TIME-FREQUENCY ANALYSIS OF EARTHQUAKE RECORDS

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SUMMARY

Reliable earthquake wave characterization is essential for better understanding wave propagation phenomena and the characterization of the local site effects subject to earthquake or man made excitations. Results of ongoing time-frequency research are presented here with the aim to compare the performance of various state-of-the-art time-frequency distributions when applied to earthquake records. In a near future, the objective is to adapt this innovative joint time-frequency signal processing technique to earthquake record analysis and parameter estimation. The time-frequency distributions studied are the Spectrogram (SP), the Wigner-Ville (WVD), the Choi-Williams (CWD), and the reduced interference (RID) distributions. Earthquake records ranging from strong to moderate/weak ground motions where used in this analysis. These accelerogram time series were recorded in the Mexico City valley. Based on our preliminary results, is our current conclusion not to recommended the use of the SP if detailed information about the changes in the signal are sought, because of its limited time resolution. The presence of interference cross-terms in the WVD limits its application for characterization of earthquake signals. The CWD suppresses the interference cross-terms but still suffers from some synchronization artifacts. The RID time-frequency distribution is, at the present stage of this study, the "best" one with a fairly good trade-off between the time-frequency resolution and suppression of numerical artifacts.

INTRODUCTION

The Fourier transform decomposes a signal into its constituent frequency components. Looking at the Fourier spectrum we can identify these frequencies; however, we can not identify their temporal localization. Time-frequency distributions (TFDs) map a one-dimensional signal into a two-dimensional function of time and frequency, and describe how the spectral content of the signal changes with time. From a strict mathematical point of view, we want to have a joint distribution, which will give us the fraction of the total energy of the signal at time t and frequency ω; i.e., we look for the function $P(t,\omega)$. The aim of time-frequency analysis is to come up with appropriate candidates for $P(t,\omega)$. Time-frequency analysis dates its origin to the first half of the 20th century, but the major developments in its understanding, practical applications, and analysis have been recently accomplished. The time-frequency concept basically describes the capability of this signal processing technique to describe the true structure of the signal when the frequency content varies with time. However, while each signal has a unique Fourier spectrum, a time-frequency analysis of a signal is nonunique in that different time-frequency methodologies yield different time-frequency distributions. In other words, many different time-frequency representations can be associated with the same data. In this research, the capabilities of some common time-frequency distributions for analyzing time series data associated with earthquakes recorded at the ground surface and/or at depth are studied.

The seismic waves observed in earthquake records manifest clearly non-stationary characteristics, as well as a wide frequency content. Those characteristics are twofold. The first characteristic involves variations with time of the intensity of the ground motion (acceleration, velocity or displacement). That is, with the arrival of the first seismic wave, it (the intensity) builds up rapidly to a maximum value for a certain time and then decreases

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slowly until it vanishes. The second characteristic involves variations with time of the frequency content, with a tendency to shift to lower frequencies as time increases. This behaviour is well known as a frequency dependent dispersive effect. This phenomenon is very complex and involves the arrival of the different seismic phases (P, S and Surface waves), the intensity of the ground motion, the magnitude of the earthquake, source and path effects, and the local soil conditions/geometry of the soil layers at the recorded site (trapped waves may produce quasi-stationary waves). In engineering seismology, as well as in seismology, the spatial variation of ground motions is not a new topic. For example Aki and Tsujiura [1959] started studies of this phenomenon in the field of seismology. Since then, related studies have identified interesting engineering concerns, including the recognition of coherent waves, where the group propagates horizontally due to the irregularity and/or heterogeneity of the soil, as well as trains of transient waves. Because the relatively high-resolution characteristics of the time-frequency signal processing technique, discussed in this paper, the identification and characterization of the time and frequency content of those masked seismic phases (contained in the earthquake time series) is possible.

Here our ongoing research has been organized and presented in four stages. Section 1 presents the theoretical overview of time-frequency distributions. In section 2, a comparison study of the performance of the different distributions, when applied to real seismic data, is presented. Section 3 presents the ongoing result of the interpretation from the TFD plots as it relates to wave propagation phenomenon. A discussion of the initial results of our findings, as well as current and further research, is finally given in section 4.

