SHEAR STRENGTH EVALUATION METHOD FOR MULTI-STORY PRECAST CONCRETE STRUCTURAL WALLS

Masato FURUTANI[1], Hiroshi IMAI[2] And Toshio MATSUMOTO[3]

SUMMARY

The purpose of this study is to propose a calculation procedure using a macro model to evaluate the shear strength of multi-story precast concrete structural walls with columns at both sides. Since the shear strength equations for the monolithic reinforced concrete are usually used in Japan even for precast concrete structural walls with horizontal joints, such multi-story precast walls are expected to slip at the horizontal joints of bedding mortar type under earthquake forces. At the moment, there are only a few studies that show a theoretical evaluation method for determining the shear strength of precast walls taking into account the slip. A new macro model is proposed and a comparison between the analytical and actual test results from 22 multi-story wall specimens with bedding mortar type horizontal joints is done.

The proposed model is composed of two shear resisting mechanisms, before and after the slip at the horizontal joint. Before the wall slips, the truss and total arch mechanisms for the whole structure resist the shear forces. However, after the slip, the shear resisting mechanism is composed of only the story arch mechanism. Thus, the total shear strength is given by the superposition of these two shear resisting mechanisms.

As a result, the proposed model gives a reliable evaluation of the shear strength of multi-story precast concrete structural walls and an adequate prediction of the resulting failure mode.

INTRODUCTION

Nowadays, precast concrete structural systems are widely used in Japan for condominium buildings. These types of structures are composed of frames in the longitudinal direction and of structural walls in the transverse direction.

To have a better workability at the construction site, the precast panels are placed using bedding mortar, without shear keys, at the horizontal joints. Therefore, under seismic actions they might slip at the horizontal joint.

Up to now, due to the lack of theoretical studies, the calculations of these types of structures have been done using a similar procedure used for monolitic reinforced concrete structures, without taking into account the slip at the horizontal joint. However, as shown in figure 1, the failure mode of the precast concrete wall is different from the monolithic one, therefore, an investigation of the proposed shear strength evaluation method for multi-story precast concrete walls is needed.

[1] *Institute of Engineering Mechanics, University of Tsukuba, Japan Email: mfurutan@ma4.justnet.ne.jp*
[2] *Institute of Engineering Mechanics, University of Tsukuba, Japan.*
[3] *Technical Research Institute, ANDO Corporation, Japan Email: LDZ06301@nifty.ne.jp*

PROPOSED MODEL

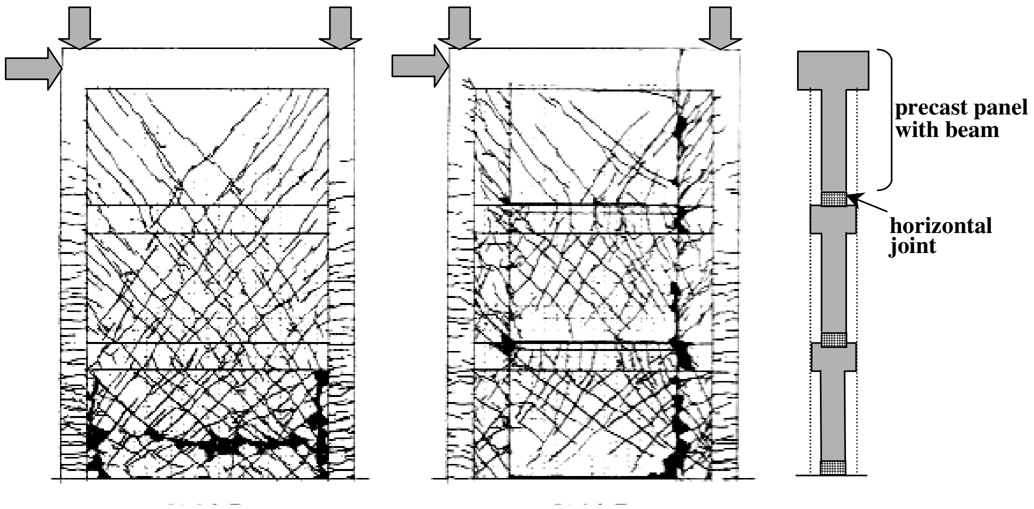
The proposed model is composed of the shear resisting mechanisms acting before and that acting after the slip at the horizontal joints. Before the panel slips, as shown in Figure 2(a), the truss and total arch mechanisms for the whole structure resist the shear forces, which is similar to that of a monolithic wall.

