ANALYSIS OF SIGNIFICANT FACTORS INFLUENCING EQUIVALENT PARAMETERS IN EQUIVALENT LINEARIZATION METHOD

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SUMMARY

By using time history analysis, nonlinear responses of an SDOF shear structure under a group of artificial ground motion excitations with the same statistical characteristics can be obtained. The equivalent parameters of an individual structure with nonlinear performance are determined based on the equivalence of power spectral density function. According to 336 pairs of equivalent parameters with different structural parameters, including natural frequency, yielding strength coefficient, and strain hardening/softening ratio, analysis of variance is done to evaluate the significant level of them. At last, the formulae of equivalent parameters are regressed. The results show that, the yielding strength coefficient, natural frequency and their interaction between them are significant factors influencing the equivalent frequency while the strain hardening/softening ratio can be ignored. The yielding strength coefficient is the most significant factor influencing the equivalent damping ratio, and the natural frequency is the secondary significant factor. The illustration shows the presented formulae have an adequate precision and a convenient approach for the application of the equivalent linearization method is presented.

INTRODUCTION

The structures subjected to strong earthquake actions often behave non-linearly, and the investigation into the response characteristics of non-linear structures is concerned by the seismic engineering scope. At present, the commonly used non-linear analysis method is time history analysis, which has the advantages of distinct physical meanings and high precision of computation. But it is not practical for the engineering application because of the large amount of computation, the random behaviors of earthquake ground montion, and the non-linear characteristics of structures, which need huge number of samples of ground motions to process. Therefore, it is of significance to find a simplified method to analyzing non-linear structures.

The equivalent linearization method is one of widely used simplified approaches for non-linear structures. The clue line is: an objective function should be constructed to link non-linear system and equivalent linearized system, then the equivalent linear system approaches non-linear system when the objective function reaches the minimum value. In the traditional equivalent linearization method, the minus of vibration equations are often selected as the objective function, which function of maximum ductility and the response energy are depicted as equivalent parameters. Because the maximum response of structures is difficult to estimate, the equivalent parameters are often determined from the response of structures with some assumptions. Hu [1988] states that the equivalent frequency F_e and equivalent damping ratio ζ_e vary with the different assumptions. In this paper, non-linear response characteristics are obtained directly from time histroy analysis. Then on the basis of the above characteristics, the parameters of equivalent linear system are determined by the transmission theory of linear system in random vibration [Lin, 1967; Yu, 1988]. The significance of the structural parameters and their interactive influence on equivalent parameters F_e and ζ_e is tested by variance analysis, and the expressions of the equivalent parameters are obtained by optimization regression.

The computational analysis shows the determination of equivalent parameters is rational. The response characteristics obtained from analysis equivalent linearied system fit well with that from non-linear analysis.

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DETERMINATION EQUIVALENT PARAMETERS

A linear system can be expressed with the vibration equation,

$$\ddot{Y}(t)+2\zeta_0\omega_0\dot{Y}(t)+\omega_0^2Y(t)=-\ddot{X}(t) \qquad \text{(Eq.1)}$$

The transmission theory of linear system in random vibration shows that, the relation between the inputs and outputs in frequency domain can be expressed as follows,

$$S_{\ddot{Y}}(f)=\left|H_{\ddot{Y}\ddot{X}}(f)\right|^2 S_{\ddot{X}}(f) \qquad \text{(Eq.2)}$$

Where $S_{\ddot{X}}(f)$ is the power spectral density (PSD) function of ground motion acceleration inputs, $S_{\ddot{Y}}(f)$ is the PSD function of acceleration response, $H_{\ddot{Y}\ddot{X}}(f)$ is the transfer function. The above relationship between the average value of inputs with the same statistical characteristics, $\overline{S}_{\ddot{X}}(f)$ and the average value of responses $\overline{S}_{\ddot{Y}}(f)$ is also correct. If the equivalent linearized system defined by Eq. 3 can be substituted for the non-linear system defined by Eq. 1,

$$\ddot{Y}_e(t)+2\zeta_e\omega_e\dot{Y}(t)+\omega_e^2Y_e(t)=-\ddot{X}(t) \qquad \text{(Eq.3)}$$

and let PSD function of acceleration responses of system expressed in Eq. 3 be equal to that of system expressed in Eq. 1, that is,

$$\overline{S}_{\ddot{Y}_e}(f)=S_{\ddot{Y}}(f) \qquad \text{(Eq.4)}$$

the transfer function $H_{\ddot{Y}_e\ddot{X}}(f)$ of equivalent linearized system can be expressed as follows,

$$\left|H_{\ddot{Y}_e\ddot{X}}\right|=\sqrt{\overline{S}_{\ddot{Y}_e}(f)\big/\overline{S}_{\ddot{X}}(f)}=\sqrt{\overline{S}_{\ddot{Y}}(f)\big/\overline{S}_{\ddot{X}}(f)} \qquad \text{(Eq.5)}$$

