A METHOD FOR EVALUATING DEFORMATION CAPACITY OF EXTERIOR R/C COLUMNS AFTER FLEXURAL YIELDING

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SUMMARY

A method for evaluating the deformation capacity of exterior columns subjected to cyclic lateral loading and a varying axial load is presented in this paper. This deformation capacity is determined by two criteria. One is defined by the strain softening of concrete, and the other is defined by the hysteretic characteristics of concrete, as well as the deformation capacity under a constant axial load which has been formulated in the authors's previous paper. Using the formulated deformation capacity under a constant axial load, the one under a varying axial load can be evaluated by the maximum axial stress ratio, while the other under a varying axial load can be evaluated by the equivalent axial stress ratio, which was introduced to take consideration of the effect of the loading pass of the varying axial load. The smaller one gives the deformation capacity of exterior columns. The proposed deformation capacity has good agreement with the experimental results.

INTRODUCTION

Exterior reinforced concrete columns of buildings are subjected to cyclic lateral loading and a varying axial load during earthquakes, and typically fail due to the crush of concrete after flexural yielding. The variation of the axial load, as well as characteristics of the stress-strain relationship of confined concrete in the columns, has a great influence on the deformation capacity of such exterior columns. However, in the previous investigations on the evaluation of the ductility of columns under a varying axial load, the maximum and minimum values in the varying axial load have mainly received attention, and the effect of the loading pass between the maximum and the minimum axial load on the ductility has not been sufficiently considered. This paper presents a method for evaluating the deformation capacity of exterior columns, taking consideration of the effects of the loading pass and the characteristics of confined concrete. This method can be easily applied to the design of exterior columns.

As for the deformation capacity of columns that fail due to the crush of concrete, the authors have proposed the following two critical conditions [Hiraishi et al., 1993]. Criterion 1 is defined by the strain softening of concrete under monotonically increasing loading. Criterion 2 is defined by the hysteretic characteristics of concrete under cyclic loading. Structural design charts and equations for the deformation capacity of columns under a constant axial load have been proposed [Inai and Hiraishi, 1996]. Using the proposed design equations, the deformation capacity under a constant axial load can be evaluated by the axial stress ratio, which is the ratio of axial stress in the column section to the compressive strength of confined concrete. These criteria are also applied to columns under a varying axial load in this paper. The deformation capacity determined by Criterion 1 under a variable axial load is formulated, using the maximum axial stress ratio in the varying axial load. The deformation capacity determined by the Criterion 2 under a varying axial load is formulated, using the equivalent axial stress ratio. This equivalent axial stress ratio is expressed in the form of a linear combination of the following three axial stress ratios: the maximum axial stress ratio, the minimum axial stress ratio and the axial stress ratio at the zero deformation, corresponding to the loading pass of the varying axial load. The adequacy of the proposed equivalent axial stress ratio is verified by the analytical moment-curvature relationships of a reinforced concrete section subjected to curvature reversals and a varying axial load. The deformation capacity of exterior columns is

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given by the smaller one determined by the two criteria. The proposed deformation capacity is compared with the experimental results, and good agreement was obtained.

