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A NEW LOOK AT THE TORSION DESIGN PROVISIONS IN SEISMIC BUILDING CODES

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SUMMARY

It is well known that torsional oscillations during an earthquake may cause severe distress in a building structure that is unsymmetric. All seismic codes therefore include some provisions for the design of structures to resist the forces induced by torsional vibrations. The torsion design provisions of a number of codes are reviewed and compared with a new set of design proposals. Results of elastic and inelastic response studies of a single storey building model show that the provisions of all of the codes studied are conservative for the design of resisting elements on the flexible side of the building. However, in some cases the code provisions may be unconservative for the design of resisting elements on the stiff side, particularly for buildings with a low value of torsional stiffness. The new provisions are seen to represent an improvement.

INTRODUCTION

Damage reports on recent earthquakes have indicated that one major cause of distress in building structures may be the torsional motion induced by the earthquake. This has renewed interest in the study of torsional response of buildings. A large number of research studies have been carried out in the past on elastic and inelastic torsional response of building models. However, perhaps due to the complexity of torsional behaviour, particularly in the inelastic range, findings of various studies have not always been consistent, leading to widely differing torsional provisions in different building codes.

A recent study by the authors [Humar and Kumar, 1998a, 1998b] has shown that certain parameters that govern the torsion response have not been given the attention they deserve. The most important of these is the torsional stiffness as measured by the ratio of uncoupled torsional frequency to the uncoupled lateral frequency. In spite of this most building codes do not contain any explicit provision in respect of the torsional stiffness, or of the frequency ratio. Based on extensive studies, the authors have recently proposed new torsion design provisions that may represent an improvement, are simple to apply, and yet are not very different from the now familiar provisions of some of the existing codes.

The objective of this paper is to review the torsional design provisions in selected building codes, and compare these with the newly suggested provisions. Five building codes have been selected for this study: (1) National Building Code of Canada, NBCC 1995, (2) Uniform Building Code, UBC 1997, (3) National Earthquake Hazard Reduction Program Recommended Provisions, NEHRP 1997 (4) New Zealand Standards NZS 4203-1992, and (5) Mexico Code 1993. A mono-symmetric, single storey, shear type of building model is studied for its elastic response to a design ground motion represented by an idealised spectrum, and inelastic response to a set of 16 ground motions. The study consists of two parts: (1) comparison of design eccentricity expressions in codes with the effective edge eccentricities obtained from response spectrum analyses, and (2) comparison of the ductility demands at the edge elements of an unbalanced model designed as per the torsional provisions of building codes. For all of the above studies, building models with a wide range of the values of eccentricity and frequency ratio have been selected.

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2. REFERENCE BUILDING MODEL

For comparative evaluation of the various code provisions, we use the simple single storey building model shown in Fig. 1. In this model, the building floor is assumed to be infinitely rigid in its own plane. The entire mass of the structure is uniformly distributed at the floor level. The origin of the coordinate axes considered in the analysis is at the mass centre, denoted by CM. The mass centre is located at the geometric centre of the floor. Forces opposing the motion are provided by vertical inplane resisting elements oriented along the two orthogonal axes. The inplane resisting elements, referred to herein as resisting planes or simply planes, may comprise columns, shear walls, braced frames or a combination thereof. The *i*th plane parallel to the x axis has an elastic stiffness k_{xi}, while the *i*th plane in the y direction has stiffness k_{yi}. The distribution of stiffness is symmetrical about the x axis, but is unsymmetrical about the y axis. Thus, in the elastic range the centre of stiffness, or centre of rigidity (CR), lies on the x axis at a distance *e* from the centre of mass, where *e* is given by

$$e = \frac{\sum_{i=1}^{N} k_{yi} x_i}{\sum_{i=1}^{N} k_{yi}} \tag{1}$$

and *N* is the number of resisting planes in the y direction. For translation in the y direction, the elastic force in a y-direction resisting plane is proportional to the plane's stiffness. Hence, in the elastic range the centre of resistance coincides with the centre of stiffness.