However, after the slip, as shown in Figure 2(b), only the story arch mechanism is the active mechanism against the additional shear force. For simplicity, it is assumed that the vertical joint failure does not have much influence on the total shear strength.

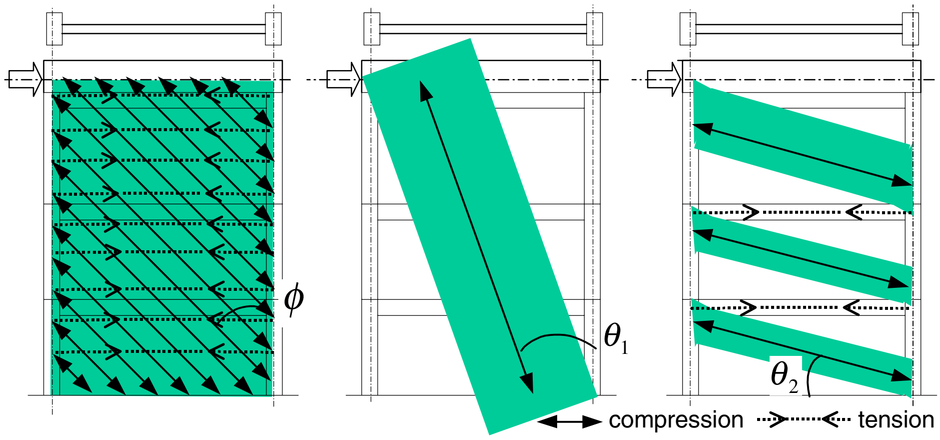
Shear resisting mechanism before slip

As recommended by the Architectural Institute of Japan (AIJ), the truss and arch mechanisms are used as the shear resisting mechanisms. The truss mechanism is represented by an analogous truss composed of the main reinforcements, shear reinforcements and concrete struts inclined with an angle parallel to the induced shear cracks. On the other hand, the arch mechanism is represented by a concrete arch lying along the main diagonal of the structural wall. In this study, in order to avoid a confusion with the arch mechanism produced in each story, the arch mechanism associated with the whole structure similar to a monolithic type is referred to as the total arch mechanism.

Moreover, the truss and arch theory has two assumptions; (1) the total diagonal compressive stress in the concrete generated by the combined truss and arch actions reaches the effective compressive strength of concrete, and (2) the stress in horizontal shear reinforcement reaches the yielding strength.



(a) monolithic **(b) precast**



(a)truss and total arch mechanisms **(b)story arch mechanism**

Shear resisting mechanism after slip
After the slip at the horizontal joint, the shear resisting mechanism against the additional shear force is composed of only the story arch mechanism. The story arch mechanism, which is different from total arch mechanism, is composed of the boundary column, the inter-story beam and the diagonal strut of a single panel. Each member can resist the acting shear force using the remaining strength available in the horizontal shear reinforcement and the concrete after the slip occurs.

In the story arch mechanism, failure of a structural wall could be due to a shear failure of the boundary column, tensile yielding of the longitudinal reinforcements of the inter-story beam, or compression failure of the diagonal concrete strut of the panel. The minimum value of the strength corresponding to these three failure modes gives the shear strength for this resisting mechanism. Therefore, the proposed model is also capable of predicting, at the same time, the failure mode.

Bending moment distribution on the boundary columns
As shown in Figure 3, the bending moment distribution produced on the boundary columns is considered as the superposition of two actions; (a) the rigid frame deformation action, and (b) the strut action of the story arch mechanism. Thus, by superposition effects as shown in Figure 3(c), the bending moment distribution is produced only at the bottom of the compression side column and at the top of the tension side column on each floor, assuming that the acting shear force **Q** of (a) is equal to that **q** of (b).