DEFORMATION CAPACITY UNDER CONSTANT AXIAL LOAD

The authors have investigated and formulated the deformation capacity of columns subjected to lateral loading and a constant axial load. The following two mechanical characteristics of concrete are considered to determine the deformation capacity: 1) the strain softening in the stress-strain relationship of concrete, 2) hysteretic characteristics of concrete in the large strain stage. The strain softening of concrete typically causes a moment reduction after the ultimate moment, and finally collapse of the column, under the monotonically increasing lateral loading [Hiraishi and Inai, 1990a and Hiraishi et al., 1990b]. On the other hand, the hysteretic characteristics of concrete cause a successive deterioration in moment resistance and axial shortening of the column under the cyclic lateral loading and a high axial load [Inai and Hiraishi, 1992]. The deformation capacities defined by these two factors has been derived as shown in Fig. 3, considering an equally reinforced core section shown in Fig. 1 and the idealized stress-strain relationship of core concrete shown in Fig. 2 [Hiraishi et al., 1993]. In Figs. 1-3, *D'* and *b'* are depth and width of core section, respectively, *fc'* is the compressive strength of confined concrete, ε_B is the strain at the compressive strength, α is a parameter representing the slope of descending branch of the stress-strain relationship of concrete, *N* is an axial load, η is the axial stress ratio and is equal to *N/(b'D'fc')*, ϕ_{SL} is the critical curvature defined by the strain softening of concrete, and ϕ_{CY} is the critical curvature defined by the hysteretic characteristics of concrete. For columns subjected to cyclic lateral loading and a constant axial load, the smaller critical curvature gives the actual deformation capacity. There is a tendency that the η-ϕ_{CY} relationships give the deformation capacity in case of a high axial stress ratio. Fig. 4 gives the adequacy of one of the η-ϕ_{CY} relationships, where four pairs of analytical moment-curvature relationships of concrete core section (218x218mm) with the same curvature amplitude and a different axial stress ratio are shown. The axial stress ratios in Case (a) - (d) are a little greater than the η-ϕ_{CY} relationship with α=0.1, while those in Case (e) - (h) are a little smaller. The moment-curvature relationships in Case (a) - (d) show a deterioration in moment resistance under cyclic loading, while those in Case (e) - (h) show very stable behavior. The adequacy of the η-ϕ_{SL} relationships has been examined using experimental results [Inai and Hiraishi, 1992].

Furthermore, the following simplified design equations for the deformation capacities of the columns under a constant axial load have been proposed, based on the above critical curvatures and a relationship of $R=\phi D'$, where *R* is the drift angel of the column [Inai and Hiraishi, 1996].
The critical drift angle of columns defined by the strain softening of concrete, *Ru*:

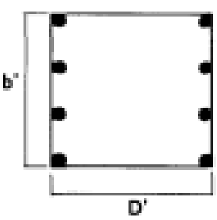


Figure 1: Core section

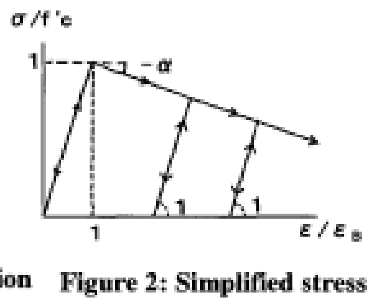


Figure 2: Simplified stress - strain relationship for concrete

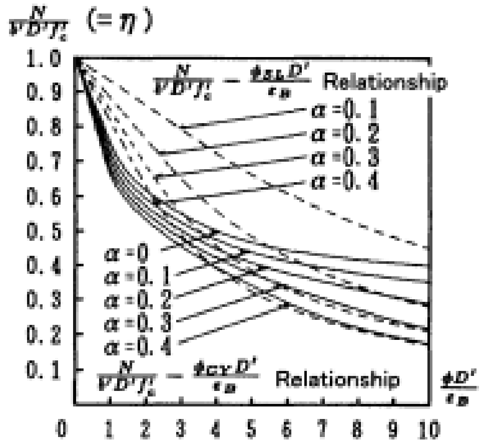


Figure 3: Critical curvatures under constant axial load

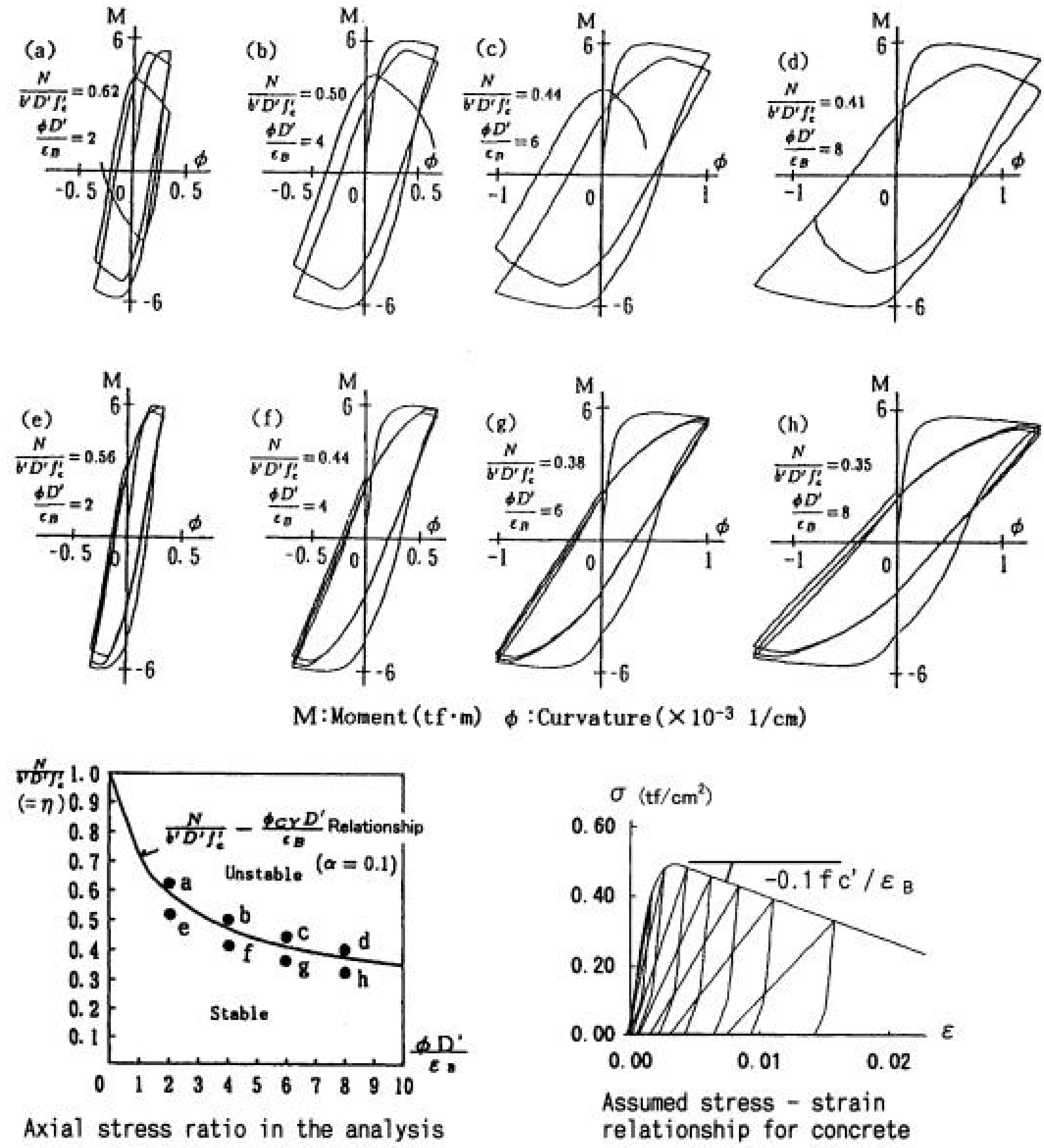


Figure 4: Analytical moment - curvature relationships under constant axial load

$Ru = (1 - h) / 24$ for $Ru \leq 1/34$, $Ru = (1 - 2h) / 14$ for $1/34 < Ru \leq 0.06$ (1)

The critical drift angle of columns defined by the hysteretic behavior of concrete, *Ru*:
$Ru = (1 - 2h) / 14$ for $Ru \leq 0.06$ (2)

Fig. 5 gives relationships between these equations and the experimental deformation capacities. Eq. (2) gives a lower-boundary of the experimental deformation capacities of the specimens subjected to cyclic lateral loading and a constant axial load. Eq. (1) gives a lower-boundary of the experimental deformation capacities of the specimens subjected to monotonically increasing lateral loading and a constant axial load, or cyclic lateral loading and a varying axial load. In Fig. 5, the experimental deformation capacity is the drift angle where the moment resistance, including the *P*-delta moment, reduces to 95% of the ultimate moment. The maximum compressive axial stress ratio is applied as the value of h in case of the specimens under a varying axial load. The reason for this will be discussed in the following section.

