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AN INTERVAL ANALYSIS APPROACH TO DEAL WITH EARTHQUAKE RESPONSE OF UNCERTAIN STRUCTURAL SYSTEMS

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SUMMARY

This work is concerned with the determination of exact bounds of structural responses of linear systems with uncertain parameters. The set of plausible parameter values is defined in bounded intervals. System response quantities such as displacement, velocity, acceleration, stress, strain, interstory drift, and system reliability become uncertain, yet bounded, in a multidimensional rectangular prism. The exact bounds for system response quantities that do not include internal extrema in the uncertain space are determined by a combinatorial interval analysis. If internal extrema are present, a global optimization technique is used to determine the maximum and minimum values of the system responses. Approximation concepts are considered for an efficient numerical implementation of the proposed methodology. The usefulness and effectiveness of the method are illustrated with a simple system. Numerical results show that great insight into the behavior of the structural system can be gained using this methodology. Finally, some extensions of the proposed formulation are presented.

INTRODUCTION

Many real systems have uncertainty in their definition. One source of uncertainty is in the structural characteristics. This uncertainty arises during the modeling of the structural behavior of the system due to many factors. For example, variation of the material properties during manufacture, inexact modeling of the material's constitutive behavior, uncertainties introduced during the construction process, inexact modeling of the boundary conditions, unmodel nonstructural components, physical imperfections, system complexities, etc. Another source of uncertainty in many analyses is in the specification of the loading characteristics. Random processes are usually used as mathematical tools for modeling the uncertain load time histories, such as those due to seismic excitation, blast loading on structures, wind excitation, water wave excitation, etc. Random vibration theory can be used to obtain probabilistic descriptions of the response such as mean, variance, higher-order statistics, and reliability estimates. Uncertainty in both structural characteristics and loading has a direct relationship to the reliability of many engineering structures, and therefore, it should be considered in order to quantify the resulting uncertainty in the structural response [Jensen and Iwan (1992), Katafygiotis and Beck (1988), Spencer and Elishakoff (1988)]. Probabilistic methods can provide the means of incorporating structural uncertainties in the analysis of the system response by describing the uncertainties as random variables. In this context, random variables are used to model uncertainties in discrete parameters in the same way as, for example, random processes are used to model the uncertain transient variation of random-like excitation [Madsen et al. (1986)]. The probability distribution assigned to each random variable describes how plausible each possible value is for the corresponding parameter. In the same manner, random fields possessing certain correlation structure can be used to quantify the uncertainty in the case of continuous variables [Lin (1967)].

A probability description is usually derived from available data or evidence. However, when there is lack of certainty in evidence or simply when it does not exist, the standard probabilistic approach is not appropriate. In this case, an interval analysis approach [Alefeld and Herzberger (1983), Moore (1979)] can be used to evaluate

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the effects of uncertainty in structural and loading characteristics on the system response. In this approach, it is assumed that the uncertainties in the system properties and loads are bounded from above and below. The system response quantities, such as displacements, internal forces, stresses, and strains become uncertain, yet bounded, in a multidimensional rectangular prism. In order to determine the bounds for the response quantities one may employ simulation methods. In general, simulation methods are quite powerful but very costly in terms of computational resources. An alternative simple and easy formulation, which employs interval algebra together with some basic inequalities and linear programming, has been suggested recently [Koyluoglu et al. (1995)]. As the number of uncertain parameters increases, the approach computes more and more conservative results for the bounds of the response quantities.

The objective of this study is to introduce an efficient methodology for determining exact bounds of structural responses of systems with uncertain parameters. The method to be described is based on combinatorial interval analysis and approximation concepts. The methodology is valid only for system response functions that do not include internal extrema in the uncertain space. If internal extrema are present, the proposed methodology is extended to determine the correct bound of structure responses by using global optimization techniques. First, the method based on combinatorial interval analysis is presented. The technique is then extended for the general case of response functions with internal extrema. Approximation concepts are introduced for an efficient implementation of the proposed methodology. Finally, a simple example problem is considered to illustrate the performance of the procedure set forth.