It is assumed that earthquake ground motion is directed along the y axis. The dimension of the floor perpendicular to the direction of earthquake is *b*, and that parallel to the earthquake is *a*. The mass of the floor is *m*; *r* is the radius of gyration of the floor about CM; $K_y = \sum_{i=1}^{N} k_{yi}$ is the total stiffness in the y direction; and $K_{\theta R}$ is the torsional stiffness about CR, given by

$$K_{\theta R} = \sum_{i=1}^{N} k_{yi} \left(x_i - e\right)^2 + \sum_{i=1}^{M} k_{xi} y_i^2 \tag{2}$$

where *M* is the number of resisting planes orthogonal to the direction of excitation, that is, in the x direction. The building model shown in Fig. 1 has three inplane resisting elements in the y direction. Plane 1 has a lower stiffness than plane 3, and will be referred to as the flexible edge plane, or the flexible plane. In a similar manner plane 3 will be referred to as the stiff edge plane. We define $\Omega_R = \omega_\theta / \omega_y$ as the ratio of uncoupled rotational frequency ω_θ to the uncoupled the translational frequency ω_y, which are given by

$$\omega_y = \sqrt{\frac{K_y}{m}}\ ; \ \ \omega_\theta = \sqrt{\frac{K_{\theta R}}{mr^2}} \tag{3}$$

3. CODE PROVISIONS

Most seismic codes specify a simple equivalent static load method for design against earthquake forces. The static load methods also include provisions for torsion induced in asymmetric buildings. These provisions usually specify values of design eccentricities that are related to the static eccentricity between the centre of stiffness and the centre of mass. The earthquake-induced shears are applied through points located at the design eccentricities. A static analysis of the structure for such shears provides the design forces in the various elements of the structure. In some codes the design eccentricities include a multiplier on the static eccentricity to account for possible dynamic amplification of the torsion. The design eccentricities also include an allowance for accidental torsion. Such torsion is supposed to be induced by the rotational component of the ground motion and by possible deviation of the centres of stiffness and mass from their calculated positions. The design eccentricity formulae given in building codes can be written in the following form

$$e_{fc} = \alpha e + \beta b \tag{4a}$$

$$e_{sc} = \gamma e - \beta b \tag{4b}$$

where e_{fc} and e_{sc} are design eccentricities, and α, β, γ are coefficients, that have different values in different building codes.

The first term in the expressions for design eccentricity represents natural torsion, while the second term is supposed to represent accidental torsion. Factors α and γ are applied to static eccentricity *e*, to take into account the effects of dynamic torque amplification. Accidental torsion, which can be assessed only in an indirect manner, is taken as a fraction of plan dimension *b*. The values of the coefficients in Eqs. 4a and 4b, for each of the building codes mentioned earlier, are given in Table 1. Besides the design eccentricity expressions given by Eqs. 4a and 4b, the building codes have some special requirements as well. These special requirements are also shown in Table 1.

Table 1: Torsional design requirements in building codes

Building Codes	α	γ	β	Special requirements
Proposed expressions	1.0	1.0 or 0.0	0.1	For $\Omega_R < 1, \gamma = 0.0$
NBCC	1.5	0.5	0.1	
UBC 1997	1.0	1.0	$0.05A_x$	
NEHRP 1997	A_x	A_x	$0.05A_x$	
NZS	1.0	1.0	0.1	Horizontal regularity criterion should be met
Mexico	1.5	0.5	0.1	For $e > 0.1b$ design base shear be increased by 25%. Restrictions on e_p

The storey shear is required to be applied at a distance e_{fc} or e_{sc}, whichever produces the higher design force, from the centre of rigidity. Usually, e_{fc} governs the design of elements on the flexible side of the building, and e_{sc} governs the design of elements at the stiff side. However, for torsionally flexible systems (low Ω_R) where the torsional response may far exceed the lateral response, e_{fc} may produce higher design forces for the elements on the stiff side of the building as well.

