear analysis in almost all cases including two-dimensional analysis. Therefore, equivalent linear method does not have advantage at all compared with truly nonlinear method even in the point of view of easiness of data preparation and computing time, both of which are important in the engineering practice.

The equivalent linear method, however, has another advantage in different stand point of view. The first and very important one is deconvolution function by which incident wave to the engineering seismic base layer is computed from the earthquake record at the ground surface or any other point when multiple reflection theory is used such as SHAKE [1972]. The other advantage is consideration of frequency dependent characteristics of wave propagation such as damping due to scattering. It is easy to consider it in the frequency domain analysis used in the equivalent linear analysis, but difficult in the time domain analysis used in the nonlinear method. In. addition, nonlinear method has disadvantage in reproducing high frequency component partly because of the numerical damping in the numerical integration scheme and partly because artificial damping such as Rayleigh damping (Stiffness proportional damping) in order to suppress the high frequency response, all of which work to

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make response in high frequency component small. Therefore, nonlinear analysis is difficult to use in order to make the input motion for the structures with high predominant frequency.

Considering these situations, equivalent linear analysis and nonlinear analysis have a nature to compensate each other, therefore they should be used depending on the purpose. In this point, improvement of equivalent linear method is important. In this paper, we propose a method to improve equivalent linear method especially focusing on the response at large earthquake.

SHORTAGE IN EQUIVALENT LINEAR ANALYSIS

When focusing on the acceleration response, there are tow significant shortage in the conventional equivalent linear analysis.

One of the author already pointed out one types of shortage [Yoshida, 1984], which is schematically shown in Figure 1. In the figure, solid line indicates shear stress τ-shear strain γ curve computed from specified shear modulus G-shear strain γ relationship. In the equivalent analysis in frequency domain, we need G and damping ratio h. They are computed from effective strain γ_{eff}, and γ_{eff} is defined to be

$$\gamma_{eff} = \alpha \gamma_{max} \tag{1}$$

where γ_{max} is maximum strain. The coefficient α is a factor to convert from maximum strain to effective strain, which is called effective strain coefficient and may be referred just α hereafter. The value of α is known not to be constant in order to obtain good simulation with observed record [Osaki, 1982, for example], but α=0.65 is usually used in the engineering practice. If the value of α is smaller than unity, stress-strain curve used in the equivalent linear analysis is linear line O-A-C in Figure 1. Therefore, maximum shear stress at γ_{max} is not τ_2 which lies on the specified stress-strain curve, but τ_1 which is larger than τ_2. In other words, maximum shear strain versus maximum shear stress relationships is dashed line O-C, which is always in upper location than specified stress-strain curve. This disagreement may not be significant under small earthquake because the difference is small. Under the large earthquake, however, this difference is significant because shear strength is overestimated, which result in larger shear stress, then acceleration. The overestimation of shear stress reaches 1/0.65=1.54 at maximum under the perfectly plastic behavior.

This overestimation is known at the early stage of the research [Finn et al., 1978: Kokusho, 1982]. The reason was explained to be resonance effect because effective stress analysis is linear analysis in these researches. This explanation is, however, not correct as shown in the following example, and the mechanism shown above is the most realistic.

Acceleration is overestimated under large earthquakes in the equivalent linear analysis because of the above mechanism. On the other hand, second shortage of equivalent linear analysis is underestimation of acceleration, which is opposite tendency to the previous shortage. Figure 2 shows comparison of amplification ratio, i.e., the ratio of Fourier amplitude of acceleration at the ground surface from the engineering seismic base motion.