From the definition of transfer function, $H_{\ddot{Y}_e\ddot{X}}(f)$ can be depicted as [Lin, 1967; Yu, 1988],

$$\left|H_{\ddot{Y}_e\ddot{X}}\right|=f^2\Big/\sqrt{(f_e^2-f^2)^2+4\zeta_e^2f_e^2f^2} \qquad \text{(Eq.6)}$$

Eq. 5 and Eq. 6 are ideatical. If $\overline{S}_{\ddot{X}}(f)$ of ground motion inputs is decided and $\overline{S}_{\ddot{Y}}(f)$ is solved by non-linear time history analysis and Fourier Transform, then the transfer function $H_{\ddot{Y}_e\ddot{X}}(f)$ of equivalent linearized system can be determined by Eq. 5. The value of F_e and ζ_e can be regressed from Eq. 6 by the Complex Form Optimization method [Xia, 1996].

The structural model in this paper is SDOF shear model. The structure hysteretic behavior is modeled by the bilinear, and the value of damping ratio is taken as 5% that is commonly used in R.C. structures. For structural dynamic parameters, the following conditions are considered.

The yielding strength ratios: ξ_y=0.2,0.25,0.3,…,0.5;

The strain hardening/softening ratios: P=0,0.02,0.05,0.1;

The natural frequencies (period) of structures: F_0 ($1/T_0$)=10(1/0.1), 5(1/0.2), 3.333(1/0.3),…, 0.8333(1/1.2).

The inputs ground motions are 30 artificial accelerogram, which are constructed under the condition of the fortification intensity of 7, site categories of II, and far-earthquake specified in Chinese seismic code [1989].

With the combination of all conditions above, there are 10080 times of non-linear dynamic time history analysis to be executed, 336 pairs of equivalent frequency F_e and equivalent damping ratio ζ_e are obtained from statistical analysis. Fig. 1 represents $H_{\ddot{Y}_e\ddot{X}}(f)$ calculated from Eq. 5 and Eq. 6, respectively, at the case of F_0=2Hz, ξ_y=0.3, and P=0. It can be seen that the regression results have perfect precision.

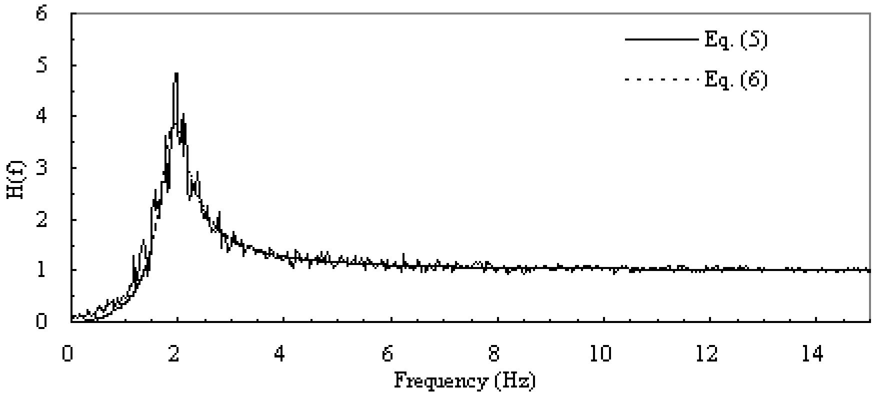


Fig. 1 Comparison of Transfer Function Value from Eq. 5 and Eq. 6

SIGNIFICANCE ANALYSIS OF INFLUENCE FACTORS

The values of equivalent parameters F_e, ζ_e are obtained from the above analysis, but there is no definite formula to express the relationship between F_e, ζ_e and the structure parameters ξ_y, P, and F_0. It will be convenient to give the expression for engineering application. The procedures of regression are as follows. First, the significance of influence factors is determined by dual factor variance analysis, and the sequence of their importance is given according to the index of significance. Then, the significance of the interaction of factors is determined by interactive dual factor variance analysis.

Dual Factor Variance Analysis

Taking equivalent parameters F_e, ζ_e as investigation object, structural parameters ξ_y, P, and F_0 as influence factor, the different value of ξ_y, P, F_0 as test level, the significance of factors can be determined by F-test given the significance level α=5%. Table 1 (a), (b), and (c) represent the significance index of F_0-ξ_y, ξ_y-P, F_0-P vs. F_e, respectively. In the tables, "SS" is the sum of square of standard deviation, "df" is the degree of freedom, "MS" is mean of square of standard deviation, "F" is the value of F-test, and "F crit." is the critical value of F-distribution with α percentage.