Provisions in UBC 1997 and NEHRP 1997 specify an amplification factor A_x to be applied to the design eccentricities. In UBC this factor is applied to the accidental torsion, while in NEHRP it is applied to both the natural and accidental torsion. Factor A_x is given by

$$A_x = \left(\frac{\delta_{\max}}{1.2\delta_{\text{avg}}} \right)^2 \tag{5}$$

where $\delta_{\max}$ is the maximum displacement of the floor produced by the equivalent static earthquake forces, and δ_{avg} is the average of the displacements of the extreme points of the structure. In calculating $\delta_{\max}$, the effect of accidental torsion must be accounted for. The UBC and NEHRP provisions do not clarify how δ_{avg} is to be calculated. It is assumed here that accidental torsion need not be included while calculating δ_{avg}. With this assumption, two separate displacement calculations must be carried out to determine A_x

It may be noted that UBC and NEHRP provisions discourage the use of structural layouts having $\delta_{\max}/\delta_{\text{avg}} > 1.4$ and, in fact, prohibit their use for seismic design categories E and F. It can be shown that the application of this restriction virtually precludes the use of structure layouts with $\Omega_R \leq 0.75$.

A smaller value of e_{sc} as given by Eq. 4b leads to a more conservative design for the stiff edge. This implies that for the stiff edge the UBC provisions in which A_x is applied only to the negative portion –0.05*b* is far more conservative than the NEHRP provision in which A_x is applied to the net eccentricity $e - 0.05b$, except in the case when *e* is smaller than 0.05*b*. Provisions of NEHRP thus lead to a very weak stiff edge plane placing a heavy ductility demand on it, particularly for a low Ω_R.

4. ANALYSIS OF ELASTIC MODELS

To assess the torsional response of elastic models a response spectrum analysis of the building model is carried out for earthquake input represented by an idealized spectrum. Two kinds of spectral shapes are used: (1) a flat spectrum, and (2) a hyperbolic spectrum. Such an analysis provides the maximum flexible edge displacement Δ_f and the maximum stiff edge displacement Δ_s. It is useful to normalize Δ_f and Δ_s by the displacement Δ_0 of the associated torsionally balanced structure (having the same K_y m, and r but coincident CR and CM) when subjected to the same earthquake motion. Thus, $\overline{\Delta}_f = \Delta_f / \Delta_0$ and $\overline{\Delta}_s = \Delta_s / \Delta_0$.

We now define an effective eccentricity e_f as the distance from CR at which the application of base shear V_0 would produce a flexible edge displacement Δ_f, and eccentricity e_s as the distance from CR at which the application of V_0 would produce a stiff edge displacement of Δ_s. It can be shown that

$$\frac{e_f}{b} = \left(\overline{\Delta}_f - 1\right) \frac{\Omega_R^2}{\left(\frac{b}{r}\right)^2 \left(0.5 + \frac{e}{b}\right)} \tag{6}$$

$$\frac{e_s}{b} = \left(1 - \overline{\Delta}_s\right) \frac{\Omega_R^2}{\left(\frac{b}{r}\right)^2 \left(0.5 - \frac{e}{b}\right)} \tag{7}$$

The effective eccentricities given by Eqs. 6 and 7 can be compared with the design eccentricities given in the various code provisions.

As stated earlier, the code provisions include an allowance for accidental torsion. Thus, for a proper comparison between the code-specified eccentricities and the effective eccentricities derived from a dynamic analysis, the latter should also include the effect of accidental eccentricity. Recent studies [De La Llera and Chopra, 1997] have shown that the effect of ground rotational motion is quite small and may be neglected. Accidental torsion induced by uncertainties in the distribution of mass and/or stiffness may be accounted for by modifying the analytical model used in the dynamic analysis. In fact two different modified models are used, one in which the CM is shifted by +0.05*b* from its original position and the second in which the CM is shifted by -0.05*b* from its original position. The larger of the forces obtained in a resisting plane from the two sets of analysis is taken as the design force.