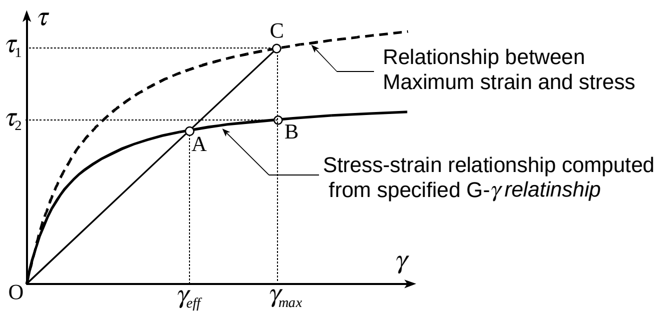

Figure 1: Schematic figure showing how equivalent linear method overestimate shear stress

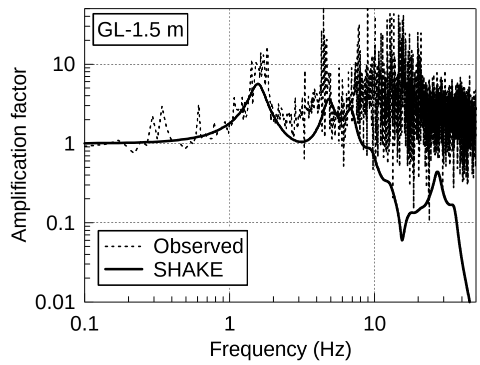

Figure 2: Comparison of amplification factor between SHAKE and vertical array observation at Tokyo Bay area

SHAKE underestimates the amplification ratio significantly compared with observed record. This is not the phenomena observed at this particular site but observed commonly. Here, it is noted that, throughout this paper, the name SHAKE is used not as the name of computer code but the method of analysis based on the of equivalent linealization and multiple reflection theory, whereas the name of equivalent linear is used with wider meaning.

This underestimation results in underestimation of peak acceleration under relatively small earthquake, because effect of high frequency component is large under small earthquake. The reason of this underestimation is clear. Shear modulus and damping ratio computed from effective stress evaluated from Eq. (1) is used in entire analysis in the equivalent analysis. This means that they are determined looking at the strain behavior near the peak behavior. Therefore shear modulus is underestimated and damping ratio is overestimate for higher frequency component, both of which yields underestimation of acceleration response. It seems that this underestimation may not significant if interested frequency range is small. This is true when computing the response under given incident wave. The problem may occur at the deconvolution process in which incident wave is going to compute from the given surface record. Because the amplification is small in the high frequency region, large incident wave is required in this region. As amplification becomes smaller, high frequency component of incident wave becomes larger. This may result in unrealistically large incident wave, and sometimes, computation does not converge, but diverge.

These two shortages that have opposite tendency comes from the same cause as easily recognized from previous discussion, i.e., the method to determine effective strain, hence shear modulus and damping ratio. In order to improve former shortage, i.e., overestimation of peak acceleration, the value of α should become larger [AIJ, 1996]. On the other hand, in order to improve the latter shortage, i.e., underestimation of acceleration, it should become smaller [e.g., Tazoh et al., 1987.]. Therefore, one cannot solve this problem at the same time.

The idea to improvement on the latter shortage is proposed by Hagiwara and Kiyota [1992] and Sugito et al. [1994]. The method shown by Hagiwara and Kiyota is to conduct modal analysis and use different material property in each mode, which is somewhat different approach from the conventional equivalent method, therefore not discussed hereafter. Sugito expressed effective strain as a function of frequency using the Fourier amplitude $F(f)$ and its maximum value F_{max} as follows:

$$\gamma_{eff}(f) = \alpha \gamma_{max} \frac{F(f)}{F_{\max}} \tag{2}$$

This method is named FDEL by the authors. They suggest α=0.65 is relevant value. The significant improvement is reported by Sugito et al. [1994] and Ueshima and Nakazono [1996]. The physical meaning of Eq. (2) is, however, not known. Moreover, since it always gives larger acceleration than SHAKE, applicability to large earthquake becomes much less than SHAKE.

As mentioned above, relevant α value depends on various cases. Two factors that have opposite tendency is supposed to be the reason why a value is not unique or simple function.

PROPOSED METHOD

Two simple questions will be arisen when Eq. (2) is employed. The first one is that, if shear modulus computed from effective strain is frequency dependent, shear wave velocity must also be frequency dependent, which is against well-known fact that shear wave velocity is constant regardless to frequency. The second one is that frequency dependent modulus and damping are against the observation in laboratory tests in which material property is reported not to be affected by frequency of loading under dynamic deformation characteristics test within the frequency range that are important in the engineering practice.

