RELIABILITY INDEXES IN EARTHQUAKE RESISTANT DESIGN OF MULTI-STOREY FRAME BUILDINGS

Luis ESTEVA[1], Orlando J DÍAZ-LÓPEZ[2] And Enrique MENDOZA[3]

SUMMARY

A reliability function β(Q) is defined as the ratio of the expected value of the natural logarithm of the safety margin of the structural system divided by its standard deviation. The safety margin is the minimum value of the ratio of the story deformation capacity to the corresponding response-associated demand, and Q is the peak ductility demand of a simplified reference system subjected to the design earthquake. A discussion is presented about the limitations of available criteria and knowledge applicable to the determination of the story deformation capacities. An exploratory study is made about the influence of several mechanical properties, and of their spatial distribution throughout the system, on the values of β(Q) for systems subjected to sets of narrow-band earthquake accelerograms similar to those recorded on soft soil in Mexico City. For the systems studied in the paper, a linearly decreasing relation is found between β and the natural logarithm of Q. The parameters of that relation are not sensitive to the fundamental period of the system for frames whose nonlinear behavior occurs concentrated at plastic hinges with bilinear constitutive moment-curvature functions. Dependence with period is found for more general types of constitutive functions.

INTRODUCTION

One of the objectives of performance-based earthquake resistant design of an engineering system is to ensure that it will be sufficiently safe when exposed to the maximum seismic excitations that may act on it during its intended lifetime. In other words, the probability that it fails during the mentioned interval must be sufficiently low. This probability, as well as its complement, the probability of survival, can be formally expressed in terms of the probability distributions of the numbers and the intensities of the seismic events that may affect the system and of the probability of failure of the system for each of those events.

The probability of failure of a given system is as an increasing function of the earthquake intensity. Because the maximum value of the latter during a given time interval is uncertain, the value of the intensity assumed for the purpose of verifying the safety level in the practice of earthquake engineering must be sufficiently high. This is achieved by stating that the value of that intensity must correspond to a sufficiently long return interval at the site where the system will be built or to a sufficiently low probability of being exceeded during a given time span. Thus, the problem of attaining a specified probability of survival after a sequence of earthquakes that may occur during a given time interval is transformed into one of attaining an adequate safety level under the action of a single earthquake with a prescribed probability of occurrence.

Typical engineering structures are multi-degree-of-freedom systems formed by members that behave in accordance with complex nonlinear constitutive functions. When subjected to earthquake ground motion, they can fail in one of several potential failure modes. Evaluating failure probabilities for such systems subjected to uncertainly known earthquake ground motion is not an easy job. The difficulties arise not only from the complexity of the mathematical tools that must be developed, but also from our imperfect knowledge about the behavior of structural materials and members. The sensitivity of the mentioned probabilities to the detailed

[1] *Institute of Engineering, UNAM, Ciudad Universitaria, 04510 Mexico, D.F., MEXICO, esteva@merlin.iingen.unam.mx*
[2] *Institute of Engineering, UNAM, Ciudad Universitaria, 04510, Mexico, D.F., MEXICO, diaz@merlin.iingen.unam.mx*
[3] *Institute of Engineering, UNAM, Ciudad Universitaria, 04510, Mexico, D.F., MEXICO*

properties of the constitutive functions, and to the uncertainties associated with them, contributes to augment those difficulties. This means that the estimates of failure probabilities and reliability measures obtained in accordance with our present knowledge and models may be skewed with respect to those that would be obtained on the basis of actual statistical distributions of mechanical properties and behavior parameters of real systems. As a consequence, those estimates may serve only the purpose of providing relative values of the safety indicators attained by the application of given design algorithms and parameters for different structural types and configurations, rather than absolute values of those indicators.