Width of the diagonal strut of the wall
Considering the strut action shown in Figure 3(b) and the equilibrium of the story arch mechanism shown in Figure 4, the width ($h_w - x$) of the diagonal strut of the panel is obtained from the following procedure. Some of the notations are listed at the last page.

$$2q = N_d \cdot \cos\theta_2 \left(\frac{h_w + x}{2h_w} - \frac{h_w - x}{2h_w} \right) \tag{1}$$

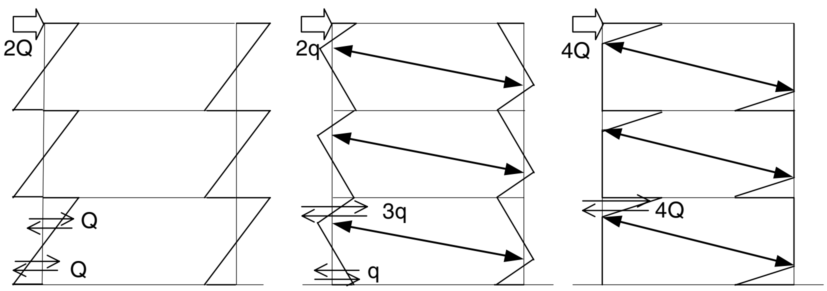
$$= \nu\sigma_B t_w \cdot (h_w - x) \cdot \frac{l_w^{\,2}}{l_w^{\,2} + x^2} \cdot \frac{x}{h_w} \tag{2}$$

where

$$N_d = \nu\sigma_B t_w \cdot (h_w - x) \cdot \cos\theta_2 \tag{3}$$

$$\cos\theta_2 = \frac{l_w}{\sqrt{l_w^{\,2} + x^2}} \tag{4}$$

Based on the lower bound theory of plastic limit analysis, eqn. (5) gives the following approximation to x while **2q** is maximum, since $l_w / h_w = 3 \sim 4$.



(a) frame deformation action (b) strut action (c) combined action(a)+(b)

$$x = l_w \left\{ \sqrt{\left(\frac{l_w}{h_w} \right)^2 + 1} - \frac{l_w}{h_w} \right\} \doteqdot \frac{h_w}{2} \tag{5}$$

where

$$\sqrt{\left(\frac{l_w}{h_w}\right)^2 + 1} \doteqdot \frac{l_w}{h_w} + \frac{h_w}{2l_w} \qquad (6)$$

Thus, as can be seen in Figure 5, the width of the diagonal strut is approximately half the height of the panel (h_w), and that the horizontal component of the resultant force of the strut affects the boundary column at a point one-fourth the height of the panel.

SHEAR STRENGTH CALCULATION

The shear strength of the multi-story precast concrete structural wall is obtained from eqn. (7). The first term on the right-hand side represents the shear force carried by the truss and total arch mechanisms before the slip occurs. The second term indicates the shear force carried by the story arch mechanism after the panel slips.

$$Vu_{(PCa)} = V_{Hj} + \min({}_cV_{cu},\ T_u,\ V_d) \qquad (7)$$

where

$Vu_{(PCa)}$: shear strength of multi-story precast structural wall

V_{Hj} : shear strength of the horizontal joint

${}_cV_{cu}$: shear strength of the boundary column

T_u : tensile yielding strength of longitudinal reinforcements of the inter-story beam

V_d : horizontal component of the compression failure strength of a concrete strut of the panel

Horizontal joint shear strength

The equation for shear strength of the horizontal joint is expressed by eqn. (8)

$$V_{Hj} = \mu \cdot a_w \sigma_{wy} + \mu \cdot N_S \qquad (8)$$

where

μ : the coefficient of friction (=0.7)

a_w : cross-sectional area of the vertical reinforcements crossing through horizontal joint

σ_w : stress of the vertical reinforcement when the wall slips

N_S : axial load which is produced by the action of the horizontal forces on the structural wall

As shown in Figure 6, N_S is obtained as eqn(9).