TIME-FREQUENCY DISTRIBUTIONS

TFD are appropriate tools for non-stationary signal analysis, synthesis, and processing. Different types of time-frequency distribution have been developed for that purpose. Two early forms of time-frequency analyses are: the Short-Time Fourier Transform (STFT), used to generate the spectrogram (SP) [Koeing, Dunn and Lacy, 1946; Allen, and Rabiner, 1977], and the Wigner-Ville distribution (WVD) [Claasen and Mecklenbrauker, 1980]. Studies of the well known linear TFD Short-Time Fourier Transform (STFT) have been published by Nawad and Quatieri, [1988], and Allen [1977] among others. The time-frequency distribution ideally describes how the energy is distributed, and allows us to estimate the fraction of the total energy of the signal at time t and at frequency ω. The above statement states that the energy should be positive.

In order to achieve fine simultaneous time-frequency resolution in a non-stationary time series, we must deal with the uncertainty principle [Williams, Brown and Hero, 1991]. The uncertainty principle restricts us from achieving arbitrarily fine resolution simultaneously in both the time and the frequency domain. The condition to satisfy in the uncertainty principle is given by the inequality $\Delta t \Delta \omega \geq \frac{1}{4\pi}$, in which the selection of Δt (time resolution) and $\Delta \omega$ (frequency resolution) are not arbitrary parameters. A trade-off between them should be considered in order to reach the desired "good" resolution. In the majority of the cases, it is dependent on the signal characteristics. Next, an overview and a brief description of the advantages/limitations of the Spectrogram (SP), the Wigner-Ville (WVD), the Choi-Williamns distribution (CWD), and the reduced interference distribution (RID) are presented.

Definitions and general concepts of the time-frequency distributions

Linearity is a desirable property of the TFD; however, quadratic TFDs have been proposed and interpreted as time-frequency energy distributions, or instantaneous "power" spectra. The TFD known as Cohen’s shift invariant general class [Cohen, 1986] distribution, combines the concepts of the instantaneous power and the spectral energy density. Special cases of this general class are the SP, the WVD, the CWD [Cohen, 1989; Hlawatsch and Boundreaux-Bartels, 1992], and the binomial distribution (BD). The last two distributions belong to the so-called reduced interference distribution (RID) and they also belong to the distribution class called Cohen's class, which is by itself an extension of the WVD [Jeong and Williams, 1992]. The WVD has been of special interest since it satisfies a large number of important properties. Every member of Cohen’s general class may be interpreted as two-dimensional filtered WVD. The SP lacks the time resolution, even though the frequencies are fairly well defined. The WVD offers a significant improvement in time-frequency resolution, but it suffers from cross-terms interference when applied to multicomponent signals such as earthquake data. The cross-term interference causes the time-frequency distribution to occasionally be negative. Furthermore, for multi-component seismic signals the presence of cross-terms make it almost impossible to carry out a detailed signal identification/characterisation, and only general characteristics are possible to identify. The CWD overcomes the WVD limitation suppressing in great amount the cross-term interference, but some time-frequency resolution is lost. Also, when different time and/or frequency components are present at the same frequency and/or time synchronisation effects occur producing singularities in the time-frequency distribution. The RID overcomes these problems to a significant extent, even though some small synchronisation and cross-terms may be present. For these reasons and the enhancement in the time-frequency resolution, the RID appears to be the "best" TFD with which to analyze earthquake waves.

Cohen's class time-frequency distributions

Cohen [1995] generalized what has come to be known as Cohen's class of TFD's. Cohen's class of TFD's can be expressed in terms of a product of a kernel and a time varying "auto-correlation" function. The general requirements of an appropriate time-frequency distribution are non-negativity, realness, time and frequency marginal, instantaneous frequency, group delay, time and frequency support, and time and frequency shift properties [Cohen, 1995]. From the Cohen's class TFD definition stated in equations 1 and 2, it should be noted that Cohen's class is a bilinear transformation of the signal, and can be interpreted as the 2D Fourier Transform of a weighted version of the ambiguity function [Cohen, 1995]. Specifically the TFD *P(t,ω)* of the signal *x(t)* is given by

$$P(t,\omega) = \frac{1}{4\pi^2}\iint A(\theta,\tau)\Phi(\theta,\tau)e^{-j\theta t - j\tau\omega}\,d\theta d\tau \qquad (1)$$

where $A(\theta,\tau)$ is the symmetrical ambiguity function of the signal *x(t)* and is given by

$$A(\theta,\tau) \equiv \int x(t+\tfrac{\tau}{2})x^*(t-\tfrac{\tau}{2})e^{j\theta t}\,dt \qquad (2)$$