Selected sets of results obtained from analytical studies of the elastic models are presented here. The effective flexible edge eccentricities obtained for an aspect ratio of 1 and a hyperbolic spectrum are shown in Fig. 2. For $\Omega_R = 0.75$ and 1.0 all of the design provisions are quite conservative. For $\Omega_R = 1.25$ the design provisions are fairly conservative, except that when *e/b* is small UBC and NEHRP provisions may be somewhat unsafe.

The effective stiff edge eccentricities for an aspect ratio of 1 and hyperbolic spectrum are shown in Fig. 3. For $\Omega_R = 0.75$ the UBC and NZS provisions are unsafe. The NEHRP provisions are conservative for intermediate values of *e/b*, but may be unsafe for high and low e/b. The NBCC provisions are adequate, but slightly unconservative for a range of eccentricities. The new provisions are quite conservative. For $\Omega_R = 1.0$ the UBC and NZS provisions as well as the new provisions are adequate, or slightly unconservative. The NEHRP provisions are unsafe, while the NBCC provisions are quite conservative. For $\Omega_R = 1.25$ all of the provisions are conservative.

5. ANALYSIS OF INELASTIC MODELS

The single-storey building similar to that shown in Fig. 1, and having three resisting planes in the y direction but only one central resisting plane along the x axis is studied for its inelastic response to a set of 16 ground motions. The following numerical data is used in the study: mass of the building floor = 400 t; mass moment of inertia = 54,000 tm^2; aspect ratio $a/b = 0.5$; floor width b = 36 m; uncoupled translational period in y-direction = 1.0 s.

Strain hardening ratio of 5% is assumed for all planes and the damping ratio is taken as 5% of critical in each of the two coupled modes. The frequency ratio Ω_R and the eccentricity ratio *e/b* are varied over a range of physically admissible values. Specified values of Ω_R and *e/b* are achieved by adjusting the values of k_1, k_2 and k_3, the stiffnesses of the planes in y-direction. Building models with eccentricity values *e/b* = 0.05, 0.1, 0.15, 0.2, 0.25 and 0.3 and frequency ratios 0.75, 1.0, 1.25 and 1.50 are considered.

The yield strength of individual planes are given by

$$V_1 = V_0 \frac{k_1}{K_y}\left[1+\frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{fc}}{b}\left(\frac{e}{b}+0.5\right)\right] \tag{8}$$

$$V_2 = V_0 \frac{k_2}{K_y}\left[1+\frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{fc}}{b}\left(\frac{e}{b}\right)\right] \tag{9}$$

$$V_3 = V_0 \frac{k_3}{K_y}\left[1-\frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{sc}}{b}\left(0.5-\frac{e}{b}\right)\right] \tag{10}$$

$$V_3 = V_0 \frac{k_3}{K_y}\left[1-\frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{sc}}{b}\left(0.5-\frac{e}{b}\right)\right] \tag{11}$$

where V_0 is the design base shear in the associated balanced building model. In determining the value of V_3 the larger of the absolute values obtained from Eqs. 10 and 11 is used.

An elastic response spectrum is obtained for each of the 16 records, normalised by its peak ground acceleration, and for a damping of 5% of critical. The total elastic strength V_e of the resisting planes in the y-direction is obtained from the mean elastic response spectrum, corresponding to a period of 1.0 s. The spectral value obtained is multiplied by 0.28 so that the spectrum is representative of an earthquake with a peak ground acceleration of 0.28*g*. The total design strength for the torsionally balanced model is taken as $V_0 = V_e/4$. This strength is distributed among the individual planes of the balanced building in proportion to their stiffness. The strengths of planes in the unbalanced building are determined from Eqs. 8 through 11. The strength distribution in an unbalanced model is different for different codes, since the expressions for design eccentricities vary from code to code. Thus, five different unbalanced models, designed according to the proposed expressions, NBCC, UBC, NEHRP, NZS and the Mexico Code, are considered. It will be noted that the associated balanced model is same for all the above codes.

To account for the effect of accidental torsion, the centre of mass CM is moved by $\pm 0.05b$ in the torsionally unbalanced buildings, to produce two modified unbalanced models corresponding to each set of *e/b* and Ω_R values. In the analytical results presented here the maximum of the response values obtained from the two modified models is reported. All of the modified unbalanced and associated torsionally balanced models are now subjected to the set of 16 earthquake records, each scaled to a peak ground acceleration of 0.28*g*.