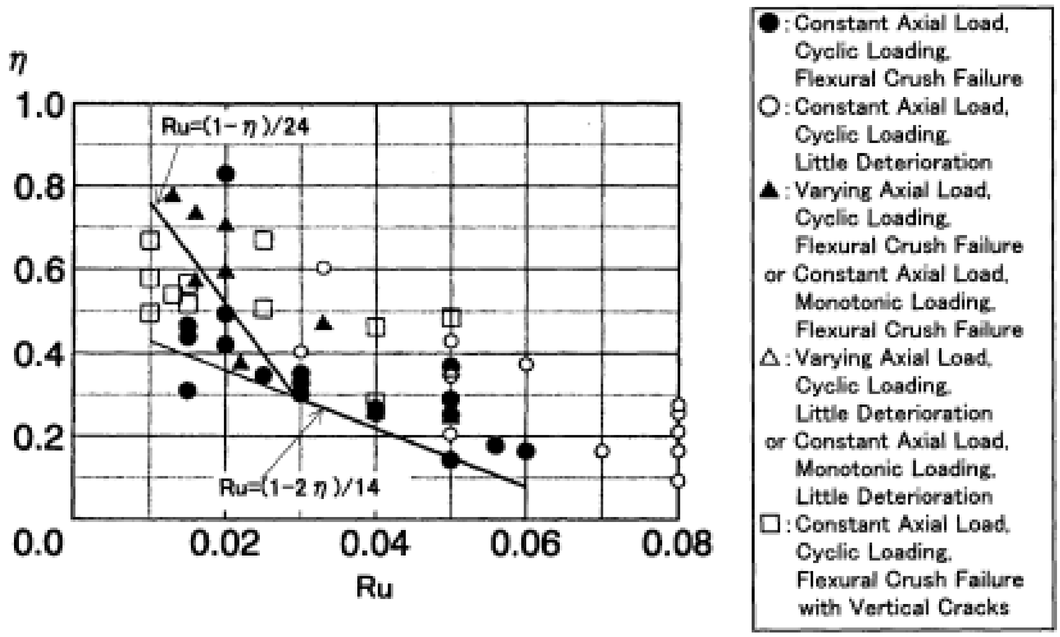


Figure 5: Relationships between design equations and experimental deformation capacities

DEFORMATION CAPACITY UNDER VARYING AXIAL LOAD

In this section, exterior columns of the buildings with a weak-beam strong-column system are considered. The deformation capacity of columns under a varying axial load is defined by the same two factors as the case under a constant axial load.

Deformation Capacity Defined by Strain Softening of Concrete

The earthquake-induced axial load of the exterior columns becomes constant after the hinges at the beam-ends are fully developed. Therefore, the deformation capacity defined by the strain softening of concrete may occur under the maximum compressive axial load, and is easily obtained from applying the maximum axial stress ratio, η_{MAX}, to the η - ϕ_{SL} relationships in Fig. 3, or Eq. (1).

DEFORMATION CAPACITY DEFINED BY HYSTERETIC CHARACTERISTICS OF CONCRETE

The deformation capacity defined by the hysteretic characteristics of concrete can be obtained from the following procedure, as well as the case under a constant axial load.

1) Consider an equally reinforced core section, shown in Fig. 1, subjected to curvature reversals with a certain amplitude and a varying axial load, where stress of concrete follows the idealized stress-strain relationship shown in Fig. 2 and the axial load is assumed to be sustained by concrete only.
2) Determine the stresses and strains in the section at the peak curvatures in the positive and negative loading directions corresponding to a given curvature amplitude and axial loads.
3) Examine the axial load carrying capacity of the section at the zero curvature after the inelastic regions have developed in both sectional edges at the positive and negative peak curvatures, assuming that the axial strain of the section at the zero curvature reaches the elastic limit of concrete.
4) The relationship between the axial loads at the three deformational stages and the curvature amplitude gives the deformation capacity defined by the hysteretic characteristics of concrete.

However, the derivation of the deformation capacity requires a very complicate process under various varying axial loads and curvature amplitude. Therefore, an alternative method is applied in the following consideration.

Fig. 6 illustrates the strain and stress distributions in the core section at the three deformational stages: the positive and negative peak curvatures and the zero curvature. The stress distributions are simplified as follows:

a) At the positive and negative peak curvatures, stresses in the region where the strain is in the elastic range are assumed to be zero, while all stresses in the region where the strain is in inelastic range are assumed to be the compressive strength, *fc'*.