$\Phi(\theta,\tau)$ is the kernel of the time frequency distribution, and θ and τ are frequency and time dummy variables of integration. In terms of the kernels, the WVD has a kernel value of 1. The kernel of the SP corresponds to the ambiguity function of the localization window *h(t)*. The kernel for the CWD is a Gaussian function with the degree of the cross-terms suppression depending on the value of σ . Finally the kernel for the RID is equivalent to a two-dimensional low pass filter. A summary, based on these kernels is presented on Table 1 for the definitions of the TFD use in our analysis.

Table 1

NAME	KERNEL $\Phi(\theta,\tau)$
SP	$\int h^*(u-\frac{1}{2}\tau)e^{-j\theta u}h(u+\frac{1}{2}\tau)du$
WVD	1
CWD	$e^{-\theta^2\tau^2/\sigma}$
RID	2D Low pass filter in θ , τ space

TIME-FREQUENCY ANALYSIS TO SEISMIC WAVES OF EARTHQUAKE RECORDS

The earthquake records used in our analysis where recorded at the Mexico City valley, on the surface or at depth in a three components (North-South, Vertical, East-West) of the ground motion. The range of earthquake magnitudes studied goes from strong (M=8.1) to moderate/weak (M=5.0) ground motions. In particular, records like the well known Mexico City earthquake of September 19, 1985, recorded at the CDAO site, and the Petatlan earthquake of May 11, 1990, recorded at PCC site among others were used in our analysis. Those seismic records were resampled for our analysis to yield a Nyquist frequency of 3.5 Hz. These accelerograms were previously corrected for instrument effects to provide absolute values of acceleration of the ground motions.

According to the geotechnical soil zonation, those sites are located at the Lake zone of the Mexico City valley. The general local site conditions in which those earthquakes where recorded are soft soils in which their thickness ranges from 55 to 40 meters. At these sites, the sub-soil conditions can be described as a sequence of an upper layer with thickness of about 5 m, followed by a superior clay layer with thickness ranges between 27 to 37 m, a thin sandy hard layer with a thickness of about 5 m, which rests in an inferior clay layer with an average thickness of 10 m. All these layers rest in what is known as the deep deposits. The average values of the shear wave velocities are 100 m/s for the upper layer, 60 m/s for the superior clay layer, 180 m/s for the hard layer, 130 m/s for the inferior clay layer, and 350 m/s for the deep deposits.

Earthquake wave analysis

As stated in section 1, the time-frequency analysis was focused on using four of the most well known and generally used TFDs. Those are: the Spectrogram (SP), the Wigner-Ville distribution (WVD), the Choi-Williams distribution (CWD), and the reduced interference distribution (RID). According to published studies, the general advantages/disadvantages of these TFDs from a pure signal processing point of view have been summarized and presented in subsection 1.1.

Because the limitation of space in this paper, we present and discuss our TFD performance analysis using only two of the 10 earthquakes studied. These can be considered as representative of the general characteristics observed. In Figure 1, both components (vertical and horizontal (E-W)) of the Petatlan earthquake time series and its frequency domain representation (|FFT|) are shown. In the frequency domain representation of the horizontal component (Figure 1(b)) at least four frequency components, located at 0.5, 0.7, 1.4 and 2.2 Hz, are clearly seen. Those frequencies were labeled with H1, H2, H3, H4, respectively. Of these frequency components, the dominant is located at 0.5 Hz. Beyond 3 Hz the spectral amplitudes are small. For the same component, the time domain representation (Figure 1(a)) indicates that the maximum amplitude is located at around 20 s. The frequency appears to change from high to low and then slightly high frequencies are observed in the time window from 17 to 30 s. A low frequency component may be observed almost all along the record. However, in certain small time window intervals a high frequency component is also evident. For the vertical component, the frequey domain representation (Figure 1(d)) indicates that as many as two dominant frequency components between 1.3 and 2.5 Hz (labeled with V1 and V2, respectively) dominate the record. The dominant one is located around 1.8 Hz. Beyond 3 Hz no high spectral amplitudes are evident. The time domain representation of this component (Figure 1(c)) shows that the maximum amplitudes are distributed in the time window of 18 to 38 s. Clearly a high frequency dominates the whole record.