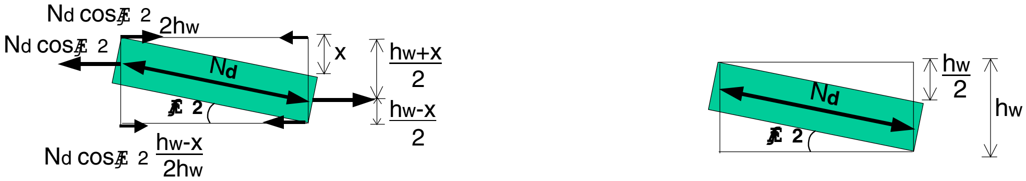
$$N_S = Vt_{(PCa)} / \tan\phi + Va_{(PCa)} / \tan\theta_1 \qquad (9)$$

where

$Vt_{(PCa)}$, $Va_{(PCa)}$: the shear forces which the truss and total arch mechanisms carry

ϕ , θ_1 : the angles of the concrete struts of the truss and arch mechanisms

To evaluate the first term of eqn.(8), eqn.(10) is taken into account.



$$\mu \cdot a_w \sigma_w = n \cdot 11.5 a_s^{0.96} \sigma_y^{0.35} \sigma_B^{0.5} (Ec / Es)^{0.42} = Do \qquad (10)$$

where

n : number of vertical joint reinforcements crossing through horizontal joint

a_s : cross-sectional area of a vertical reinforcement

σ_y : yield strength of the vertical reinforcements

σ_B : compressive concrete strength

Ec , Es : the Young's modulus of concrete and vertical reinforcements

Now, by supposing $\phi = 45^\circ$ and substituting eqn.(9)and(10) for eqn.(8), we obtain eqn. (11) is obtained.

$$V_{Hj} = Do + \mu \cdot (Vt_{(PCa)} + Va_{(PCa)} / \tan\theta_1) \tag{11}$$

Eqn.(11) can be transformed into eqn.(12).

$$V_{Hj} = \left(D_o \cdot Vt / Vu + \mu \cdot Vt_{(PCa)}\right) + \left(Do \cdot Va / Vu + \mu \cdot Va_{(PCa)} / \tan\theta_1\right) \tag{12}$$

The first and second terms of eqn.(12) mean the shear strengths of the horizontal joint which the truss and total arch mechanisms carry.

On the other hand, since the shear force is carried by the truss and total arch mechanisms before the panel slips, eqn.(13) is obtained.

$$V_{Hj} = Vt_{(PCa)} + Va_{(PCa)} \tag{13}$$

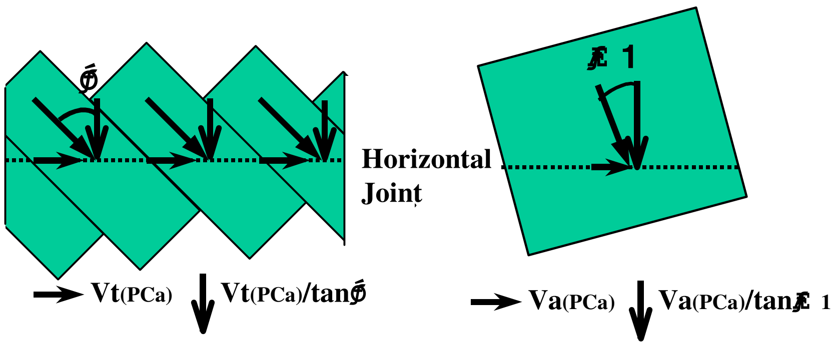
Considering eqn.(12) and eqn.(13), eqn.(14) is given in respect to the truss mechanism. Eqn.(15) is obtained by solving eqn.(14).

