EXTENSION OF SEISMIC ZONATION MAPS FOR DESIGN AT ANY PRESELECTED FAILURE PROBABILITY

A H HADJIAN[1]

SUMMARY

A general methodology is presented that could be used to select seismic load factors to the NEHRP 1997 mapped values to achieve any pre-selected target failure probability. A simple algebraic expression is derived that estimates the median resistance, R_{50}. Given the statistics of the resistance (mean, COV, mean to nominal ratio), the median resistance is adjusted to obtain the nominal code design capacity, R_n. Results are tabulated for selected parameters

INTRODUCTION

Performance-based design requires, among other constraints, that failure probabilities be commensurate with the consequences of failure. Failure is defined herein as the condition beyond any pre-specified limit state. The motivation for this work stems from the rather ad hoc manner used in the NEHRP 1997 Provisions to specify performance-based requirements. A simplified procedure is developed that would allow the use of the NEHRP 1997 national maps to achieve any preselected failure probability. Given the confines of this paper, the main thrust is on the conceptual elements of the process rather than the development of hard numerical criteria.

FAILURE PROBABILITY

Earthquake failure probability can be calculated from Eq. 1 [Kennedy and Short, 1994]

$$P_F = \int_o^\infty H(a)\; f(a)\, da \qquad (1)$$

where the median seismic hazard curve, H(a), is the complimentary Cumulative Distribution Function (CDF) of the ground motion parameter under consideration, and f(a) is the Probability Density Function (PDF) of the conditional failure probability of the system (usually referred to as the fragility), given that load level **a** occurs. For the purposes of this paper, the peak ground acceleration **a** will be used as the ground motion parameter.

Assuming a lognormally distributed fragility curve with median F_{50} and variance ζ^2, Eq. 1 can be written as

$$P_F = \int_o^\infty H(a) \frac{1}{a\,\zeta\sqrt{2\pi}}\, e^{-\frac{(ln\frac{a}{F_{50}})^2}{2\zeta^2}}\, da \qquad (2)$$

Defining M = ln F_{50} and x = ln **a** (**a** = e^x and d**a** = e^x dx) and appropriately changing the limits of the integration, Eq. 2 can be written as

[1] *Member, Sr. Tech. Staff, DNFSB, Washington, DC. asadourh@DNFSB.gov*

$$P_F = \int_{-\infty}^{+\infty} H(a) \frac{1}{\zeta\sqrt{2\pi}} e^{-[\frac{(x-M)^2}{2\zeta^2}]} dx \quad (3)$$

To perform the closed-form integration of Eq. 3, Kennedy and Short (1994) approximate the hazard curve by

$$H_a = K_1 a^{-K_H} \quad (4)$$

where K_1 is a constant and K_H is a slope parameter defined by $K_H = \frac{1}{\log(A_R)}$, wherein A_R is the ratio of ground

motions corresponding to a ten-fold reduction in exceedance probability. Substituting Eq. 4 into Eq. 3 results in

$$P_F = \frac{K_1}{\zeta\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-K_H x} e^{-[\frac{(x-M)^2}{2\zeta^2}]} dx \quad (5)$$

Performing the above integration, the failure probability is obtained as

$$P_F = K_1 e^{-K_H M} e^{\frac{(K_H\zeta)^2}{2}} \quad (6)$$

Defining H_D as the annual frequency of exceedance of the median Design Basis Earthquake (DBE) ground motion level, and substituting Eq. 4 into Eq. 6, gives the final expression for P_F [Kennedy and Short, 1994]:

$$P_F = \frac{H_D e^{1/2(K_H\zeta)^2}}{(F_{50}/DBE)^{K_H}} \quad (7)$$

Eq. 8 gives a second characterization of H(a).

$$\log H(a) = ba^n \quad (8)$$

where b and n are constants. The objective of an analytical representation of the hazard curve is to facilitate the numerical integration of Eq. 2 for several H(a). Figure 1 shows a family of hazard curves based on Eq. 8 in two sets. The full line hazard Curves 1, 2 and 3 are considered to be representative of high seismicity hazard curves, and the dashed hazard Curves, 4, 5 and 6 are considered to be representative of low seismicity hazard curves. The bolded dot-dashed hazard curves A and B are the High and Low Seismicity Curves A and B, respectively, of Fig. 2-1 of Kennedy and Short (1994). The match of Hazard Curves 2 and 5 with A and B is adequate. The b and n values for all six hazard curves are listed in Table l (together with their A_R and K_H values calculated at $H_D=10^{-4}$). Figure 2 is a plot of A_R calculated at the midpoint of the decade for all six hazard curves of Fig. 1 as a function of H_D. The wide range spanned by Curves 1 and 3, and Curves 4 and 6 would qualify these sets of curves as a reasonable basis for a generalized solution.