Figure 3 is a schematic figure showing the mechanism why frequency dependent nature appears in the time history, in which hysteresis curves at large shear strain amplitude A and small amplitude B caused by small unloading-reloading cycle are shown. Under large amplitude behavior, apparent shear modulus is small and apparent damping ratio is large. On the other hand, shear modulus is large and damping ratio is small under small amplitude behavior. Therefore, frequency dependent nature will appear when looking at the time history.

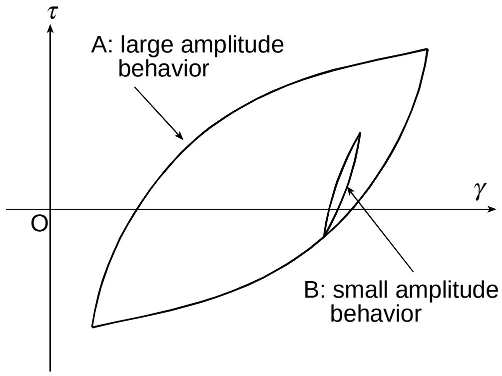

Figure 3: Stress-strain relation under large amplitude and small amplitude

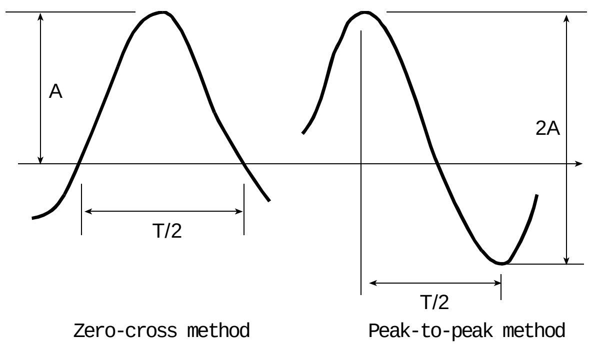

Figure 4: Zero-cross method and Peak method

These phenomena are a behavior in time domain or transition phenomena. On the other hand, frequency dependent characteristics using Eq. (2) is a behavior in frequency domain or steady state behavior. Therefore, they cannot be discussed in the same way. If, however, there are similar tendency or correlative relationship between them, it becomes possible to convert time domain behavior into frequency domain behavior.

In order to compare the time domain behavior into the frequency domain behavior, amplitude of shear strain versus period relationship computed from the shear strain time history by zero-cross method is computed. Figure 4 is an example of this kind, which comes from the example shown later. Here, *A* denotes amplitude and *T* denotes period. Smoothed Fourier spectrum is also shown in Figure 4 for comparison purpose. As can be seen, there seems correlative relation between them, therefore improvement of equivalent linear method seems possible.

In the above-discussed sense, the method employed in FDEL has physical meaning. When improving equivalent linear method, however, FDEL cannot be said better because of several reasons. Firstly, they suggest to use α=0.65. In addition, as discussed in the proceeding, FDEL will overestimate accelerations under large earthquake much more than SHAKE which already overestimates it. Secondly, although smoothing procedure of the Fourier spectrum is necessary in applying FDEL, there is no index how large smoothing should be necessary. Thirdly, as seen in Figure 4, frequency at which Fourier amplitude becomes peak does not coincide with the frequency at which zero-cross amplitude becomes peak value.

As discussed above, it is necessary to set α=1.0 when applying equivalent linear method to large earthquake. It is also obvious that setting α=1.0 in Eq. (2) does not work well because α=0.65 is recommended by the authors through the comparison between earthquake observation in addition to the above mentioned reasons. In this paper, we propose the following procedure. Firstly, instead of using Eq. (2), we just express the effective strain as a function with respect to frequency. The function should be a good approximation with the strain amplitude versus frequency relationship computed from time history. In the comparison above, we employed zero-cross