Table 1(a) ξ_y-F_0

Source	SS	df	MS	F	F crit.
ξ_y	0.287882	6	0.04798	33.0881	2.23948
F_0	384.1202	11	34.92	24081.5	1.93696

Table 1(b) P-ξ_y

Source	SS	df	MS	F	F crit.
P	8.4E-05	3	2.79E-05	0.417779	3.15991
ξ_y	0.18453	6	0.030755	460.2457	2.6613

Table 1(c) F_0-P

Source	SS	df	MS	F	F crit.
P	3E-04	3	1E-04	0.86978	2.892
F_0	227.2	11	20.65	157304	2.093

As shown in Table 1, both the natural frequency F_0 and the yielding strength ratio ξ_y have significant influence on the equivalent frequency F_e, while the strain hardening/softening ratio P has no significant influence. The significance order of the factors is $F_0>\xi_y>>P$.

Table 2 (a), (b), and (c) represents the significance index of F_0-ξ_y, ξ_y-P, F_0-P vs. ζ_e, respectively. It is obvious that ξ_y and F_0 have significant influence on ζ_e while P has no significant influence. The significance order of the factors is $\xi_y>F_0>>P$.

Table 2(a) ξ_y-F_0

Source	SS	df	MS	F	F crit.
ξ_y	0.103	6	0.017	236.5	2.239
F_0	0.038	11	0.003	48.4	1.937

Table 2(b) P-ξ_y

Source	SS	df	MS	F	F crit.
P	0.0001	3	4E-05	7.0503	3.1599
ξ_y	0.0329	6	0.0055	904.97	2.6613

Table 2(c) F_0-P

Source	SS	df	MS	F	F crit.
P	0.001	3	3E-04	19.1	2.892
F_0	0.062	11	0.006	310.4	2.093

Dual Factor Variance Analysis Considering the Interaction

The interaction of factors is investigated by variance analysis of repeated tests. For the equivalent frequency F_e, both F_0 and ξ_y are both significant factors and their interaction is shown in Table 3. The values listed in Table 3 show that the interaction of ξ_y and F_0 has significant influence on F_e, but far less than that of ξ_y or F_0.

Table 3 ξ_y x F_0

Source	SS	df	MS	F	F crit.
ξ_y	1.058	6	0.176	1653.54	2.135
F_0	1541	11	140.1	1313125	1.827
Interrelation	0.313	66	0.005	44.4869	1.358

The factors ξ_y, F_0, and P have significant influence on equivalent damping ratio ζ_e, only the interaction of ξ_y and F_0 is investigated since the significance of P is far less than that of ξ_y and F_0. The results are shown in Table 4. The results show that, the interaction of ξ_y and F_0 has significance on ζ_e and its significance is far less than that of ξ_y and F_0.

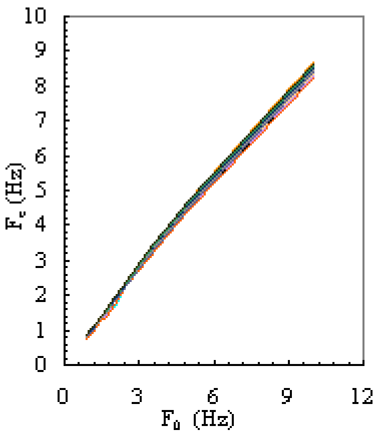
Table 4 ξ_y x F_0

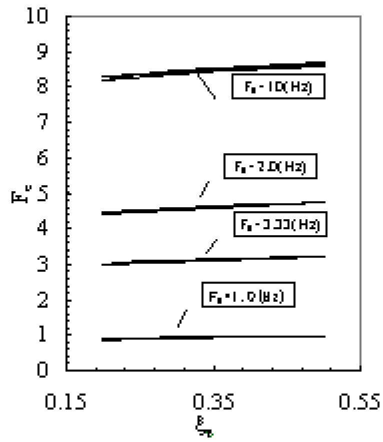
Source	SS	Df	MS	F	F crit.
ξ_y	0.30832	6	0.0513859	5135.0843	2.1468125
F_0	0.11645	8	0.0145568	1454.6847	1.987658
Interrelation	0.01459	48	0.0003039	30.374223	1.4252564

In summary, the significance order of the structural parameters influencing on equivalent frequency F_e is F_0>ξ_y>>F_0 x ξ_y >P, and the significance order of the structural influencing on equivalent damping ratio ζ_e is ξ_y> F_0>>F_0 x ξ_y >P.