Comparison of Results - Equations 4 and 8

Before proceeding any further it would be useful to compare the failure probabilities obtained from Eq. 2 for the two characterizations of H(a), Eqs. 4 and 8. Figure 3 shows an example for ζ=0.4 and the following constants, using Hazard Curve 2 of Fig. 1 as the basis of the comparison:

$$a)\quad H_D = 10^{-3}, DBE = 0.291, A_R = 2.29,\ K_H = 2.78, F_{50} = 0.582, \frac{F_{50}}{DBE} = 2.0$$

$$b)\quad n = 0.406,\ b = -4.96,\ F_{50} = 0.582$$

Table 1: Characteristics of the six hazard curves of figure 1

		High Seismicity			Low Seismicity		
		1	2	3	4	5	6
	b	-6.28	-4.96	-4.22	-6.78	-5.81	-5.06
H_D	n	0.429	0.406	0.384	0.343	0.333	0.321
10^{-4}	A_R	1.80	1.86	1.92	2.08	2.13	2.19
10^{-4}	K_H	3.93	3.72	3.52	3.15	3.05	2.94

The three curves shown in Fig. 3 are the hazard (dashed line), the fragility (dotted line), and the product of the two parameters (full line). The area under the product curve is the failure probability, P_F. The main difference in the two results shown in Fig. 3 is the curvature of the hazard curves (dashed lines). The hazard curve in Figs. 3a is the actual (in log-log plot), whereas that in Fig. 3b is a linear replacement of, Hazard Curve 2 of Fig. 1. The resulting failure probabilities (area under the product curve - full line) are given below (Cases a and b):

Case		**ζ= 0.2**	**ζ= 0.4**	**Comments**
a	Equations 2 & 8	1.34×10^{-4}	2.38×10^{-4}	
b	Equation 5	1.70×10^{-4}	2.70×10^{-4}	A_R at vicinity of H_D
c	Equation 5	1.30×10^{-4}	2.45×10^{-4}	A_R below H_D

As is to be expected from Fig. 3b, Eq. 5 will always result in a larger P_F because the concavity of the true hazard curve is lost and hence the product with the fragility PDF will always be larger than its counterpart in Fig. 3a. Another observation in Fig. 3 is the fact that the product curve is very similar to the fragility curve and hence the contribution of the hazard curve to P_F outside of a fragility band, say at 1% of its peak value is, therefore, minimal.

The K_H value used in the above examples is calculated at the vicinity of $H(a) = 10^{-3}$, from the A_R for the decade from $10^{-3.5}$ to $10^{-2.5}$. If the decade from 10^{-4} to 10^{-3} were used, A_R would equal 2.03 instead of 2.29 and K_H would equal

3.25 instead of 2.78. Thus, for $\zeta = 0.4$, from Eq. 7, $P_F = 10^{-3}\dfrac{e^{1/2(3.25\,x\,0.4)^2}}{2^{3.25}} = 2.45x10^{-4}$ instead of 2.70×10^{-4},

and P_F would equal 1.30×10^{-4} for $\zeta = 0.2$ instead of 1.70×10^{-4}. As compared in the above table (Case c), these results are closer to those obtained directly by use of Eqs. 2 and 8 (Case a). Although the choice of A_R calculation at the vicinity of H_D makes conceptual sense (in that K_H is analogous to the slope of the hazard curve), it seems that the errors due to the linearization of the hazard curve and the calculation of A_R from below H_D counterbalance each other to produce the above encouraging results. Therefore, in the remaining, A_R are calculated from below H_D.

USE OF NEHRP 1997 MAPS

The NEHRP 1977 maps are for the recurrence of ground motion parameters that would occur with a 2% probability in 50 years. Assuming a Poisson process for the occurrence of peak ground motion parameters, this translates to about a mean return period of 2500 years. From Fig. 2 and Table 1, the A_R below H_D (H_D = 1/2500 = 4×10^{-4}) is approximately estimated for the decade from $10^{-4.5}$ to $10^{-3.5}$ (or equivalently at the vicinity of H_D = 10^{-4}). From Eq. 7, the Failure Return Period (FRP =$1/P_F$) is expressed as

$$FRP = \frac{1}{P_F} = \frac{2500\,(DF_{50}\;)^{K_H}}{e^{1/2(K_H\;\zeta\;)^2}} \tag{9}$$

where the Design Factor (DF) is defined as $DF_{50} = F_{50}/DBE$. DF is analogous to load factors used in load and resistance factor design. Figure 4 is a plot of the FRP for all six hazard curves and for several ζ, when $H_D = 4 \times 10^{-4}$.