The maximum ductility demand in a plane in any torsionally unbalanced model subjected to a given earthquake is denoted by μ_u while the maximum ductility demand for the associated torsionally balanced model is denoted by μ_b. The ratio of the two ductilities, $r_\mu = \mu_u/\mu_b$, provides a measure of the effect of torsional motion. The mean value of the ratio of ductilities for the flexible edge $\bar{r}_{uf}$, obtained for the set of 16 earthquakes, is plotted against *e/b* in Fig. 4 for selected values of Ω_R. The value of $\bar{r}_{uf}$ is less than 1 for all the codes and all cases, implying that the flexible edge ductility in a torsionally unbalanced model is less than that in the associated torsionally balanced model. The provisions of the Mexico Code are most conservative of all.

The mean value of the ratio of ductilities for the stiff edge, $\bar{r}_{us}$, obtained for the set of 16 earthquakes, is plotted against e/b in Fig. 5 for several values of Ω_R. It is seen that $\bar{r}_{us}$ is higher than 1 for NZS for $\Omega_R = 0.75$. The ratio $\bar{r}_{us}$ is also higher than 1 for NZS, the proposed expressions, and UBC for $\Omega_R = 1.0$, particularly for higher values of e/*b*. However, the difference is small (less than 20%) and the provisions of NZS, proposed expressions and UBC may be considered adequate for $\Omega_R = 1.0$. As stated earlier, NEHRP provisions may lead to an unsatisfactory design for the stiff edge plane. This will be evident from the high values of $\bar{r}_{us}$ obtained for $\Omega_R = 1.0$ and 1.25.

6. SUMMARY AND CONCLUSIONS

Analytical results are presented for the elastic and inelastic response of single storey torsionally unbalanced models. The results of elastic studies are compared with the design provisions of different building codes. The results presented here show that the provisions of NBCC, NEHRP and Mexico code are overly conservative for the design of elements on the flexible side of the building. The NEHRP provisions deviate most from the dynamic analysis results. When $\Omega_R \leq 0.75$ the torsional provisions of all the codes are unconservative for the design of elements on the stiff side of the building. Only the proposed expressions provide an adequate assessment of the stiff edge responses. For higher values of Ω_R all code provisions are either adequate or conservative, the NBCC and the NEHRP provisions being the most conservative.

The results of inelastic response to recorded motions indicate that the provisions of all the codes and of the proposed expressions are conservative for the design of flexible edge, the value of $\bar{r}_{\mu f}$ being less than 1 in all cases. Provisions of Mexico code and NEHRP are most conservative. These results also indicate that the provisions of certain codes may be unsafe for the elements on the stiff side of the building, in certain situations. The proposed expressions generally give an adequate design. For $\Omega_R = 1.0$, $\bar{r}_{\mu s}$ is somewhat more than 1 for the models designed according to the proposed expressions, NZS and UBC, particularly for large values of eccentricity. However, $\bar{r}_{\mu s}$ is only slightly greater than 1 and these provisions may be considered adequate even for $\Omega_R = 1.0$.

Humar and Kumar [1998a, 1998b] have studied the application of the new provisions in the design of elastic as well as inelastic single and multistorey buildings. The design provisons work well with single storey buildings. They also work well with multistorey buildings as long as the frequency ratio does not vary significantly across the height. Variation in the frequency ratio across the height can be treated as a sign of vertical irregularity. Additional studies are needed to develop a more precise definition of such irregularity.