From the standpoint of the practice of structural design, there is a need for easy to apply design requirements, expressed in terms of formats and methods capable of leading to consistent safety levels for the different cases covered by them. For the purposes of deriving these formats and methods and determining the parameters to be used in combination with them, systematic studies must be performed along two main lines. One of them is the development of methods for the reliability analysis of complex structural systems subjected to seismic excitations. Another is the study of the possible relations between quantitative measures of the reliability levels of given systems under the action of seismic excitations of given intensities and some simple indicators of the amplitudes of the structural response normalized with respect to adequate indicators of structural capacity.

The present paper summarizes the results of some recent studies along the two lines mentioned in the last paragraph before this. The ground motion excitations used for the purpose of illustration are realizations of a stochastic process model with statistical properties similar to those shown by the acceleration time history designated here as SCT890919EW. This corresponds to the EW acceleration record obtained on soft ground at the SCT site in Mexico City during the 19 September 1985 destructive earthquake [Mena et al, 1986].

SEISMIC EXCITATIONS: STOCHASTIC MODEL

The model used in this paper to represent random ground motion time histories follows closely one previously developed by Yeh and Wen [1989]. It considers an earthquake accelerogram as the realization of a gaussian non-stationary stochastic process modulated in amplitude and frequency:

$$\xi(t) = I(t)\zeta(\varphi(t)) \qquad [1]$$

Here, $\xi(t)$ is the ground acceleration as a function of time t, $I(t)$ is a deterministic amplitude modulation function, $\varphi(t)$ is a transformation of the time scale, intended to modulate frequencies, and $\varsigma(\varphi(t))$ is a unit-variance filtered white noise, stationary with respect to φ. The statistical properties of the model are determined by the forms and parameters of the amplitude and frequency modulation functions, as well as by $S_0(\omega)$, which is the spectral density of $\zeta(t)$ evaluated at a reference time, t0. In the process of Monte Carlo simulation of sets of synthetic ground motion records, the first step consists in determining the values of the parameters of functions $I(t)$, $\varphi(t)$ and $S_0(\omega)$. This is usually accomplished by curve fitting to the empirical functions associated with a given record; in this case, with SCT850919EW. This record is characterized by its long effective duration, as well as by its great energy content in a narrow frequency band. The pseudo-acceleration response spectrum for 0.05 damping shows a pronounced peak at a natural period of 2s, with a maximum ordinate of about 1g.

From a conceptual standpoint, a better alternative for the determination of the statistical parameters of the stochastic model of ground motion consists in estimating those parameters on the basis of “generalized intensity attenuation functions” (GIAF’s). These are empirical functions that express the mentioned parameters in terms of earthquake magnitude and source-to-site distance, including correction factors that account for the influence of local soil conditions. These two approaches will be designated as Methods A and B, respectively. Both were sequentially used in this paper for the generation of sets of acceleration time histories employed in the system reliability studies.

Alamilla et al [1999] present details of the modified version of Yeh and Wen’s model adopted here, as well as about the estimation of its parameters through the applicable GIAF’s. The uncertainties affecting this estimation are significant. As a result, the variability shown by the global characteristics of the records generated in accordance with Method B is much wider than that corresponding to the records that resulted from the application of Method A. In both cases, the resulting accelerograms were normalized to the same intensity. This was achieved multiplying the ordinates of each time history by an adequate scaling factor. Other normalization criteria can also be applied, which may lead to reliability indexes different from those obtained here.

RELIABILITY ANALYSIS OF BUILDING FRAMES

The probability of failure of a multistory frame structure, like that of any complex structural system, is equal to the probability of occurrence of at least one of the possible failure modes. In order to calculate its value, it is necessary to identify those failure modes with the largest probabilities of being reached and to calculate the probability of occurrence of at least one of them. In the case of multistory frame systems, the relevant failure modes usually correspond to the occurrence of relative story displacements in excess of the corresponding story deformation capacities. For any story, the available ductility is defined as the ratio of the ultimate deformation capacity to the corresponding yield deformation. The latter can be obtained dividing the shear strength of the story by its lateral stiffness. Neither the values of the strength nor those of the stiffness of adjacent stories are mutually independent variables in properly designed building frames, because in those cases the occurrence of failure mechanisms involving only structural members from the same story is prevented. However, reasonable estimates of their expected values and uncertainty measures can be derived with the aid of a push-over analysis. The problem remains of estimating the value of the available ductility at any story and, hence, of the corresponding deformation capacity. In the lack of either empirical or theoretically derived values of available story ductility, in this paper the assumption is made that those values can be described in probabilistic form by a mean value equal to$\mu^* \exp(1.65V_\mu)$, where μ^* is the nominal design value of μ, the uncertainly known available ductility, and V_μ its variation coefficient [Esteva and Ruiz, 1989]. In the cases studied here, μ^* and V_μ were taken equal to 4 and 0.25, respectively.