The following are the significant parameters affecting FRP, in decreasing order of importance:

(a) ζ. The impact of ζ for the range of 0.2 - 0.6 is dramatic. ζ is the more dominant parameter than K_H in relation to FRP and DF_{50} . Adequately determined ζ is important in failure probability estimations.

(b) The significance of High (Hazard Curves 1, 2, 3) and Low (Hazard Curves 4, 5, 6) Seismicity becomes a concern at RFPs greater than about 10,000 years ($P_F = 10^{-4}$). For structures designed to building codes, such as IBC 2000, a failure probability $P_F = 2x10^{-4}$ could be considered as the lower bound of interest.

(c) The differences in P_F are minor within both the high and low seismicity group hazard curves. Based on this observation, in the following analyses Hazard Curves 2 and 5 will be used as representative curves for High and Low seismicities, respectively.

Figure 5 shows the above curves (for Hazard Curves 2 and 5 only) on semi-log plot. Figure 5 can be used as a design tool by entering the ordinate at a target FRP (= $1/P_F$) and reading off the associated DF_{50} for any applicable ζ. The F_{50} of the required fragility curve then is simply the product of the DF_{50} with the NEHRP 1997 mapped value.

Considering observation (b) above, it would seem possible to draw a conservative envelope on the lower side of the curves of Fig. 5 for each ζ such that the variability due to A_R (i.e. High and Low Seismicity) is eliminated. It is noted that the dependence on seismicity is not strong below FRP of 10,000 years. It would thus be possible to establish a conservative envelope that can be used generically. For the range of FRP from 500 to 10,000 years, the following simple expression provides a good prediction of these envelopes:

$$DF_{50} = 0.34\,\zeta^{0.7}\,FRP^{0.27} \tag{10}$$

FROM FAILURE PROBABILITY TO RESISTANCE ($P_F \rightarrow R$)

Having determined DF_{50}, one can then obtain the required fragility curve (in terms of $F_{50} = DF_{50}.DBE$ and ζ) and determine the required design resistance, R. It is important to note that the fragility curves, with ζ and F_{50} parameters, incorporate the median values and variances of the resistance as well as all applicable loads. ζ is a parameter that covers many sins. But primarily it is a measure of the considerable uncertainty in the hazard curves, as well as the quality in analysis, design and construction. The variance of the fragility curve can be obtained from $\zeta^2 = \zeta_R^2 + \zeta_S^2$, where ζ_R^2 and ζ_S^2 are the variances of the resistance and loads, respectively. ζ_S^2 could represent either the variability in the hazard curve and the analysis model, or the variability of all concurrently acting loads, such as seismic, dead and live loads. Thus, different fragility curves would be obtained when the loading is due only to the seismically induced load or when the loading is due to all concurrent loads.

Assuming initially that the fragility curve is due to seismically induced loads only, e.g. when the earthquake lateral loads are resisted primarily by shear walls, $\zeta^2 = \zeta_R^2 + \zeta_E^2$. Depending on the quality of construction and the limit state of interest, ζ_R could potentially vary from 0.1 - 0.15. Determining ζ_E is more of a challenge, and beyond the scope of this paper. Conservatism introduced in response analyses is an important issue but cannot be treated herein due to space limitations. Therefore, it is explicitly assumed that the response analysis provides best estimate results and ζ_E reflects both hazard and model variabilities.

Resistance, R

The next step is to translate F_{50} into a resistance-related parameter. The conditional failure probability $F_{P/a}$ (any one point on the fragility curve) is calculated by the convolution of a load, S_{50}^{a} (for each sequentially selected **a**), and applicable resistance distribution function. When the ratio $\frac{S_{50}^{a}}{R_{50}}$ is small, the failure probability would be small.