7. REFERENCES

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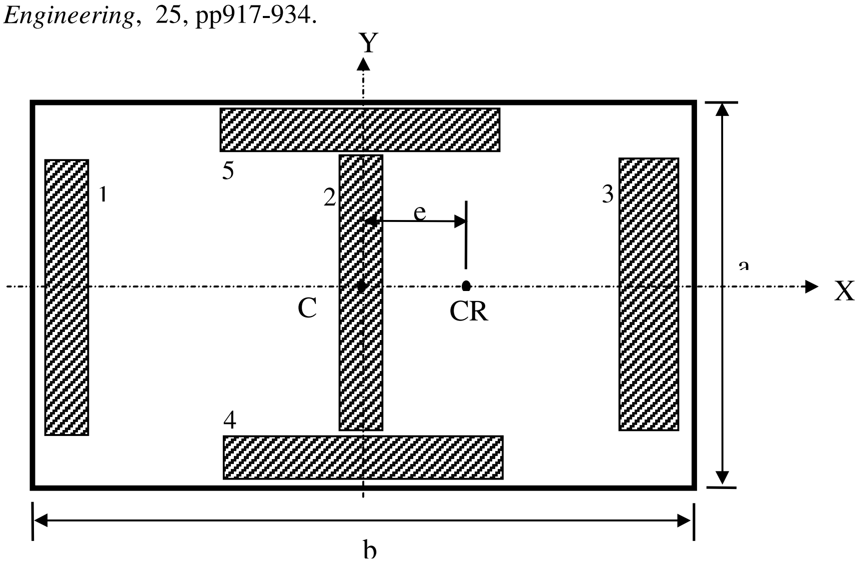


Fig. 1 Single storey building model

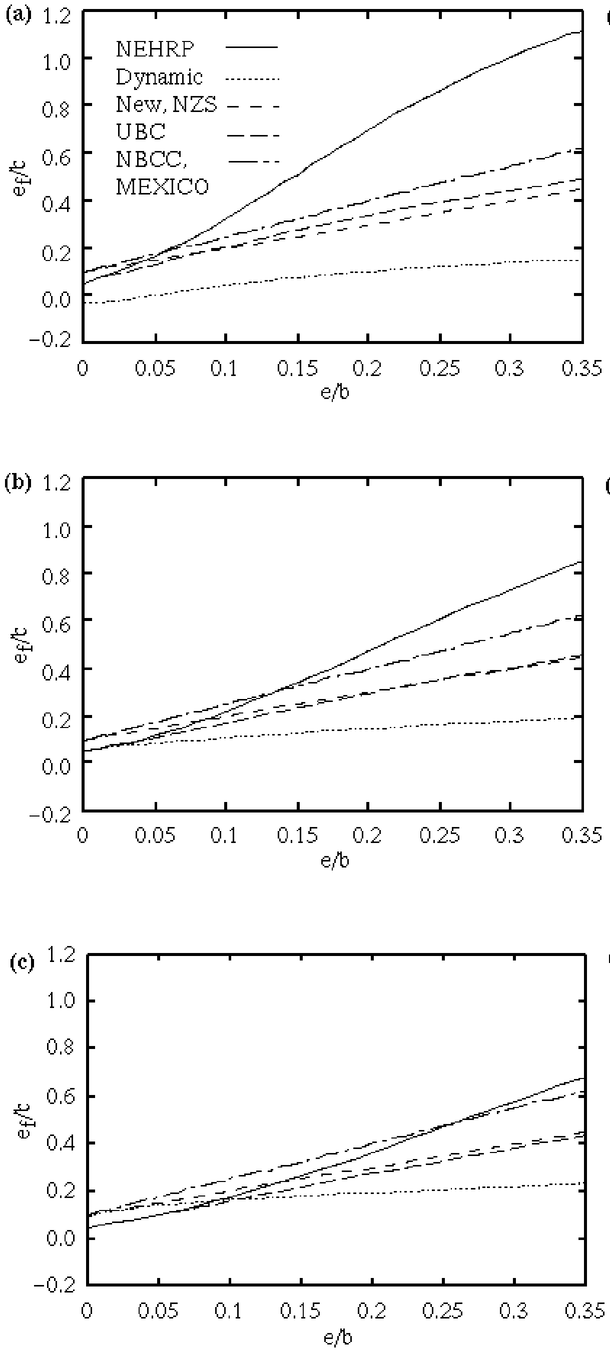


Fig. 2 Variation of effective flexible edge eccentricity with the static eccentricity for a hyperbolic spectrum, aspect ratio = 1