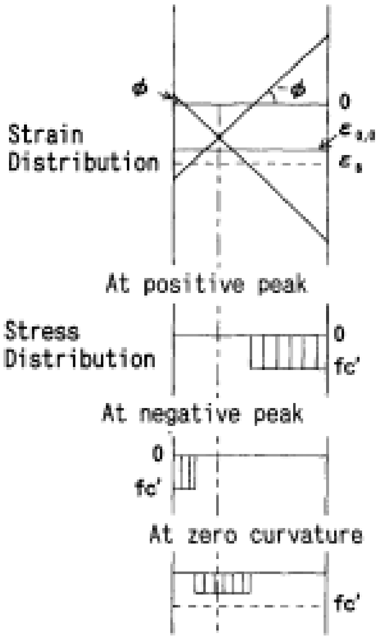


Figure 6: Assumed stress distributions

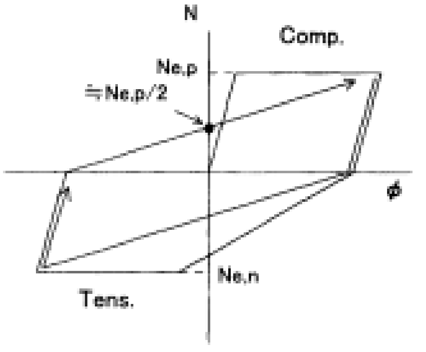


Figure 7: Assumed earthquake-induced axial load - curvature relationship

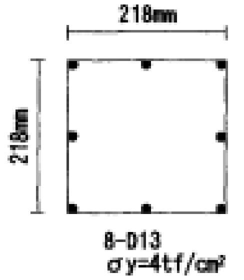


Figure 8: Core section considered

b) At the zero curvature, stresses in the region that has experienced the inelastic strain at the peak curvatures are assumed to be zero.

The strain of the section at the zero curvature reaches the elastic limit of concrete at the deformation capacity. On the basis of these simplifications, the following equation comes into existence at the deformation capacity, and is valid for any curvature amplitude and varying axial load.

$$(\eta_c + \eta_{e,p}) + (\eta_c - \eta_{e,n}) + \eta_0 = 1 \tag{3}$$

where, $\eta_c=Nc/(b'D'fc')$; $\eta_{e,p}=Ne,p/(b'D'fc')$; $\eta_{e,n}=Ne,n/(b'D'fc')$ for $\eta_{e,n}<\eta_c$, $\eta_{e,n}=\eta_c$ for $\eta_{e,n}>\eta_c$; $\eta_o=No/(b'D'fc')$; *Nc* is the long-term axial load, and positive for compression; *Ne,p* is the earthquake-induced axial load at the positive peak deformation, and positive for compression; *Ne,n* is the earthquake-induced axial load at the negative peak deformation, and positive for tension; *No* is the axial load at the zero deformation, assumed to be compression, and positive for compression. ($\eta_c+\eta_{e,p}$) and ($\eta_c-\eta_{e,p}$) usually express the maximum and the minimum axial stress ratio, respectively.

In case that $\eta_{e,p}=\eta_{e,n}=0$ and $\eta_c=\eta_o=\eta^*$ in Eq. (3), Eq. (3) expresses the critical condition under a constant axial load, and becomes $3\eta^*=1$. From equalizing Eq. (3) and $3\eta^*=1$, the following equation is obtained.

$$\eta^* = (2\eta_c + \eta_{e,p} - \eta_{e,n} + \eta_0)/3 \tag{4}$$

The η^* in Eq. (4) expresses the axial stress ratio to be considered in case that the critical condition under a varying axial load is represented by the critical condition under a constant axial load, and is referred as the equivalent axial stress ratio.

The earthquake-induced axial load of the exterior columns is generally determined by the restoring-force characteristics of the beams in the buildings. In the following consideration, the curvature in the critical section of the exterior column is assumed to be in proportion to the deformation of the beams, and the earthquake-induced axial load given by the beams as illustrated in Fig. 7 is used. In this case, the earthquake- induced axial load at the zero deformation of the exterior column becomes nearly *Ne,p*/2. Therefore, by substituting ($\eta_c+\eta_{e,p}/2$) into η_o in Eq. (4), the equivalent axial stress ratio is expresses as follows:

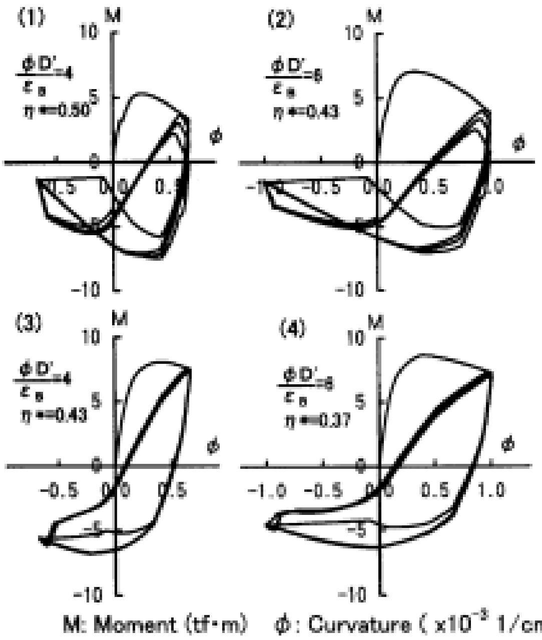


Figure 9: Analytical moment - curvature relationships under varying axial load

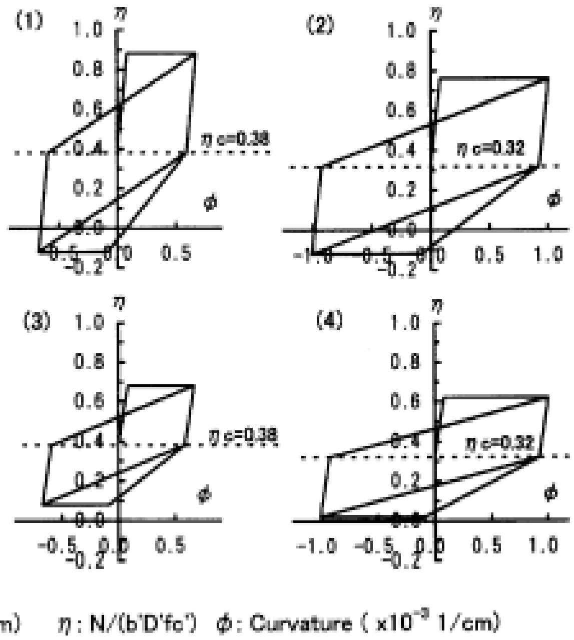


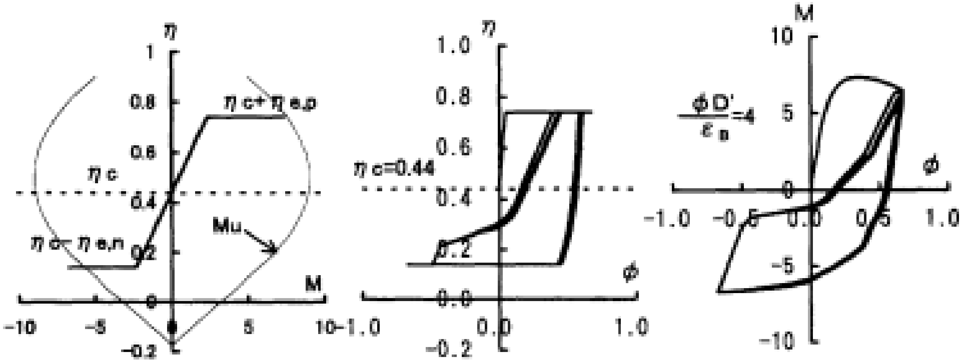
Figure 10: Assumed varying axial load - curvature relationships

$$\eta^* = \eta c + \eta_{e,p} / 2 - \eta_{e,n} / 3 \tag{5}$$

In order to verify the adequacy of the equivalent axial stress ratio, η^*, in Eq. (5), the analyses of cyclic moment-curvature relationships of a reinforced concrete section shown in Fig. 8 under a varying axial load were conducted. The sectional dimension and the used stress-strain relationship of core concrete are the same as those in the analyses under a constant axial load shown in Fig. 4. However, reinforcing bars are considered in the section because a tensile axial load is applied to the section in this analysis. The bi-linear model is used to represent the stress-strain relationship of the reinforcing bars. Two pairs of the analytical moment-curvature relationships with the same curvature amplitude and a different equivalent axial stress ratio are shown in Fig. 9. The loading histories of axial load in each analysis are shown in Fig. 10. The equivalent axial stress ratios given by Eq. (5) in Case (1) and (2) are a little greater than the η-ϕ_{CY} relationship under the constant axial load, and are the almost same as those in Case (b) and (c) in Fig. 4, respectively. The equivalent axial stress ratios in Case (3) and (4) are a little smaller than the η-ϕ_{CY} relationship, and are the almost same as those in Case (f) and (g) in Fig. 4, respectively. A successive deterioration in moment resistance due to the hysteretic characteristics of concrete is observed in the positive loading direction in Case (1) and (2), while the behavior of Case (3) and (4) are stable under four curvature reversals. Therefore, the deformation capacity defined by the hysteretic characteristics of concrete under a varying axial load can be obtained from applying the proposed equivalent axial stress ratio, η^*, to the η-ϕ_{CY} relationships in Fig. 3, or Eq. (2).