$$Vt_{(PCa)} = Do \cdot Vt / Vu + \mu \cdot Vt_{(PCa)} \tag{14}$$

$$Vt_{(PCa)} = \frac{Do}{1-\mu} \cdot \frac{Vt}{Vu} \tag{15}$$

Moreover, since $\tan\theta_1$=0.22□0.67 and μ =0.7, in the second term of eqn.(12), the frictional force $\mu \cdot Va_{(PCa)} / \tan\theta_1$ produced by the vertical component of force of the arch mechanism is over estimated than $Va_{(PCa)}$ itself. So the second term of eqn.(12) is replaced by $Va_{(PCa)}$ not to be contradicted. As for $Va_{(PCa)}$, eqn.(16) gives.

$$Va_{(PCa)} = Va \cdot \frac{Vt_{(PCa)}}{Vt} \tag{16}$$



(a) truss mechanism **(b) arch mechanism**

Finally, substituting eqn.(15) and (16) for eqn.(13), eqn.(17) is given.

$$V_{Hj} = \frac{D_o}{1-\mu} \tag{17}$$

The shear strength equation for the story arch mechanism

Based on the truss and arch theory, the shear strength equation for the boundary column is given by eqn. (18).

$$ {}_cV_{cu} = D_c b_c p_{ch} \sigma_{ch} \cot\phi + \tan\theta_c (1-\beta_c)\cdot D_c b_c \nu\sigma_B / 2 \qquad (18)$$

where

p_{ch} : shear reinforcement ratio

σ_{ch} : yield strength of the shear reinforcement

$$\tan\theta_c = \sqrt{(h_w/4D_c)^2+1} - h_w/4D_c \qquad (19)$$

$$\beta_c = (1+\cot^2\phi)\cdot p_{ch}\sigma_{cy}/(\nu\sigma_B) \qquad (20)$$

The first term on the right-hand side of eqn. (18) represents the shear force carried by the truss mechanism, and the second term indicates that carried by the arch mechanism. The tensile yielding strength of the longitudinal reinforcements of the inter-story beam is obtained from eqn.(21).

$$T_u = A_b \sigma_{by}(1-\alpha) \qquad (21)$$

where

$$\alpha = V_{Hj}/Vu \qquad (22)$$

A_b : cross-sectional area of the longitudinal reinforcement in a floor

σ_{by} : yield strength of the longitudinal reinforcement

To calculate the compression failure strength of the diagonal concrete strut of the panel, eqn. (23) is used.

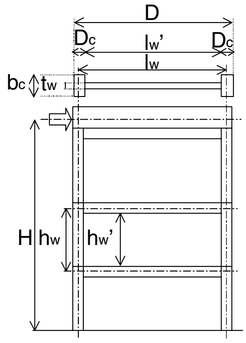
$$V_d = \nu\sigma_B(1-\alpha)\cdot h_w/2\cdot t_w\cdot\cos^2\theta_2 \qquad (23)$$

THE SPECIMENS

In this study, to discuss the validity of using the proposed model, a comparison between the analytical and actual test results from 22 specimens of multi-story precast shear wall with the horizontal joint of bedding mortar type is done. The features of each specimen are summarized in Figure 7 and Table 1.

All of the specimens are subjected to varying lateral forces, which are applied in a cyclic manner at the top of specimens. The 22 specimens are about 1 / 3 scaled actual walls and have precast panels. Some of the specimens have 2 or 3 precast panels in a span.

As shown in Figure 8, all specimens have the mortar bedding type horizontal joints, bedding mortar connects the bottom end of panel and the top of the beam.