For the purpose of measuring the safety level of a given multistory frame structure subjected to an earthquake ground motion of a specified intensity, use is made of the well known reliability index β proposed by Cornell. In our case, this index was calculated as follows:

$$\beta = \frac{E(Z)}{\sigma_Z} \qquad [2]$$

Here, E(Z) and σ_Z are respectively the mean and standard deviation of Z, which is in turn equal to the minimum value, along the building height, of the natural logarithm of the ratio of the story deformation capacity to the peak deformation demanded by the nonlinear structural response to the assumed ground motion. The values of these statistical parameters are obtained by Monte Carlo simulation. Uncertainties associated with the values of the gravitational loads, member strengths and stiffness properties, are randomly selected from statistical distributions proposed by several researchers on the basis of empirical information [Mirza and McGregor, 1979; Meli, 1976]. Ground motion time histories of specified intensities were generated in accordance with the criteria and methods described above.

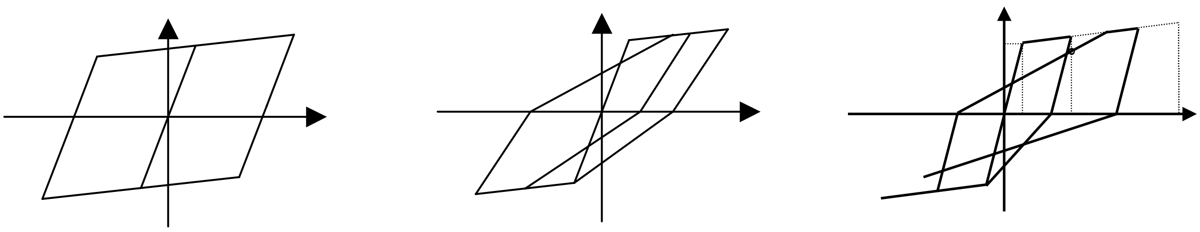
Three types of moment curvature constitutive functions were assumed at the critical sections of the frame members. They will be identified here as BL, TK and MWS, which stand for bilinear, Takeda and modified Wang and Shah, respectively (Figs. 1a-c). Functions of type BL assume that no reductions of strength or stiffness take place as a consequence of cyclic behavior; that is, the influence of damage accumulation is neglected. The first branch of the curve is obtained from an analysis that accounts in detail for the cross section dimensions, the amount of reinforcement and the stress-strain curves for both, concrete and steel reinforcement. A cracked section model is assumed for this part of the curve. The post-yield segment is characterized by a positive slope, which accounts for strain hardening of the reinforcement.

According to its simplest version, model TK assumes that the stiffness values of both the unloading and the reloading branches of the cyclic moment curvature functions may suffer reductions that depend only on the maximum deformation amplitudes reached in previous cycles. The severity of these reductions is determined by the parameters α_T and β_T (Fig.1b), which are taken here equal to 0.15 and 0.4, respectively.