As S_{50}^{a} is increased and becomes equal to R_{50}, then ln (R_{50} /S_{50}) = ln 1 = 0 and hence the conditional failure probability is 0.50 (irrespective of ζ). In other words, at F_{50}, the median load and median resistance are equal, and $R_{50} \approx F_{50}$. The median resistance is thus given by

$$R_{50} = DF_{50}\ .\ DBE \qquad (for\ all\ \zeta\ ) \tag{11}$$

This step transfers the problem of determining the resistance from fragility space into resistance space. Any further manipulation of results should consider only the variance of the resistance, i.e. ζ_R^2, and not ζ^2 of the fragility curve.

Design is usually performed with nominal values of loads and resistances. Designating the ratio of the mean resistance, $\overline{R}$, to the nominal resistance, R_n, as $N_R = \overline{R} / R_n$, and noting that for the lognormal distribution the mean and median are related by $x_{50} = \frac{\overline{x}}{\sqrt{1+COV_x^2}}$ (COV = Coefficient of Variation), Eq. 11 can be re-written for R_n as

$$R_n = \left(\frac{\sqrt{1+COV_R^2}}{N_R} . DF_{50} \right) DBE \tag{12}$$

Given the confines of this paper, and as mentioned earlier, only seismic loads are considered. The design equation in code format is $\phi R_n = \alpha_E E_{50}$ (a load combination similar to $\phi R_n = 1.4 D_n$) where α_E is the sought for design load factor on the median seismic load. We will continue to characterize the DBE as the specific median value from the hazard curve rather than a nominal seismic load. From Eq. 12, the seismic load factor becomes

$$\alpha_E = \frac{\phi \sqrt{1+COV_R^2}}{N_R} .DF_{50} \tag{13}$$

<u>Example</u>

Given that $\zeta_R \approx COV_R = 0.13$, $\phi = 0.90$, $N_R = 1.12$ (all for reinforced concrete flexure), and $\zeta_E = 0.38$, calculate the load factor α_E for $P_F = 10^{-3}$ or FRP = 1000 years.

$\zeta = \sqrt{0.38^2 + 0.13^2} = 0.4$, and, from Fig. 5 (Curve 5 controls), DF_{50} = 1.08, and hence $\alpha_E = \frac{0.9\sqrt{1 + 0.13^2}}{1.12} \, x\, 1.08 = 0.87$.

When α_E =2/3, as in the NEHRP 1997 provisions, DF_{50}=0.823 and from Fig. 5 (for ζ=0.4) FRP=400 years for High, and FRP=700 years for Low hazard seismicities.

Design codes specify different ϕ for different actions. The ACI code, for example, specifies $\phi = 0.9$ for flexure and 0.6 for shear in high seismic regions. In addition to reflecting a larger COV (0.20 vs 0.13), the smaller ϕ for shear more importantly reflects the design profession's intent to make seismically induced shear failure significantly less likely . Thus, it may be prudent not to meddle with currently used ϕ values and to normalize α_E with the largest ϕ (i.e. 0.9), as in the above example. The expected relative reliabilities for other ϕ (actions) would thus be maintained.

Design Conservatism in R_n

Lognormal distribution of the resistance is characterized by the median capacity R_{50} and its variance. Given the general expression of the distribution, the resistance can be **equivalently** characterized by any other resistance probability R_P through the following relationship:

$$\frac{R_P}{R_{50}} = e^{-X_P \zeta_R} \tag{14}$$

where X_P is the associated standard deviation of the cumulative standardized normal distribution. Thus, the inherent conservatism introduced in design codes, in addition to the explicit load factors, can be quantified. For the above example R_n / R_{50} equals 0.9, and thus X_p, when ζ=0.13, equals 0.81, and the exceedance probability is 21%.

SUMMARY RESULTS AND CONCLUSIONS

A simple yet viable procedure is presented to calculate the seismic load factor**,** α_E (no other concurrent load is considered herein), to the NEHRP 1997 mapped values, leading to the estimation of the required nominal resistance, R_n, for any specified failure probability. Based on this procedure, Table 2 provides load factors (normalized to reinforced concrete flexure) for several failure return periods. From a designer's perspective the data indicates that α_E is ζ sensitive. The determination of ζ thus becomes an important design consideration.