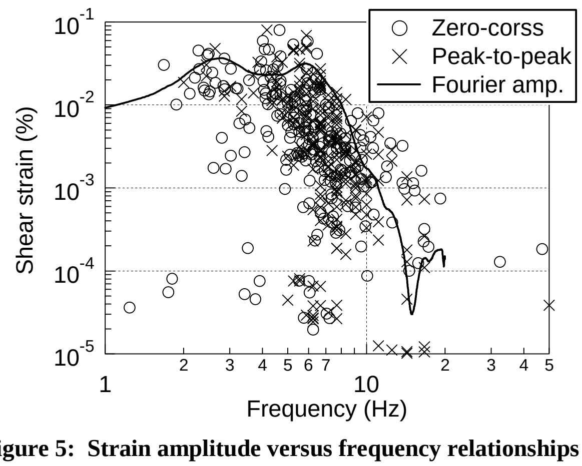

Figure 5: Strain amplitude versus frequency relationships

method to compute it. This method, however, is not relevant in this purpose because frequency component cannot be computed if shear strain drifts, which is quite different nature compared with acceleration, for example, that vibrates crossing the zero-axis. Instead of zero-cross method, we employed peak-to-peak method that is also schematically shown in Figure 3. The result is also shown in Figure 4 and general tendency is the same with the relationship by zero-cross method.

Then, we express the shape of the function temporary as follows (The meaning of "temporary" will be described in concluding remarks.):

$$\begin{aligned} \log\gamma_{eff} &= A(\log f - \log f_p)^m + \log\gamma_{max} & f \geq f_p \\ \gamma_{eff} &= \gamma_{max} & f < f_p \end{aligned} \tag{3}$$

were f_p is a frequency when shear strain amplitude versus frequency relationship obtained by zero-cross method shows peak value and γ_{max} is a maximum shear strain. We temporary set m=2, and the value of coefficient A is computed by least-square method.

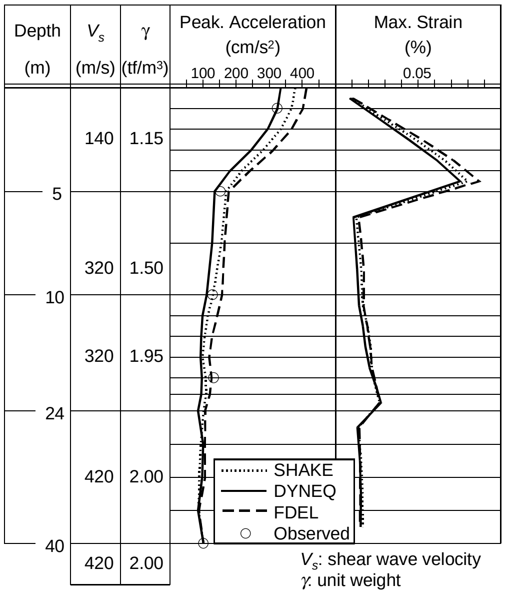

Figure 6: Soil profiles and peak response value

NUMERICAL EXAMPLE AND DISCUSSION

The vertical array records at the Chiba site of Industrial Institute of Science during the 1987 Chiba-ken Toho-oki earthquake (M=6.7) is used as an example. Material properties shown by Liu et al. [1989] for the analysis of this site are also used. The computer code DYNEQ developed by authors [Yoshida and Suetomi, 1996] are improved for calculation. Therefore, all the calculation are conducted by DYNEQ, but the names SHAKE, FDEL and SHAKE in order to distinguish the method.

Peak acceleration and maximum strain are shown in Figure 6 and acceleration time history at GL-1m (ground surface hereafter) are compared in Figure 7. The maximum shear strain is observed in the layer 5 (GL-4~5m); the value is a little less than 0.1%. Therefore, the simulation can be said to be the range that equivalent linear analysis is applicable [Ishihara, 1982], but the shortages of the equivalent linear method can be clearly seen as described in the followings. Looking at the surface response where effect of nonlinear is easiest to appear, peak response is the largest by FDEL, then SHAKE and DYNEQ. DYNEQ gives almost perfect simulation whereas FDEL overestimates peak acceleration of about 25 %. This proves the discussion in the preceding true.