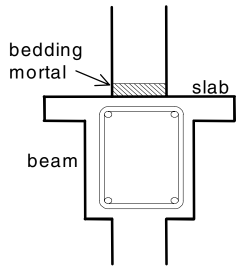


Figure 7: Specimen Figure 8: Details of the

Table 1: Details of specimens

Refer.	Specimen	Dc <bc> (cm)	lw <tw> (cm)	H <hw> (cm)	p_{wh} (%)
[Matsumoto et al,'97]	No.2 No.3 No.4 No.5	21 <34>	219 <7.2>	336 <122>	0.38
[Matsumoto et al,'98]	No.7	21 <34>	219 <7.2>	336 <122>	0.80
[Inoue et al,'93]	WS1 WS2 WS3 WF	20 <60>	250 <7.5>	312 <100>	0.37
[Kuramoto et al,'94]	WF1 WF2 WF3 WF4				
[Inukai et al,'91]	PCWALL1	30 <30>	300 <10>	700 <116.6>	0.61
	PCWALL2			424.7 <116.6>	0.16
[Yanase et al,'94]	PW0 PW1 PW2	30 <30>	170 <8.5>	240 <128.25>	0.43
	PW3	35 <30>	165 <8.5>		0.46
[Iida et al,'90]		35 <46>	385 <6.8>	172 <96>	0.41
[Tanaka et al,'87]	PCW-6101 P W 6103	22.5 <22.5>	234 <6>	180 <96>	0.67

COMPARISON WITH TEST RESULTS

To compare with conventional equations, the shear strength equation modified by Hirosawa[AIJ, 1990a], given in eqn. (24), and that recommended by the AIJ[AIJ, 1990b], given in eqn. (25), were used. The modified Hirosawa's equation is an empirical equation and is used in Japan for the design of monolithic reinforced concrete shear walls which includes the precast type. On the other hand, the AIJ equation is a theoretical equation based on the truss and arch theory.

$$V_{su} = \left\{ \frac{0.068 p_{te}^{0.23} (\sigma_B + 180)}{\sqrt{M/(QD) + 0.12}} + 2.7\sqrt{\sigma_{wh} p_{wh}} + 0.1\sigma_0 \right\} \cdot b_e \cdot j \tag{24}$$

$p_{te} = 100 \cdot a_t / (b_e \cdot lw')$

a_t : sum of the cross-sectional area of the longitudinal reinforcements of the tensile boundary column

σ_B : compressive concrete strength

$M/(QD)$: shear span-to-depth ratio

σ_{wh} : yield strength of the shear reinforcement

p_{wh} : shear reinforcement ratio

σ_0 : averaged constant axial stress, $d = D - Dc/2$

$b_e = (2Dc \cdot bc + l_w' \cdot t_w)/D$, $j = 7/8 \cdot lw'$

$$Vu = t_w l_w p_{wh} \sigma_{wh} \cot\phi + \tan\theta_1 (1-\beta) \cdot t_w D \nu \sigma_B / 2 \tag{25}$$

where

$$\beta = (1 + \cot^2\phi) \cdot p_{wh} \sigma_{wy} / (\nu \sigma_B) \tag{26}$$

$$\tan\theta = \sqrt{(H/D)^2 + 1} - H/D \tag{27}$$

$\nu = 0.7 - \sigma_B / 2000$: effective factor for the compressive strength of concrete

Figures 9-11 show the results of the analyses. The calculated and experimental shear strengths of multi-story precast concrete structural walls were normalized by the ultimate flexural strength Vmu expressed in eqn. (28), and were plotted in the horizontal and vertical axes, respectively.

$$Vmu = (a_t \sigma_y + 0.5 a_w \sigma_{wy} + 0.5N) \cdot l_w / H \tag{28}$$

σ_y : yield strength of the longitudinal reinforcements

a_w : cross-sectional area of the vertical reinforcements

σ_{wy} : yield strength of the vertical reinforcements

In the case of V_{Hj} being larger than Vu, the multi-story precast structural wall specimens are regarded as monolithic ones and that the shear strength is given by eqn. (25).

The calculation using the modified Hirosawa's equation provides higher values compared to the actual strengths, while the AIJ's equation gives results close to the mean value. When both equations, which are in general used for monolithic walls, are applied to multi-story precast walls, calculations show that the strengths of the specimens having horizontal joints of the mortar bedding type are over estimated than the other types of specimens.

Specimens, which have more than 2 panels in a span, are over estimated by the modified Hirosawa's and the AIJ's equation.

With regard to the proposed evaluation method, most of the estimated shear strengths were close and, moreover, conservative compared to the actual strengths even for the specimens which have more than 2 panels in a span.