(a) Ω_R = 0.75 (b) Ω_R = 1.0 (c) Ω_R = 1.25

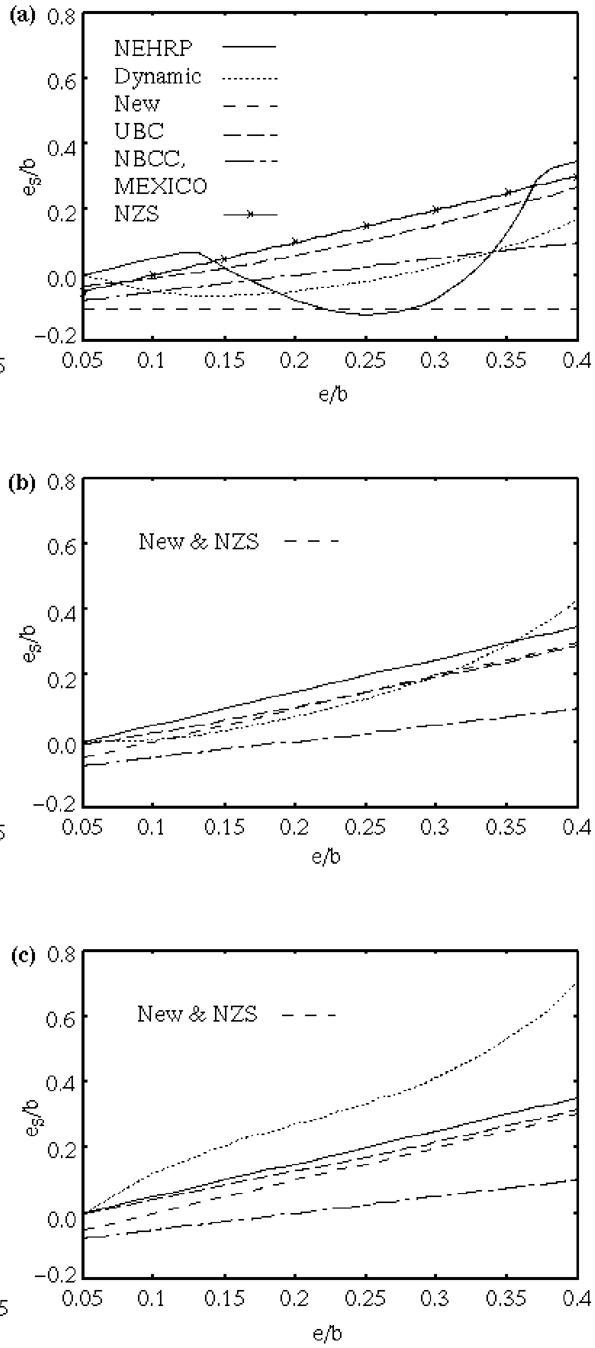
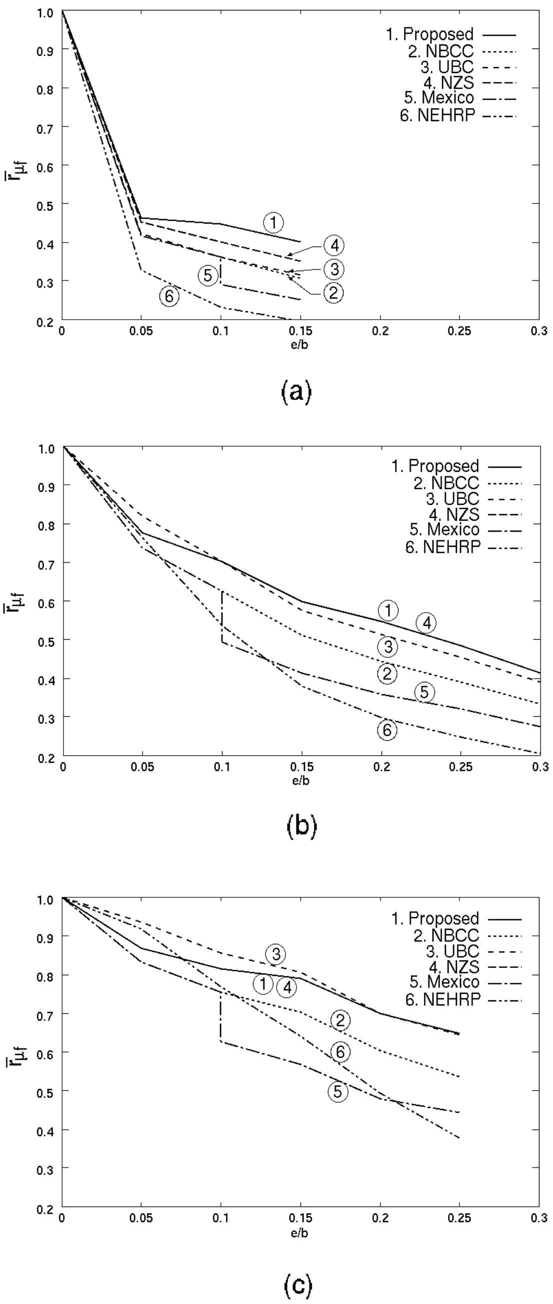