In order to make the detailed comparison in Figure 7 easy, arrow is drawn at five points A to E in each figure. Here, the arrow points at the location of peak value in observed records. It is clearly seen that FDEL and

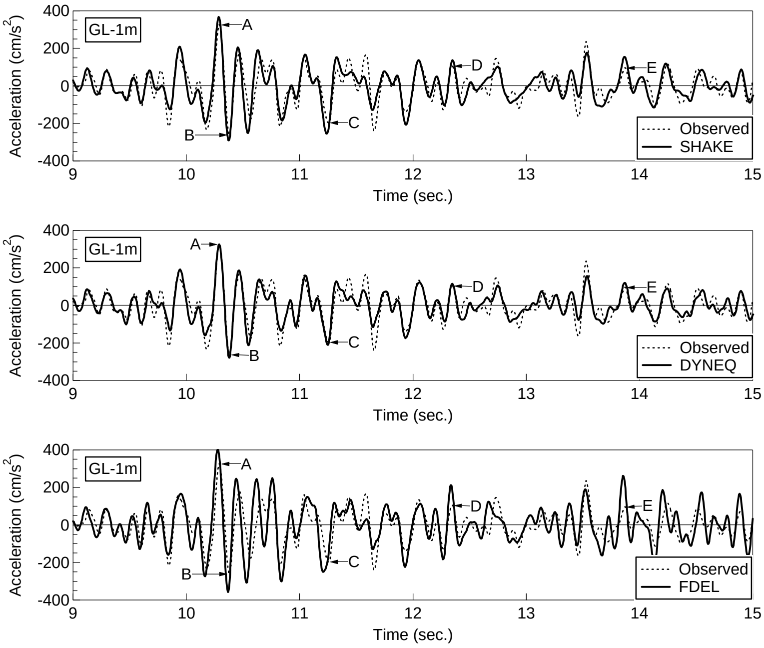

Figure 7: Comparison of acceleration time histories at ground surface

SHAKE overestimate acceleration only near the peak. If overestimation occurs because of resonance effect, the difference should appear entire time history, but difference appear especially only at the peak. This proves the discussion in the preceding true. Slight disagreement is seen beside point A to E, but they are common in all analyses. Therefore, DYNEQ gives much better agreement with observed record in general compared with conventional equivalent linear methods.

There is another difference in the comparison in Figure 7. The phase by SHAKE and FDEL is earlier than the observed one whereas that by DYNEQ agrees with it. This indicates that SHAKE and FDEL overestimate shear wave velocity, hence shear stiffness. This can be recognized by looking at Figure 1; conventional equivalent linear method gives larger stiffness. This observation again shows the mechanism for overestimation of peak acceleration is the one shown in Figure 1, but not by resonance effect.

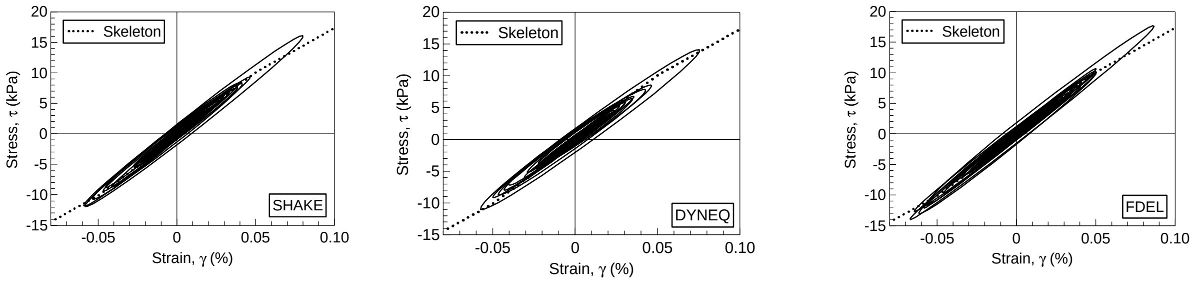

Figure 8: Stress-strain relations at th layer (GL-4 - 5 m)

Figure 8 compares stress-strain relationship including complex stiffness in the 5th layer (GL-4 to 5 m) where shear strain is the largest. Dashed line in the figure is the specified stress-strain curve. Again SHAKE and FDEL overestimate shear stress near maximum strain, whereas DYNEQ agrees with specified relations.