METHODOLOGY

Let the vector $\{q\}(q_i,\ i = 1,...,m)$ represents all uncertain system parameters. Each parameter q_i is assumed to have uncertainties with constant envelope bounds $q_i^l \leq q_i \leq q_i^u$, $i = 1,..., m$, where q_i^l is the minimum value that the uncertain parameter can take, and q_i^u its maximum value. The general equation of equilibrium of a n-degree of freedom linear structural system subjected to external forces can be cast in the form

$$[M\{q\}]\{\ddot{x}(\{q\},t)\}+[C\{q\}]\{\dot{x}(\{q\},t)\}+[K\{q\}]\{x(\{q\},t)\}=[P\{q\}]\{f(t)\} \qquad [1]$$

where $\{x(\{q\}t)\}$ is the displacement response vector of dimension n, $[M(\{q\})]$, $[C(\{q\})]$, and $[K(\{q\})]$ are the mass, damping and stiffness matrices of dimension n x n, $\{f(t)\}$ is the excitation vector of dimension N, and $[P(\{q\})]$ is a matrix of dimension n x N that couples the excitation $\{f(t)\}$ to the degrees of freedom of the structure. A system response quantity such as displacement, velocity, acceleration, stress, strain, interstory drift, and system reliability can be written in general form as $h(\{x(\{q\},t),\{q\})$, where h is an explicit function in $\{x\}$ and $\{q\}$. The explicit dependence of h in $\{q\}$ arises when, for example, the response quantity is a stress component and the modulus of elasticity involved in the computation of the stress is assumed to be uncertain. The response quantity h is in general a nonlinear implicit function of the uncertain parameter set $\{q\}$. The solution of equation 1, for a given value of the parameter set $\{q\}$, can be solved directly using some numerical integration technique or modal analysis. Then, the system response quantity h at time t can be considered as an algebraic expression containing m variables, that is

$$g(q_1,q_2,...,q_m) = h(\{x(\{q\},t)\},\ \{q\}) \qquad [2]$$

It is clear that some of the variables q_i, $i = 1,..., m$ may appear more than once in the algebraic expression g. In that case, the interval operations involving the variable must be handled simultaneously, otherwise, the identity of the variable in its occurrences in the expression is lost [Dong and Wong (1987)]. The resulting interval is wider than it should be and contains the correct interval (conservative bounds). As previously stated, the uncertain parameters q_i, $i = 1,..., m$ have corresponding envelope bounds $[q_i^l, q_i^u]$ The $2m$ end-points can be combined in 2^m distinct combinations or permutations of an array $(q_1, q_2,..., q_m)$, where q_i can be either q_i^l or q_i^u , $i = 1,..., m$. Denote these arrays by $\alpha_1,...,\alpha_{2^m}$, where $\alpha_1 = (q_1^l,..., q_m^l)$, $\alpha_2 = (q_1^u,..., q_m^l)$, ..., $\alpha_{2^m} = (q_1^u,..., q_m^u)$. If the algebraic expression g does not include internal extrema inside the intervals $[q_i^l, q_i^u]$, $i = 1,..., m$, then its bounds are given by the interval

$$[g^l, g^u] = [Min\{g(\alpha_1),..., g(\alpha_{2^m})\}, Max\{g(\alpha_1),..., g(\alpha_{2^m})\}] \qquad [3]$$

It is noted that when multiple occurrences of a variable are counted more than once, the number of permutations increases from 2^m to 2^{m+p} , where p is the additional occurrences of variables considered independently. From equation 3, it is clear that the interval for g resulting from the 2^{m+p} permutations can be larger than the interval corresponding to the 2^m permutations. The true interval is always contained in the incorrect interval obtained by the unconstrained analysis. The implementation of the combinatorial interval analysis approach given by equation 3 is based on the Fuzzy Weighted Average (FWA) algorithm [Dong and Wong (1987)]. This scheme is a discrete version of fuzzy mathematics that uses an interval analysis at discrete membership values. In this context, each membership value corresponds to a given level of uncertainty of the system parameters. The method provides exact bounds of general system responses in a very efficient and simple manner.