Model MWS is a modified version of one previously proposed by Wang and Shah [1987]. This model is expressed in terms of six parameters (Fig.1c): F_y, K_y, α_K, X_F, C and α. The first three of these define the curve of the envelope load-deflection curve for monotonically increasing load, and the other three describe the consequences of damage accumulation. A fatigue damage index β_D is calculated for load direction in accordance with the following equation:

$$\beta_D = C\sum_{i=1}^{N}\frac{X_i}{X_F} \quad [3]$$

Here, N is the number of deformation cycles; Xi, i = 1,..,N are their amplitudes, X_F is the amplitude leading to failure under monotonic load and C is an empirically determined parameter. This fatigue index determines the reduction in stiffness experienced by segment FH of the reloading curve shown in Fig.1c. According to this figure, the mentioned segment intersects at point G the vertical line going through point B, corresponding to the maximum amplitude experienced in previous response cycles. The ordinate of point G is $F_D(X_i)$, which is equal to the ordinate of point B, lying on the virgin curve, multiplied by the damage-related factor (1-D), where D = 1-exp ($-\alpha\beta_D$) and α is a constant that may be taken equal to 0.0671, on the basis of experimental evidence.



a) Bilineal b) Takeda c) Modified Wang and Shah

Fig.1 Moment curvature constitutive functions

CASES STUDIED

Two groups of systems were studied. Group 1 includes standard building frames, with beam/column stiffness ratios considered to be typical of those found in practical cases. They were obtained by straightforward application of the basic criteria and methods specified by Mexico City seismic design regulations. However, alternate designs were obtained for several values of the base-shear coefficients, different from those required by the regulations applied. This was done with the objective of studying the variation of the reliability index β with the lateral-strength seismic design coefficient, while keeping constant the value of the ground motion intensity. For each frame included in Group 1, a frame was generated for Group 2, keeping the same values of the story stiffness of its counterpart, but modifying the value of the stiffness of each individual structural member. The objective was to obtain in each case a frame having the same global linear response properties as its counterpart in Group 1, but with a larger value of the beam/column stiffness ratio, while maintaining the beam/column strength ratio necessary to comply with the strong-column-week-beam design philosophy. The resulting values of the beam/column stiffness ratio for the frames included in Group 2 were in general of the order of five times those of their counterparts in Group 1.

Each of the two groups mentioned above included four cases, according to the number of stories: five, ten, fifteen and twenty. Each frame was assumed to belong to a square-plan building. Five- and ten-story buildings had two bays in each direction; for fifteen- and twenty-story buildings the numbers of bays were equal to three and four, respectively. The bay widths were respectively equal to 5.0 and 6.2m, for the five- and ten-story buildings, and 6.5m for the others. The fundamental periods of vibration were equal to 0.641, 1.096, 1.544 and 1.628s, respectively, for the two groups considered, irrespective of the values of the beam/column stiffness ratio. It is easy to notice that the difference between the periods corresponding to fifteen and twenty stories do not reflect the differences in height. This is a consequence of adjusting the values of the story stiffness in the taller buildings, in order to comply with the maximum-displacement limitations established in the design regulations applied.

Figures 2 and 3 show the variation of the reliability index β with the nominal value of the base shear coefficient that would apply to linear response. These values are significantly lower than the observed peak of 1g mentioned above regarding the pseudo-acceleration response spectrum of the SCT850919EW record for linear behavior and 0.05 damping. This apparent inconsistency is justified in terms of the over-strength margins that are implicit in all structural systems as a consequence of the safety factors adopted in design. The nominal values of the seismic design coefficients that were actually applied are equal to one fourth of those shown in the horizontal axis of each figure. This reduction is intended to account for the capability of ductile behavior of the systems considered, as well as for the over-strength margin mentioned above.