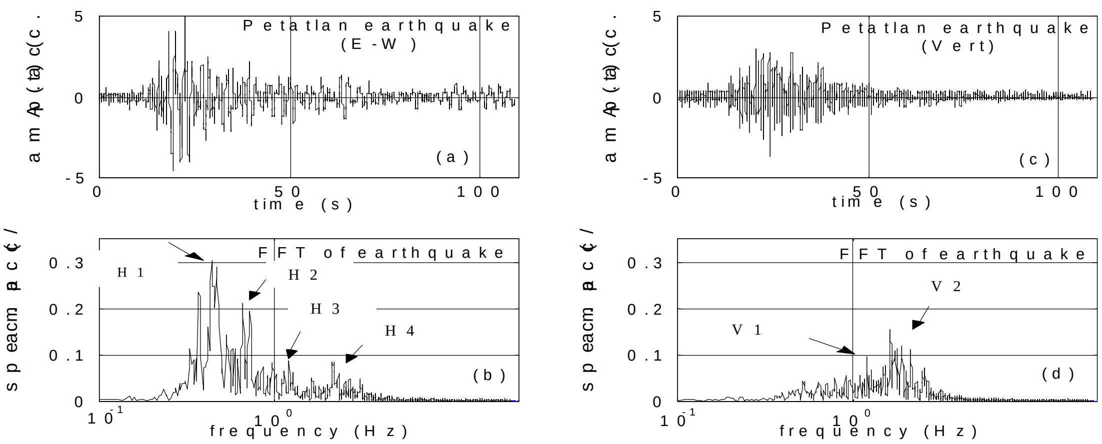

Figure 1. Time series and Fourier spectra of Petatlan earthquake. (a) E-W time series, (b) corresponding |FFT|, (c) vertical time series, and (d) corresponding |FFT|

Earthquake wave TFD analysis

The TFDs of both horizontal and vertical components are shown in Figures 2 and 3, respectively. At the top of Figures 2 and 3 the time series of the earthquake to be analysed are also included. The sequence of plots labeled with (a), (b), (c), and (d) corresponds to the TFDs of the SP, WVD, CWD, and RID, respectively. The relevant frequency components to be discussed in the following analysis are labeled with H1, H2, H3, and H4, and indicated with arrows. For simplicity, the labels H1, H2, H3 and H4 are shown only on Figures 2(a), and 3(a). Because the relative position of those frequencies prevails in all the sequence of figures, only the arrows are indicated in the Figures labeled (b), (c), and (d). In Figure 2(a), at least three smooth well-defined frequency components can be observed. Those are labeled with H1, H2, and H4 in the same Figure. The strongest in amplitude and apparent large duration (labeled as H1) is located at the lowest frequency contained in the earthquake record. In this case the large duration is in relative comparison with the other observed frequency components. However, we must be cautious when making such an interpretation because of the well-known limitations of the SP, which is lacking in time resolution, even though the frequency resolution is fairly satisfactory. A significant increase in time and frequency resolution is obtained when the WVD is applied. As can be seen in Figure 2(b) sharp spiky details are now present in the WVD time-frequency representation. However that increase in resolution is unfortunately blurred by cross-term interference that makes a realistic interpretation of the characteristics of the different waves, and their frequency components, almost impossible. The CWD is shown in Figure 2(c), the normalization parameter σ of its kernel was set equal to one. It can be seen that many of the cross-terms have been eliminated. However synchronisation effects (i.e., the vertical lines) prevail and some of the details gained in the WVD are lost due to the trade-off between suppression of the cross-terms and the auto-component terms. Figure 2(d) shows the TFD obtained with the RID. In this case both the synchronisation, and cross-terms are smaller than observed in the CWD and WVD, respectively. This improvement facilitates the identification/interpretation of the frequency components of the earthquake waves. It may be feasible to interpret some seismic signals in terms of frequency dispersive characteristics. It is also possible to observe that the times when the maximum amplitudes occur in the time domain of the earthquake record also correspond to the times where several time frequency characteristics of the signal "converge".