EXPRESSIONS FOR EQUIVALENT PARAMETERS

The principle of equivalent parameters changing with the corresponding factors is investigated based on the significance order obtained in the above section, and the expressions of equivalent parameters are determined accordingly.





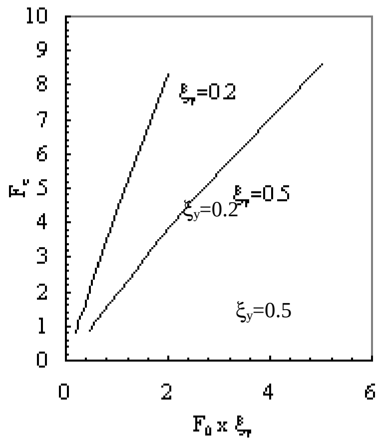


Fig. 2 Relation Curve of F_0-F_e **Fig. 3 Relation Curve of F_e-ξ_y** **Fig. 4 Relation Curve of F_e-F_0 x ξ_y**

From the correlation curves of F_0-F_e at the case of P=0 as shown in Fig. 2, we find F_e is linear to F_0 approximately,

$$F_e = X(1)F_0 + A \quad \text{(Eq. 7)}$$

The correlation curves of F_e-ξ_y, as shown in Fig. 3, can be depicted as quadratic equation,

$$F_e = X(2)\xi_y^2 + x(3)\xi_y + B \quad \text{(Eq. 8)}$$

The relation between F_e and F_0 x ξ_y is linear (see Fig. 4) also,

$$F_e = X(4)F_0\xi_y + C \quad \text{(Eq. 9)}$$

Combining Eq. 7, Eq. 8 and Eq. 9, the relationship between F_e and the structural parameters can be expressed as,

$$F_e = X(1)(1 + X(2)\xi_y)F_0 + X(3)\xi_y^2 + X(4)\xi_y + X(5) \quad \text{(Eq.10)}$$

in which X(1) to X(5) are constants to determine.

In the same way, the relationship between ζ_e and the structural parameters is investigated according to three groups of ζ_e-ξ_y, ζ_e-F_0, and ζ_e-ξ_y x F_0 as show in Fig. 5, Fig. 6, and Fig. 7, respectively. The expression of ζ_e may be written as,

$$\zeta_e = X(1)\xi_y^2 + X(2)\xi_y + X(3)F_0^3 + X(4)F_0^2 + X(5)F_0 + X(6) \quad \text{(Eq. 11)}$$

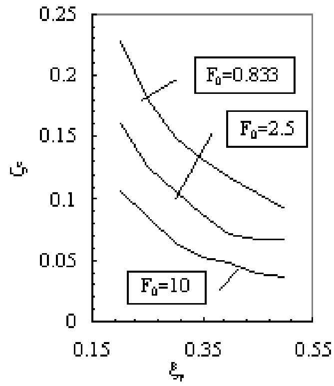


Fig. 5 Relation Curve of ζ_e-ξ_y

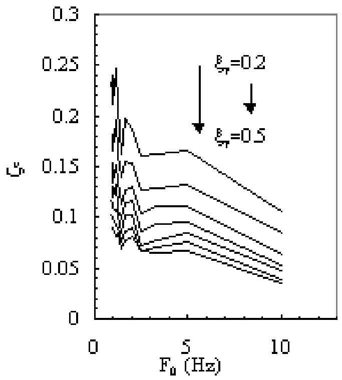


Fig. 6 Relation Curve of ζ_e-F_0

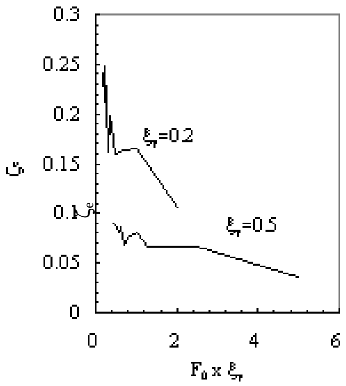


Fig. 7 Relation Curve of ζ_e-F_0 x ξ_y

By using Complex Form Optimization method, the parameters X(i) are evaluated. So, the expressions of F_e and ζ_e can be written as the following,

$$F_e = 0.855(1+0.226\xi_y)F_0 - 1.182\xi_y^2 + 0.952\xi_y - 0.157 \quad \text{(Eq. 12)}$$

$$\zeta_e = 0.876\xi_y^2 - 0.935\xi_y - 0.000612F_0^3 + 0.0096F_0^2 - 0.0448F_0 + 0.3765 \quad \text{(Eq. 13)}$$