The previous implementation is only valid for response functions that do not include internal extrema in the uncertain space. This is because only the endpoints of the uncertain system parameter intervals are used in the computation. If internal extrema are present, the FWA scheme can be extended to determine the correct bounds of the system responses. In this case, the approach relies on a subdivision of the intervals of the uncertain parameters and a subsequent recomputation of system responses with the new intervals. The complexity of the algorithm is dependent on the subdivision structure and the placement of the internal extrema with respect to the subdivisions. Such a dependency causes the upper bound on complexity to greatly exceed that of the standard FWA algorithm. Alternatively, global optimization techniques can also be used to find the maximum and minimum values of the system responses in the uncertain space. In this study, an algorithm based on multistart techniques is implemented to find the global optimum for general nonconvex problems [Betro and Schoen (1987)]. In this method, initial random points are generated in the uncertain region, and local optima are determined using standard nonlinear programming techniques. Stopping rules based on a Bayesian approach and regions of attraction are used to stop the multistart procedure. In the Bayesian approach, each local optimum that a local search finds is viewed as an observation from an unknown distribution. After k local searches, the available data is used to decide whether to continue with new searches or to stop [Boender and Rinnooy-Kan (1987)]. On the other hand, the stopping rules based on regions of attraction use an estimate of the relative sizes of the regions of attraction that the algorithm has not detected. The algorithm stop as soon as the posterior expected relative size of the detected regions, exceeds a user specified number [Rinnooy-Kan and Timmer (1986)].

NUMERICAL IMPLEMENTATION

The system response function h is in general a nonlinear implicit function of the parameter set $\{q\}$. The combinatorial interval analysis and the global optimization approach described in the previous section require the evaluation of the system response function many times. This can be very costly in terms of computational resources because system response functions of real systems are evaluated only in an algorithmic or numerical way, for instance, by means of a finite element model. In order to develop an efficient numerical implementation of the methodology, approximation concepts are considered in the proposed formulation. The basic ideas used in the approximation concept method [Barthelemy and Haftka (1993)] are extended in a straightforward manner for the evaluation of response functions of multiple degree of freedom linear structural systems. A general system response h can be rewritten as

$$h(\{x(\{q\},t)\},\{q\}) = r\,(t,\{w(\{z(\{q\})\})\},\{z(\{q\})\},\{q\}), \qquad [4]$$

where $\{w\}(w_i, i\in I)$ denotes intermediate response quantities, $\{z\}$ $(z_j, j \in J)$ denotes intermediate parameters, and I and J are sets of indices. In equation 4, it is assumed that 1) r is explicit in $\{w\}$, $\{z\}$, $\{q\}$ and t; 2) w_i, $i \in I$ are implicit functions of $\{z\}$; and 3) $z_j, j \in J$ are explicit functions of $\{q\}$. The evaluation of the intermediate response quantities (w_i, i $\in$ I) can be very costly in terms of computational resources. In this formulation, analytical approximations of the intermediate response quantities are used and they are constructed by approximating the functions w_i, $i \in I$ explicitly in terms of the intermediate variables $\{z\}$, to give $\tilde{w}_i$, $i \in I$. Once the approximations have been obtained, the system response $h(\{x(\{q\},t)\},\{q\})$ can be approximated and written explicitly in terms of the set of original variables $\{q\}$ due to the explicitness of the function r. The approximation of the response functions of a n-degree of freedom linear structural system can be based on the direct solution of the equation of motion or by modal analysis. It has been shown that the direct solution approach has convergence problems because it gives poor approximations for the system responses [Iwan and Jensen (1993)]. In the modal solution approach, the dynamic response is represented by a truncated linear combination of complex mode shapes and modal participation coefficients. The modal participation coefficients can be written explicitly in terms of the modal energies [Sepulveda and Thomas (1993)]. These terms, which are implicit non-linear functions of the uncertain system parameters, are taken as intermediate response quantities and approximated by using a convex linearization with respect to selected intermediate variables [Fleury and Braibant (1986)], that is