Table 2: DF_{50} (from Eq. 10) and α_E for selected Failure Return Periods

	ζ=0.3		ζ=0.4		ζ=0.5		ζ=0.6	
FRP (yr)	DF_{50}	α_E	DF_{50}	α_E	DF_{50}	α_E	DF_{50}	α_E
500	0.78	0.6	0.96	0.8	1.12	0.9	1.27	1.0
1000	0.95	0.8	1.16	0.9	1.35	1.1	1.54	1.3
2000	1.14	0.9	1.39	1.1	1.65	1.3	1.85	1.5
5000	1.46	1.2	1.79	1.5	2.09	1.7	2.37	1.9

Table 3 shows results from a reverse process. It lists estimates of failure return periods for three Importance Factors and incorporates the 2/3 (α_E?) factor of the NEHRP 1997 provisions applied to the Maximum Considered Earthquake. It is interesting to note that for all ζ the failure return periods approximately double as the I factor is increased to 1.25 and 1.5. It should be noted that FRPs could increase, particularly for low seismicities, when dead and live load effects are considered (acting concurrently, as in moment resisting frames) since significant reserve resistance ($1.4D_n$) would then be available. For this class of structures, D_{50}/E_{50} is a critical parameter.

Table 3: Failure Return Periods (nearest 100 yrs) for selected Importance Factors (DF_{50} from Eq. 9)

I Factor	ζ=0.2	ζ=0.3	ζ=0.4	ζ=0.5	ζ=0.6
1.0	900	600	400	200	100
1.25	2100	1500	900	500	200
1.5	3900	2900	1800	1000	500

REFERENCE

Kennedy, R. P. and Short, S. A. (1994) Basis for Seismic Provisions of DOE-STD-1020" UCRL-CR-111478 and BNL-52418, LLNL and BNL.

Disclaimer: *The views expressed in this paper are solely those of the author, and no endorsement of the paper by the Defense Nuclear Facilities Safety Board is intended nor should be inferred*

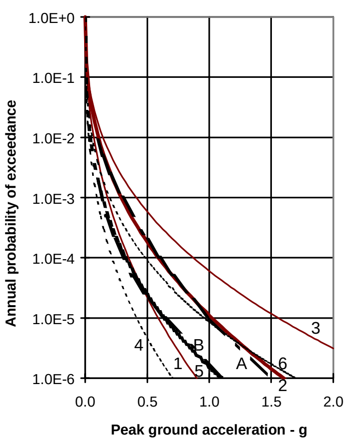

Figure 1: Idealized High (1, 2, 3) and Low (4,5,6) seismicity hazard curves

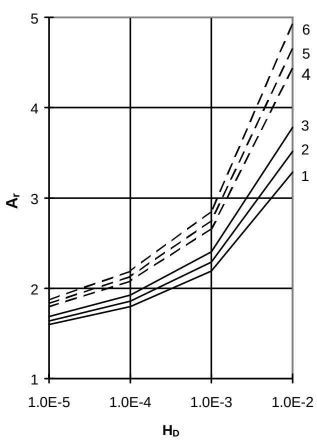

Figure 2: Variation of A_r along hazard curves 1-6

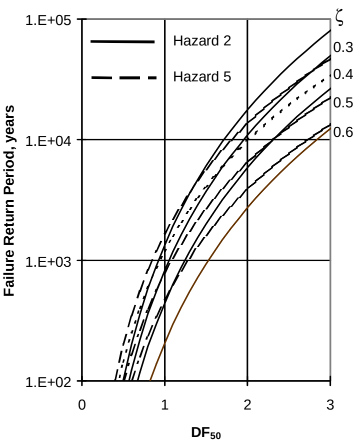

Figure 5: Design Factor vs Failure Return Period for hazard curves 2 and 5 for several ζ

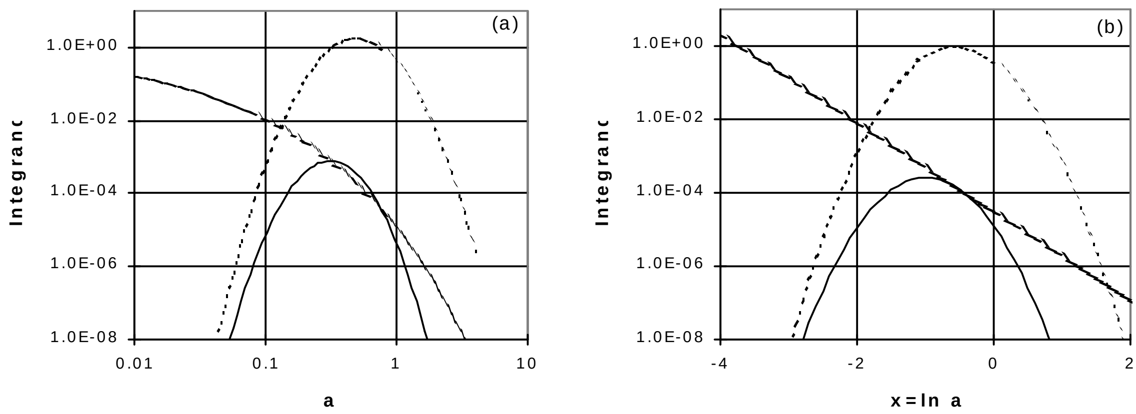

Figure 3: comparison of the two characterizations of hazard curves, equations 8 and 4