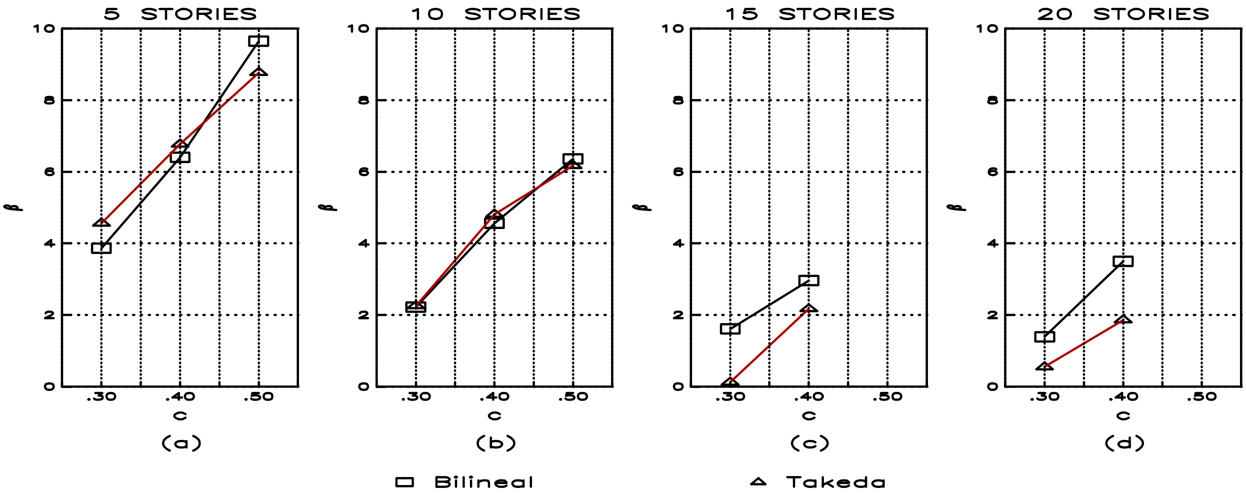


Fig. 2 β vs. c for simulated records with method A for an intensity of 1g.

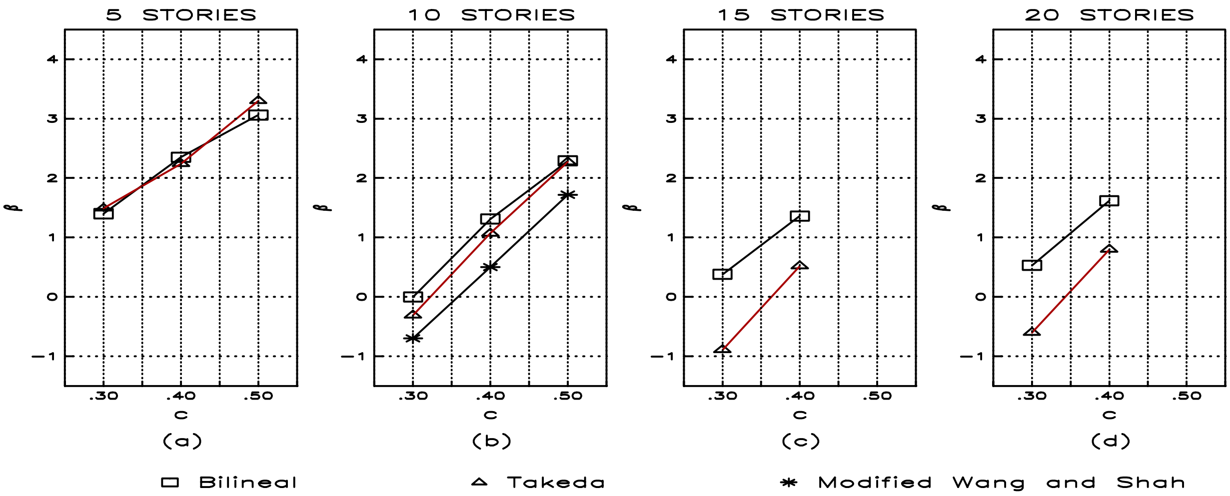


Fig. 3 β vs. c for simulated records with method B for an intensity of 1g.

The curves shown in Figs. 2 and 3 cover only a fraction of the number of cases studied. They consider the three behavior models described in the previous section, but they are restricted to systems belonging to Group 1.Figs. 2 and correspond respectively to sets of synthetic ground motion records simulated in accordance with methods A and B, respectively. All the records were normalized so as to lead to the same value of the maximum ordinate of the linear-response spectral pseudo-acceleration for 0.05 damping: 1g, which is the value obtained for the SCT850919EW record. As expected, the wide variability observed on the global characteristics of the records generated in accordance with Method B gave place to values of β much lower than those associated with sets of records simulated in accordance with method A.