$$\tilde{w}_i(\{z\}) = w_{i0} + \sum_{(+)} \frac{\partial w_i(\{z_0\})}{\partial z_j}\left(z_j - z_{j0}\right) + \sum_{(-)} \frac{\partial w_i(\{z_0\})}{\partial z_j}\frac{z_{j0}}{z_j}\left(z_j - z_{j0}\right), \qquad [5]$$

where $w_{i0} = w_i(\{z_0\})$, $\{z_0\} = \{z(\{q_0\})\}$, and $\{q_0\}$ corresponds to the vector of uncertain parameters $\{q\}$ when the values of the components are equal to their nominal values, $\sum_{(+)}$ means summation over the variables for which $\frac{\partial(\cdot)(\{z_0\})}{\partial z_j}$ is positive, and $\sum_{(-)}$ contains the remaining variables. An attractive property of this linearization is that it yields the most conservative approximation among all the possible combinations of direct/reciprocal variables. The partial derivatives used in the approximations are evaluated assuming that the mode shapes are invariant. This makes sensitivity calculation (derivatives) very inexpensive from a computational point of view. Numerical results have shown that this procedure generates high quality approximations for dynamic system responses [Jensen and Sepulveda (1998)].

EXAMPLE PROBLEM

The example problem involves a simple 10 meters bending structure, which is modeled with 10 equal beam type finite elements. The beam is constrained so that only in-plane horizontal motion is allowed. Nominal material properties are E=7.1 x 10^6 N/cm^2, and ρ = 2.77 x 10^{-3} kg/cm^3. A concentrated mass is located at the top of the structure. The structure is subjected to a harmonic base excitation of nominal magnitude P and frequency Ω. Two cases are considered here. In the first case, the forcing frequency is chosen away from the resonance peaks of the nominal system while in the second case is near the fundamental frequency. To complete the model, some amount of damping is added to the system. The magnitude of the load and the bending stiffness of the structural system are imprecisely defined in a range of possible values. A variation between 0% and 20% of the load magnitude and bending stiffness with respect to their nominal values is considered in this application. For example, a system parameter variability of 10% indicates that the value of the upper bound is equal to 1.1 times the parameter nominal value and the lower bound equal to 0.9 times its nominal value. The bending stiffness of the system is represented by the stiffness of three different segments. Segment I corresponds to the section near the base of the structure, segment II to the middle section, and segment III to the section near the top of the structure.

Figure 1 shows the steady-state top displacement response bounds with respect to variation of the uncertain system parameters. In this case, the forcing frequency is away from resonance. The response bounds are computed by the combinatorial interval analysis approach described in the previous section. The figure shows the bounds obtained by using the exact system response and the approximate response. It is seen that the approximate response case is almost coincident with the exact response case. Therefore, the convex linearization is able to capture the global behavior of the response function in the specified range of variation of the uncertain system parameters. On the other hand, the computational cost using approximations is only a small fraction of the time required for the exact system response case. The relative importance of the uncertain parameters on the response bounds is shown in figure 2. In this figure, line 1 represents the case of uncertainty in the load magnitude and bending stiffness of the entire system, line 2 the case of uncertainty in the bending stiffness of segment I, line 3 the case of uncertainty in the bending stiffness of segment II, and line 4 the case of uncertainty in the load magnitude. At this figure shows, the stiffness properties of the system near the base have an important influence on the response upper bounds. In fact, the uncertainty in this parameter is more important than the uncertainty in the other parameters. Contrarily, the uncertainty in the load magnitude shows more importance than the other parameters on the response lower bounds. The influence of the uncertainty in the bending stiffness near the top (segment III) is very small and therefore it is not shown in this figure.