In this analysis, there is no influence on the shear strengths by joint reinforcements at vertical joints.

Results show a good agreement between the actual and the theoretical failure modes. Of the 22 specimens considered, very good correspondence is observed for 10 specimens.

CONCLUSIONS

The proposed evaluation model, which takes into account the slip at the horizontal joint of multi-story precast walls, provides a good conservative estimation of the shear strength. With regard to the failure mode, the analyses results show good agreement between the actual and the theoretical failure modes.

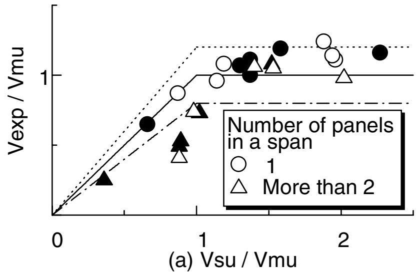


Figure 9: Modified Hirosawa eqn.

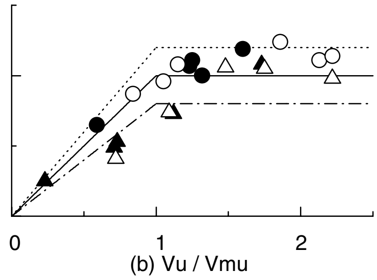


Figure 10: AIJ eqn.

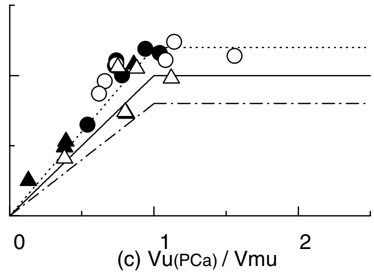


Figure 11: Proposed model

REFERENCES (ALL IN JAPANESE)

AIJ (1990a), *Ultimate Strength and Deformation Capacity of Buildings in Seismic Design*

AIJ (1990b), *Design Guidelines for Earthquake Resistant Reinforced Concrete Buildings Based on Ultimate Strength*

Furutani, et al (1998), "Evaluation Method of Shear Strength of Multi-Story Precast Concrete Structural Walls", *Summaries of Technical Papers of Annual Meeting AIJ*, C, Structures IV, pp. 807-810

Iida et al (1990), "Development of High-Rise Building Constructed With Precast Reinforced Concrete Members (Part 5)", *Summaries of Technical Papers of Annual Meeting AIJ*, C, Structures II, pp. 445-446

Inoue et al (1993), "Cooperative Researches on WR-PC Construction Method (Part 1, 2)", *Summaries of Technical Papers of Annual Meeting AIJ*, C, Structures II, pp. 529-532

Inukai et al (1991), "Experimental Study on Shear Walls of High-Rise Precast Reinforced-Concrete Frame Buildings (Part 1, 2)", *Summaries of Technical Papers of AIJ*, C, Structures II, pp. 787-790

Kuramoto et al (1994), "Cooperative Researches on WR-PC Construction Method (Part 3, 4)", *Summaries of Technical Papers of Annual Meeting AIJ*, C, Structures II, pp. 327-330

Matsumoto, et al (1997), "A Study on Shear Resistant Mechanism of Multi-story Shear Walls with Infilled Precast Concrete Panels", *ANDO Technical Research Report*, Vol. 3, pp. 33-42

Matsumoto, et al (1998), "A Study on Shear Resistant Mechanism of Multi-story Shear Walls with Infilled Precast Concrete Panels (Part 2)", *ANDO Technical Research Report*, Vol. 4, pp. 63-74

Suzuki et al (1996), "Experimental Study on Shear Behavior in Connection of Pre-cast Concrete", *Proceedings of the Japan Concrete Institute*, Vol. 18, No.2, pp. 1199-1204

Yanase et al (1994), "Experimental Study on Shear Behavior of Precast Concrete Shear Wall", *Reports of Sato Kogyo Engineering Research Institute*, No. 20, pp. 53-60

AIJ : Architectural Institute of Japan