Fig. 3 Variation of effective stiff edge eccentricity with the static eccentricity for a hyperbolic spectrum, aspect ratio = 1

(a) Ω_R = 0.75 (b) Ω_R = 1.0 (c) Ω_R = 1.25



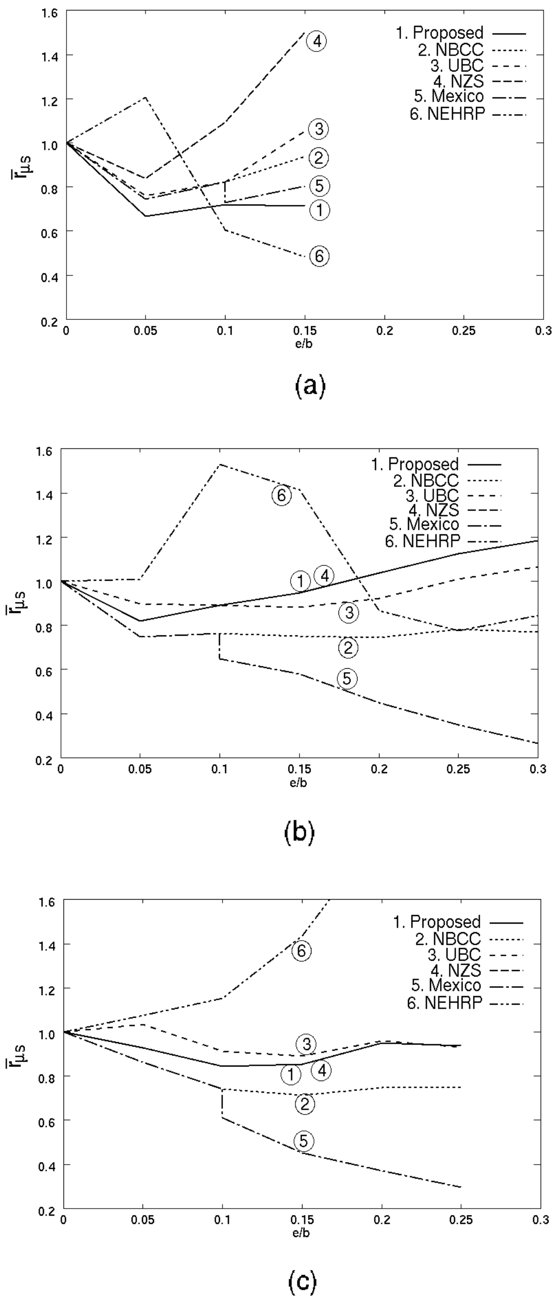


Fig. 4 Ratio of flexible edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$, (c) $\Omega_R = 1.25$

Fig. 5 Ratio of stiff edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$, (c) $\Omega_R = 1.25$