The case when the forcing frequency is near the first frequency of the nominal system is presented in figure 3. This figure shows the steady-state top displacement response bounds with respect to variation of the system parameters. In this case, internal extrema are present and therefore the global optimization approach is used to obtain the response bounds. Once again, the results obtained by using approximation concepts agree very well with the exact solution. The main difference is in the number of structural analyses required to obtain the system response bounds (a factor of 100 in this case). This difference indicates that the combination of global optimization techniques with approximation concepts is very attractive in obtaining maximum structural responses for the general case. The sensitivity of the response bounds with respect to the uncertain parameters for the resonance case is presented in figure 4. In this plot, line 1 represents the case of uncertainty in the load magnitude and bending stiffness of the entire system, line 2 the case of uncertainty in the bending stiffness of segment I, line 3 the case of uncertainty in the bending stiffness of segment II, and line 4 the case of uncertainty in the bending stiffness of segment III. It is seen that the uncertainty in the stiffness properties of segment I is more important than the uncertainty in the other parameters. This is due to the strong influence of the stiffness properties near the base of the structure on the spectral characteristics of the system. Also, it is found that the influence of the uncertainty in the load magnitude and bending stiffness of segment III are similar and very small in this case.

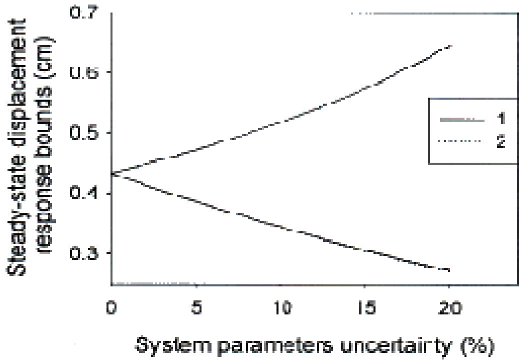

Figure 1: Response bounds with respect to variation of the system parameters. 1)Exact response. 2) Approximate response

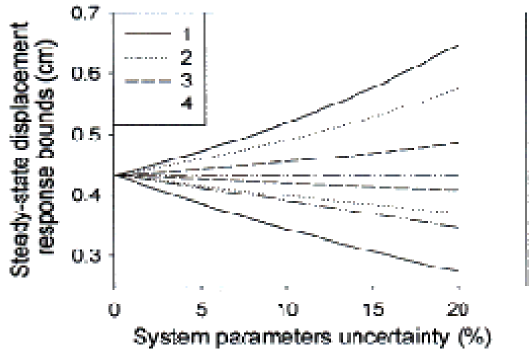

Figure 2: Response bounds with respect to variation of the system parameters. 1)Uncertainty in all parameters. 2)Uncertainty in the bending stiffness of segment I. 3)Uncertainty in the bending stiffness of segment II. 4)Uncertainty in the load magnitude

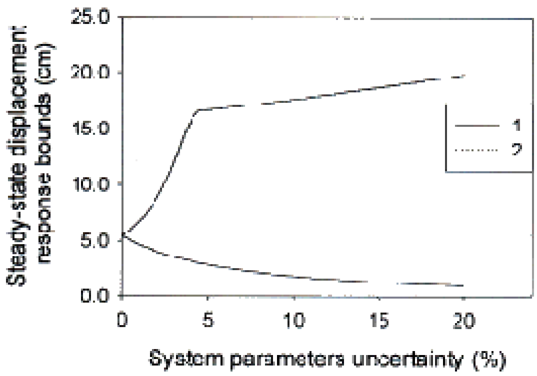

Figure 3:Response bounds with respect to variation of the system parameters (resonance condition). 1) Exact response. 2)Approximate response