The higher values of β shown in Figs. 2 and 3 for five- and ten-story buildings are a natural consequence of the lower values of the expected spectral ordinates for the low period range. The low sensitivity of β to the type of moment-curvature constitutive function constitutes an expression of the low level of nonlinear response attained. The higher sensitivity shown for the ten-story buildings subjected to the records generated by method B is a consequence of the larger uncertainty that affects the resulting spectral ordinates. The values of β for the fifteen- and twenty-story buildings are not very different from each other. This is an expected result, in view of both, the small difference between the expected values of the fundamental periods for both groups of structures and the flatness of the pseudo-acceleration response spectra for bilinear behavior.

STORY DEFORMATION CAPACITIES

An important weakness of the foregoing studies lies on the lack of validation of the criteria used to estimate the story deformation capacities. In order to judge about the validity of those criteria and gain some information based on better grounds, a brief exploratory study was undertaken with the aid of mathematical models of the stories of the building frames reported above. The study consisted in modeling each story as isolated from the rest of the system and subjecting it to a monotonically increasing horizontal load. In each case, the system analyzed included the story of interest and the half-lengths of the columns immediately above and below it. The inflection points of bending in the columns were assumed to lie at the ends of those half-lengths, with the exception of the first story columns, for which the inflection points were assumed to lie at 0.6 of the column length above the foundation.

As in the other studies reported here, plastic hinges were assumed to form at the ends of beams and columns. The maximum local rotation capacity of each hinge was calculated using the plane-cross-section assumption of the conventional theory of nonlinear bending. The values used for the maximum deformation capacities of concrete and reinforcement were based on the results of laboratory tests. The internal moment at a hinge was assumed to become equal to zero when the maximum rotation capacity of the hinge was reached. In this manner, a curve relating horizontal shear with lateral deformation was obtained for each story. Values of the lateral yield deflection and of the deformation capacity were computed on the basis of this information. Results for the ten-stories building are summarized in Fig. 4. This figure presents values of the ratios between the yield deflection and the deformation capacity calculated in this manner to the values derived from the simplified criteria described in previous sections.

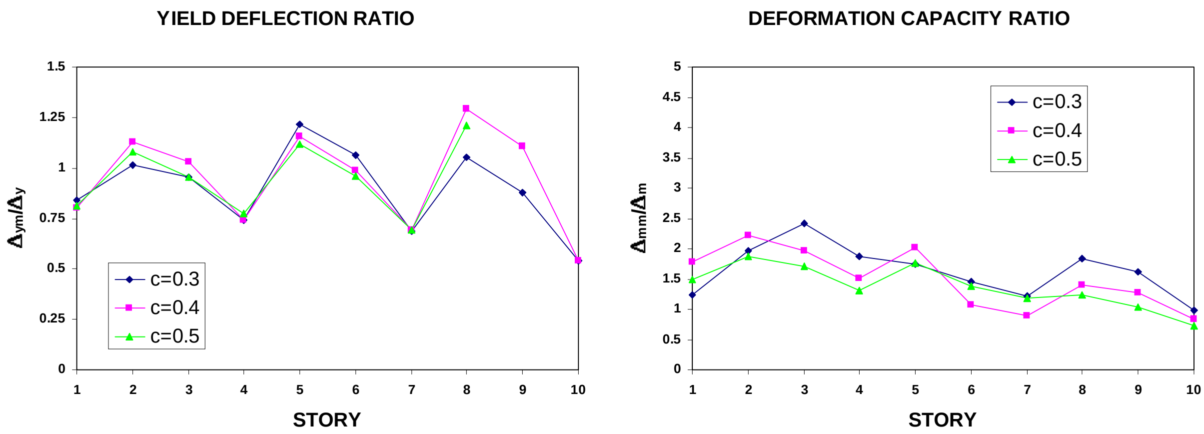


Fig. 4 Yield deflection and deformation capacity ratios for ten stories building

Figure 4 shows that, in general, the story deformation capacities are larger, and the yield deformations are smaller, than those assumed in the system reliability studies that gave place to results similar to those reported in Figs. 2 and 3. This means that both, the capacities for ductile deformation and the reliability indexes are larger than those obtained in the studies mentioned in the section on CASES STUDIED.