Results of the TFD analysis of the vertical component of the same earthquake are presented in Figure 3. The objective of presenting at least these two cases is to demonstrate that the characteristics described for the E-W component are not an artifact of the numerical algorithm of the TFDs studied here. Figure 3(a) shows the SP, in which four frequency components (H1, H2, H3, and H4) of shorter time duration, compared with the ones observed in the E-W component, are located between the frequencies of 1.5 Hz and 3 Hz. Passing through the sequence of Figures 3(b) (WVD), 3(c) (CWD), and 3(d) (RID) a similar pattern is observed regarding the increase/reduction of the time-frequency resolution, and the suppression of cross-term interference and synchronisation effects, as was discussed for the E-W component.

ANALYSIS/INTERPRETATION

The joint time-frequency characteristics of the earthquake seismic waves studied here exhibit interesting features from the seismologist interpretation point of view. Lets first describe the interpretation we may obtain if using only the SP time-frequency distribution. Global characteristics about the frequency distribution are quite clearly evident, but details about its time duration are limited, as well as any inference related with energy concentration due to the (probable) interaction between different seismic wave phases. When using the WVD time-frequency plot, energy concentrations are possible to observe. This is mainly due to the significant increase of the time-frequency resolution, but at the same time a large amount of cross-term interference masks the true signal of the earthquake record. As indicated in Figure 2(b) there is an apparent concentration of energy at ~ 1.4 Hz that is not observed in the FFT spectrum or in the SP. This artifact is due to interference effects. Without being conclusive some time-frequency distribution trends are possible to identify. While using the CWD, cross-term interference is reduced, making it possible to identify more clearly the spots where the concentration of energy occurs. However, continuous trends of the signal in time and frequency are not possible to follow with reasonable detail. The RID time-frequency distribution is the one that appears to offer the best trade-off between the signal time-frequency distribution and suppression of numerical artifacts (such as cross-term interference and synchronisation effects) of the algorithm. Both energy concentration spots and the time-frequency trends seem to be somewhat clearer and also consistent with the partial information seen in the distributions previously described. Because the time-frequency trends here are neater and consistent, some dispersive characteristics may be interpreted, even though currently this interpretation of earthquake wave identification may not be conclusive.

DISCUSSION AND CONCLUSION

For global frequency distributions in which no fine time resolution is necessary, the SP is well suited to identify the dominant frequencies contained in the seismic signals. The time resolution of the SP is limited, and restricts its use for highly time resolved earthquake wave signal characterisation. Even though the WVD has significant