The term of F_0^3 and F_0^2 can be ignored since their coefficients are very small. The expression of ζ_e may then be simplified as,

$$\zeta_e = 0.878\xi_y^2 - 0.941\xi_y - 0.0062F_0 + 0.344 \quad \text{(Eq. 14)}$$

EXAMPLES FOR VERIFICATION

Given ξ_y=0.45, $F_0=1/T_0$=1/0.65=1.538(Hz), and P=0, the average PSD functions $\overline{S}_{\ddot{Y}}(f)$ of responses can be computed by time history analysis. Substituting structural parameters into Eq. 12 and Eq. 14, $\overline{S}_{\ddot{Y}_e}(f)$ can be solved from Eq. 6. $\overline{S}_{\ddot{Y}}(f)$ and $\overline{S}_{\ddot{Y}_e}(f)$ are both plotted in Fig. 8.

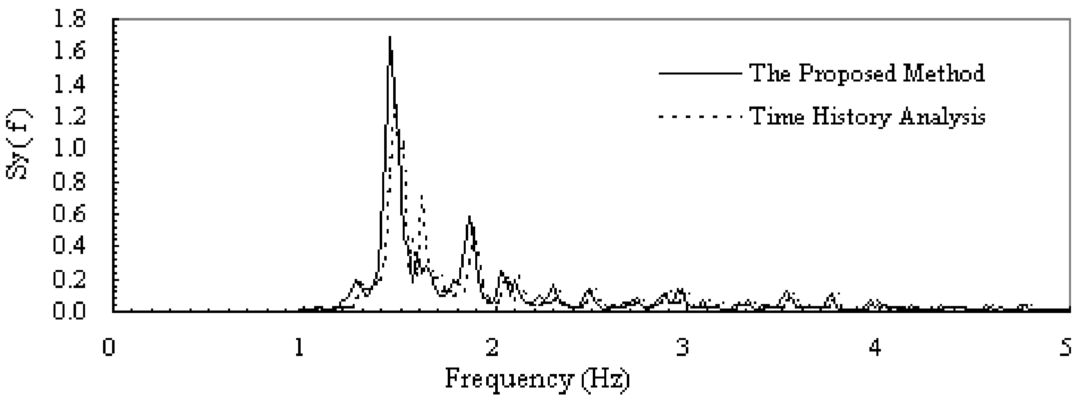


Fig. 8 Comparison of Response PSD Function Calculated by Time History Analysis and the Proposed Method

The difference between $\bar{S}_{\ddot{Y}}(f)$ and $\bar{S}_{\ddot{Y}_e}(f)$ can be described by the following two relative errors. One is relative error of peak value,

$$R_p = \frac{\left|\bar{S}_{\ddot{Y}}(f)_{\max} - \bar{S}_{\ddot{Y}_e}(f)_{\max}\right|}{\bar{S}_{\ddot{Y}}(f)_{\max}} \approx 25\% \qquad \text{(Eq.15)}$$

and the other is the relative error of spectra values averaged in the whole frequency domain,

$$R = \frac{1}{n}\sum_{i=1}^{n} \frac{\left|\bar{S}_{\ddot{Y}}(f_i) - \bar{S}_{\ddot{Y}_e}(f)\right|}{\bar{S}_{\ddot{Y}}(f_i)} \approx 7.6\% \qquad \text{(Eq.16)}$$

It should be emphasized that the significance of a PSD function is mainly on the distribution of power but not an individual value of power. Though Eq. 16 shows good agreement between $\bar{S}_{\ddot{Y}}(f)$ and $\bar{S}_{\ddot{Y}_e}(f)$ in the whole range while R_p described in Eq. 15 is large, the method is still acceptable in general sense.

CONCLUSIONS

Based on the statistical characteristics of non-linear response and linear transmission theory, the equivalent parameter of the equivalent linear system is obtained. The following conclusions can be addressed from the results of variance analysis. The order of significant of the factors influencing on the equivalent frequency F_e may be the natural frequency F_0, the structural yielding strength ratio ξ_y, and their interaction. The expression of F_e is described as Eq. 12. The order of significant of the factors influencing on the equivalent damping ratio ζ_e may be ξ_y, F_0 while the influence of interaction of ξ_y and F_0 can be ignored. The expression of ζ_e is given in Eq. 14. The example illustrates that, the PSD function of acceleration responses calculated by the proposed method agrees well with the result calculated by time history analysis, and the procedure can be used for practical application.

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