Figure 9 compares spectral ratio. Here amplification ratio of the observed record is smoothed by triangular window. Same with Figure 2, SHAKE underestimates amplification in frequency range greater than 10, but improvement is clearly seen by FDEL and DYNEQ. DYNEQ shows lower amplification compared with FDEL and SHAKE at low frequency range, which comes from the low acceleration value at the ground surface by DYNEQ.

Figure 10 compares frequency dependent shear modulus and damping ratio at the 5th layer (GL-4~5m). The result by SHAKE is, of course, expressed by horizontal line, and results by other methods shows frequency dependency. Since elastic shear modulus of this layer is 22540kN/m^2, the ground behaves in an elastic manner in frequencies lower than 1 Hz and higher than 10 Hz in FDEL. In other word, nonlinear behavior occurs only in narrow medium frequency range. On the other hand, lowest shear modulus and highest damping ratio are used in low frequency range in DYNEQ. The difference between SHAKE and DYNEQ in this frequency range comes from the assumption that SHAKE uses effective strain coefficient of 0.65. Elastic behavior occurs in frequencies larger than about 20Hz.

CONCLUDING REMARKS

A new equivalent linear technique is proposed, which overcomes two significant shortages that conventional equivalent linear method has. The shortages are overestimation of peak acceleration under large earthquake, and underestimation of amplification factor, hence peak acceleration, at high frequency range. The latter effect becomes predominant under small earthquake in which high frequency component affects peak acceleration. It may become predominant in the deconvolution analysis resulting in huge incident wave on the engineering seismic base motion or divergence of numerical analysis.

Numerical example clearly showed the shortage of the conventional method, and proposed method improves the simulation significantly.

As seen in this approach, response of the ground by the equivalent linear method may changes depending on the effective strain expressed with respect to frequency. In this study, we assume 2nd order equation and least-square method in determining the shape of the function. This, however, may not be the best feature of the function; there may be better functional shape, which is the reason why we add the word temporary in defining Eq. (3). This will be carried in the subsequent research.

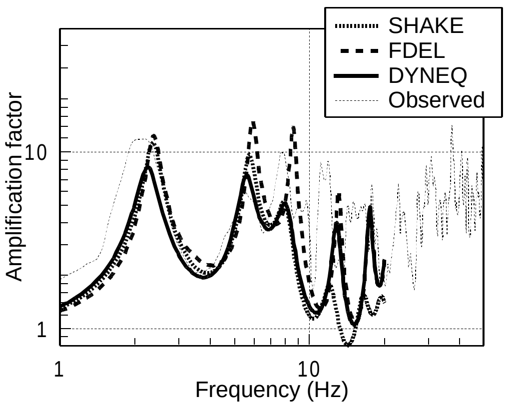

Figure 9: Amplification factor

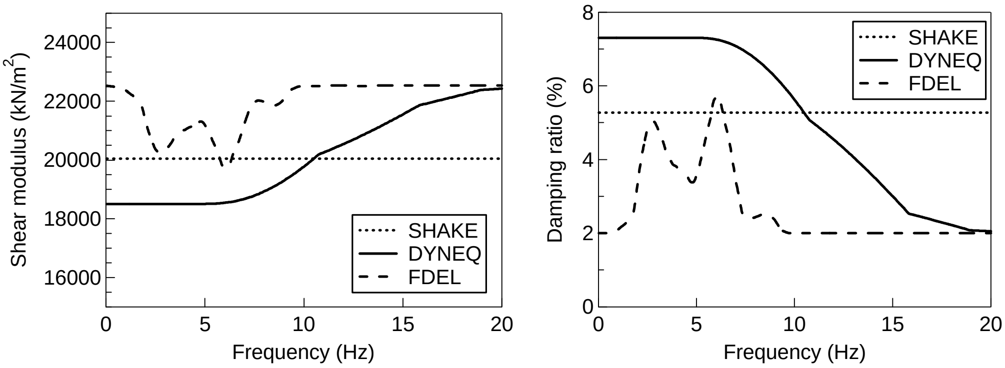

Figure 10: Strain dependent modulus and damping ratio in 5th layer

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