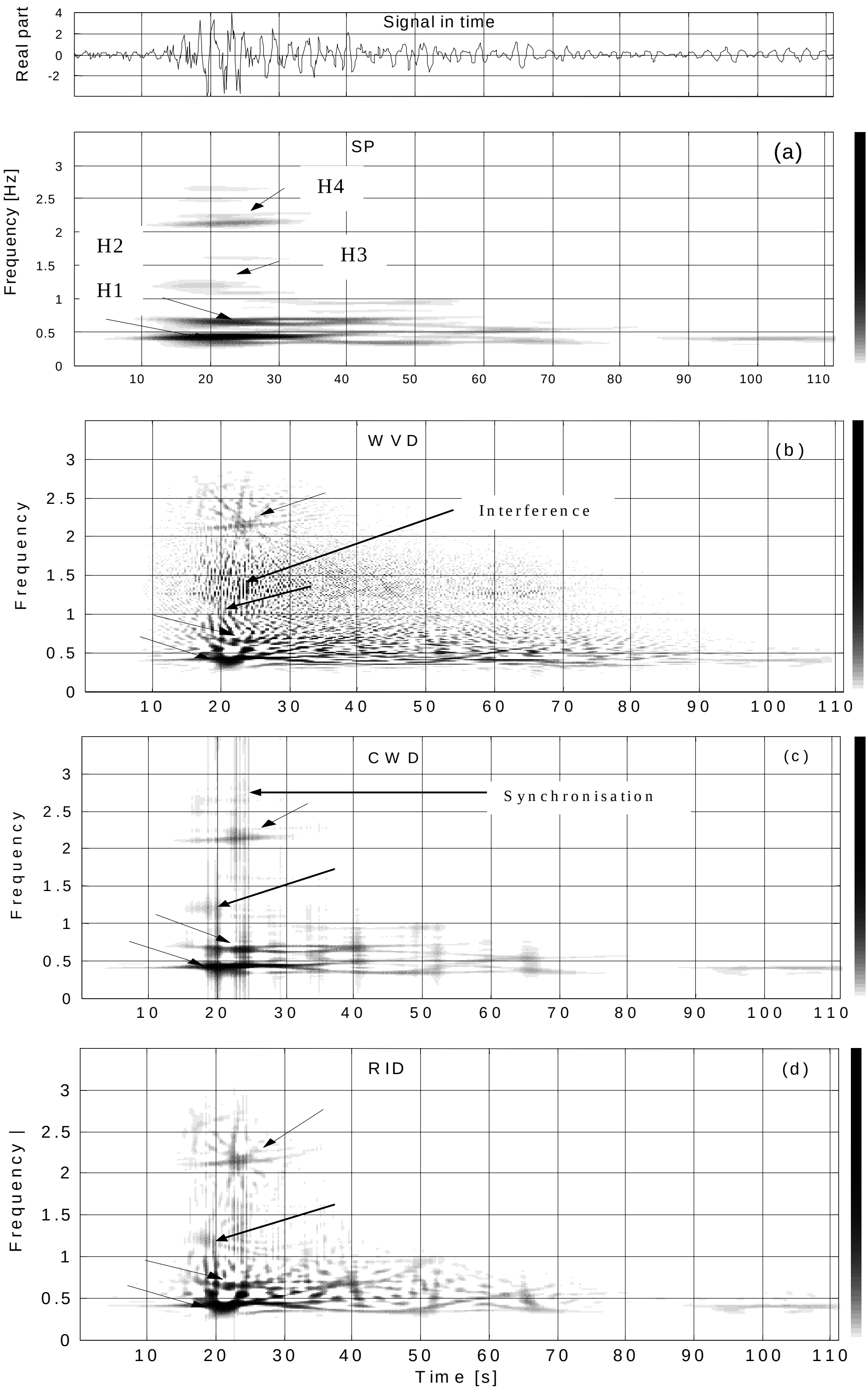

Figure 2. Time-frequency distributions for the (E-W) component of the Petatlan earthquake: (a) spectrogram, (b) Wigner-Ville distribution, (c) Choi-Williams distribution, and (d) reduced interference distribution.