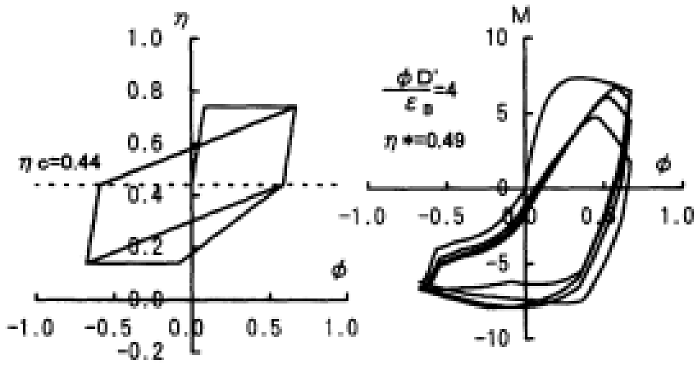
Deformation Capacity of Exterior Columns and Comparison with Experimental Results

The deformation capacities defined by the strain softening and the hysteretic characteristics of concrete under a varying axial load can be evaluated by the maximum axial stress ratio and the equivalent axial stress ratio, respectively. The smaller one gives the deformation capacity of exterior columns subjected to cyclic lateral lading and a varying axial load. Fig. 11 shows one of the effects of the loading pass on the deformation capacity of exterior columns, using the sectional analyses of the reinforced concrete section shown in Fig. 8. The moment-curvature relationship in Fig.11 (a) was obtained under the condition that the axial load varied proportionally to the moment of the section up to the maximum or minimum axial load. On the other hand, the moment-curvature relationship in Fig. 11 (b) was obtained under the loading pass described in the previous section. The maximum and minimum axial loads and the curvature amplitude are identical in the both cases. The analytical moment-curvature relationship in Fig. 11 (b) shows a successive deterioration in moment resistance



M: Moment (tf·m) η: N/(b'D'fc') ϕ: Curvature (x10^{-3}1/cm)

(a) Proportional to moment



M: Moment (tf·m) η: N/(b'D'fc') ϕ: Curvature (x10^{-3}1/cm)

(b) Proportional to deformation

Figure 11: Effect of loading pass of varying axial load

due to the hysteretic characteristics of concrete, while that in Fig. 11 (a) shows no deterioration. This is because the section was subjected to the high axial load only around the peak curvature in the positive loading direction in the case of Fig. 11 (a). Most of experimental deformation capacities under a varying axial load shown in Fig. 5 were derived from the specimens subjected to the varying axial load in proportion to the moment as shown in Fig. 11 (a). It is indicated that the strain softening of concrete rather than the hysteretic characteristics of concrete determined these experimental deformation capacities. Therefore, Eq. (1) with the maximum axial stress ratio gives a good lower-boundary of the experimental deformation capacities under cyclic lateral loading and a varying axial load, as well as those under a monotonically increasing lateral loading and a constant axial load.

CONCLUSIONS

The following conclusions can be drawn from this study.

1) The deformation capacity of exterior columns is determined by two criteria. One is defined by the strain softening of concrete. The other is defined by the hysteretic characteristics of concrete. The smaller one gives the deformation capacity of exterior columns.
2) The deformation capacity defined by the strain softening of concrete under a varying axial load can be evaluated by the maximum axial stress ratio, using the η-ϕ_{SL} relationships in Fig. 3 or Eq. (1).
3) The deformation capacity defined by the hysteretic characteristics of concrete under a varying axial load can be evaluated by the equivalent axial stress ratio given by Eq. (5), using the η-ϕ_{CY} relationships in Fig. 3 or Eq. (2).
4) The proposed deformation capacity has good agreement with the experimental data.

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