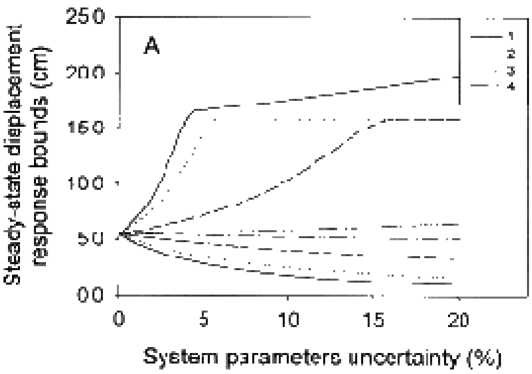

Figure 4:Response bounds with respect to variation of the system parameters (resonance condition). 1) Uncertainty in all parameters. 2)Uncertainty in the bending stiffness of segment I. 3)Uncertainty in the bending stiffness of segment II. 4)Uncertainty in the bending stiffness of segment III.

The relative importance of the uncertain system parameters on the response of the system can also be evaluated by a sensitivity analysis. A sensitivity measure based on a measure of fuzziness is used in this formulation [Dubois and Prade (1980)]. This measure can be used to determine the relative coupling between uncertain inputs (parameters) and outputs (system responses). Table 1 lists the results for the example problem. Case I corresponds to the case when the forcing frequency is away from the resonance peaks of the nominal system, and case II when the forcing frequency is near the first frequency of the nominal system. When an uncertain system parameter has the greatest qualitative importance, the numerical measure produces a normalized value of one. As the measure decreases in value, the corresponding input has little effect in the performance of the system. This sensitivity information is analogous to the usual sensitivity, but applies to the uncertain parameters, and represents the entire range of the parameters, not a single operating point. From the table, it is clear that the uncertainty in the bending stiffness of segment I is the most significant in both cases. On the other hand, the load magnitude shows an important influence in case I but a very little influence in case II.

Table 1: sensitivity measure

System Parameter	Case I	Case II
Bending stiffness of segment I	1.00	1.00
Bending stiffness of segment II	0.39	0.53
Bending stiffness of segment III	0.08	0.05
Load magnitude	0.46	0.04

The difficulties of the combinatorial interval analysis approach in the case of internal extrema are illustrated in figure 5. In this figure, line 1 represents the response bounds obtained by the global optimization approach while line 2 represents the bounds obtained by the interval analysis approach. This case corresponds to a resonance condition and therefore maximum responses occur inside the intervals that characterized the uncertain system parameters. From this figure, it is clear that the results obtained by the standard combinatorial interval analysis are not correct while the ones obtained by the multistart procedure are the correct system response bounds.

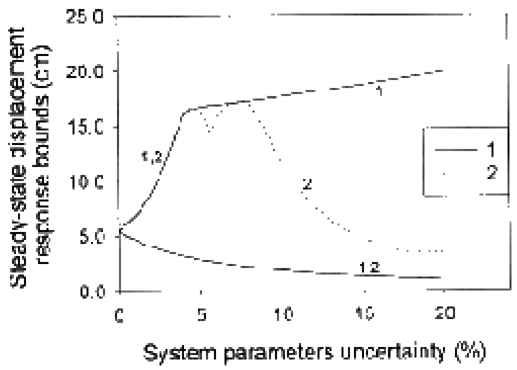

Figure 5: Response bounds with respect to variation of the system parameters (resonance condition). 1) Global optimization approach. 2) Combinatorial interval analysis approach.

CONCLUDING REMARKS

In this work, a simple and efficient methodology for determining exact bounds of system responses has been presented. The method is based on combinatorial interval analysis and global optimization techniques. Approximation concepts are considered for an efficient numerical implementation of the proposed methodology. The method can identify the more influential parameters on the structural behavior of the system. This sensitivity information can be very useful since provides valuable information for rational decision making in the analysis of structural system with uncertain parameters. The method can also be used during the modeling stage, since it can serve to identify those parameters that because of their influence in the system response need to be determined with more precision. In general, great insight into the behavior of the structure can be gained using the methodology. Extension of this technique to consider general dynamic load conditions and complex structural systems is currently under investigation by the author and will be treated in detail in an upcoming report.

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