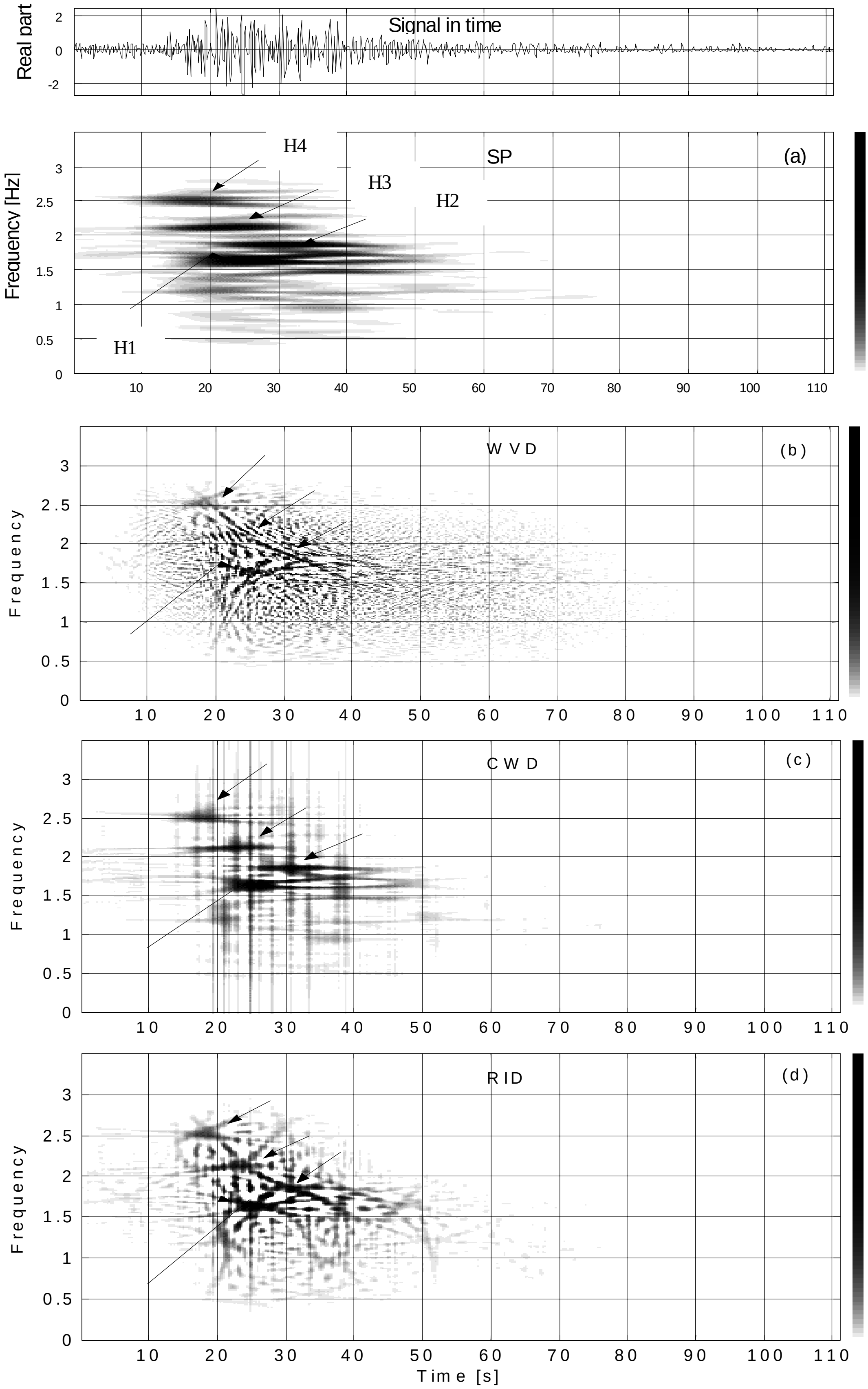

Figure 3. Time-frequency distributions for the vertical component of the Petatlan earthquake: (a) spectrogram, (b) Wigner-Ville distribution, (c) Choi-Williams distribution, and (d) reduced interference distribution.

time-frequency resolution, the information that the TFD exhibits is blurred by the interference cross-terms. This poses difficulties for any well-supported and realistic interpretation of the time-frequency characteristics of the different earthquake waves. The CWD suppresses much of the cross-term interference. However synchronisation effects prevail and some of the details gained in the WVD are lost. The RID facilitates the identification/interpretation of time and frequency components of the seismic waves, even though a small synchronisation effect is still present for some components of the signal. It is now possible to interpret some seismic signals which exhibit frequency dispersive characteristics. Also, particular frequency components can be identified with the respective times that match with the maximum amplitudes in the time domain of the earthquake record. Those maximum amplitudes correspond to a point in the time-frequency plane where several time-frequency characteristics of the signal concentrate. These are the results of our ongoing study and because of the problems already identified, current and future research will be focused on the study of the sensitivity and resolution of the TFDs with changes in the kernel parameters. In addition, an investigation of the feasibility of using optimal radial Gaussian kernels, as well as the use of optimal adaptive kernels, is in progress and will be continued. Finally, the physical interpretation of the TFDs and characteristics of the earthquake waves in the time-frequency plane will be further studied in order to provide new physical insight into seismic wave propagation phenomena and soil properties.

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