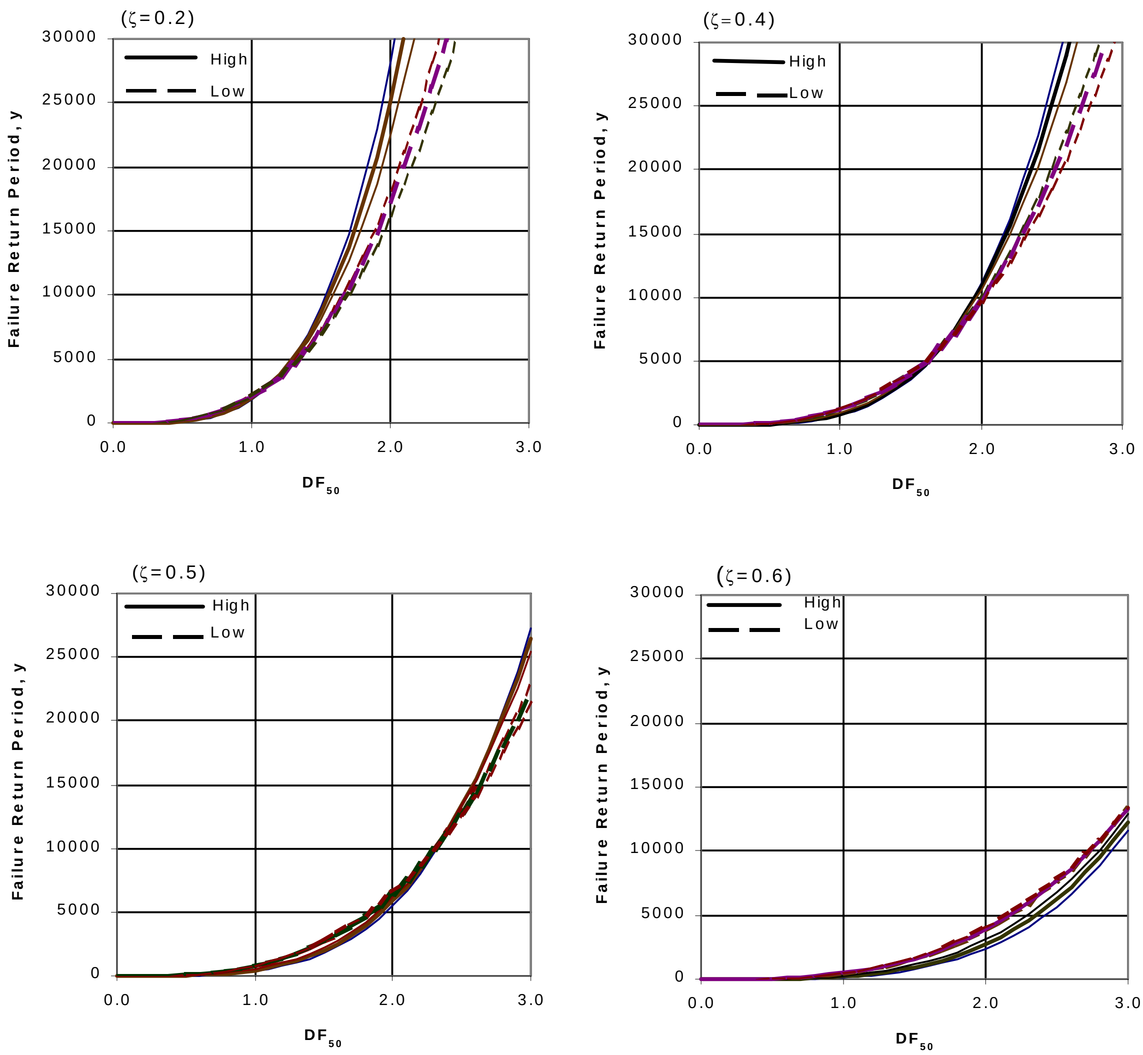

Figure 4: Failure Return Period vs DF_{50} for High and Low seismicities and several ζ