SEISMIC RELIABILITY FUNCTIONS

For different applications it is important to characterize a complex structural system in terms of a simple quantitative description of its capacity to resist earthquakes. The best option would be to express that capacity by means of a parameter defining the minimum earthquake intensity capable of leading to structural failure. In this manner, the probability of failure under the action of an earthquake with uncertainly known intensity would be calculated as the probability that the intensity of an earthquake acting on the structure were larger than the earthquake resistant capacity of the latter. Obtaining this probability is not difficult when dealing with a single-degree-of-freedom system, but it is a major undertaking for a complex one. Some of the difficulties are associated with the need to consider the interaction between several potential failure modes; others are concerned with the practical impossibility to define the failure conditions for all the possible failure modes in terms of simple indicators of the expected structural response. Just for illustration, consider the difficulties involved in formulating the failure conditions for a story of a building frame subjected to cyclic deformations, starting from the failure conditions at the critical sections of beams and columns.

An alternative to dealing with the problem of determining the probability distribution of the earthquake intensity that can be resisted by a structural system is obtaining a function that links the probability of failure with the earthquake intensity or with a measure of the structural response. This is the approach adopted here, under the assumption that reliability indexes can be calculated in terms of story ductility demands and capacities, the latter calculated as described in the preceding sections. It is also assumed that the variable Z that appears in Eq.2 is the reciprocal of a damage index D_Z that can be related to the normalized response of a sdof reference model of the system (SRM) according to Eq.4. The reference model is one obtained by means of a push-over analysis of a detailed model of the system.

$$D_Z = \xi \frac{S_d}{\delta_y} \qquad [4]$$

Here, ξ is a random factor that transforms the response of the SRM into the maximum story deformation of the detailed model; δ_y is the yield deflection of the SRM and Sd is the ordinate of the nonlinear displacement response spectrum. The ratio $Q = S_d/\delta_y$ is a design parameter. Replacing Eq.4 into Eq.2, a linear relation is obtained between β and ln Q:

$$\beta = -(E \ln\xi + \ln Q)/\sigma \ln\xi \qquad [5]$$

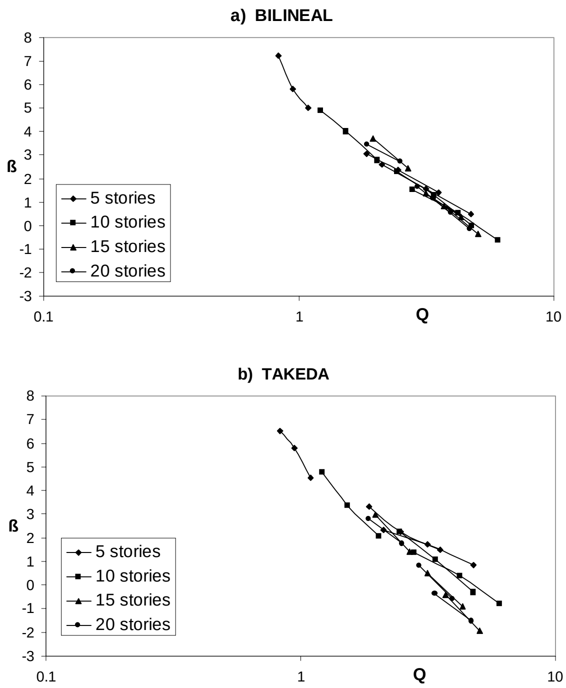


Fig. 5 β vs. Q for frames of Group 1

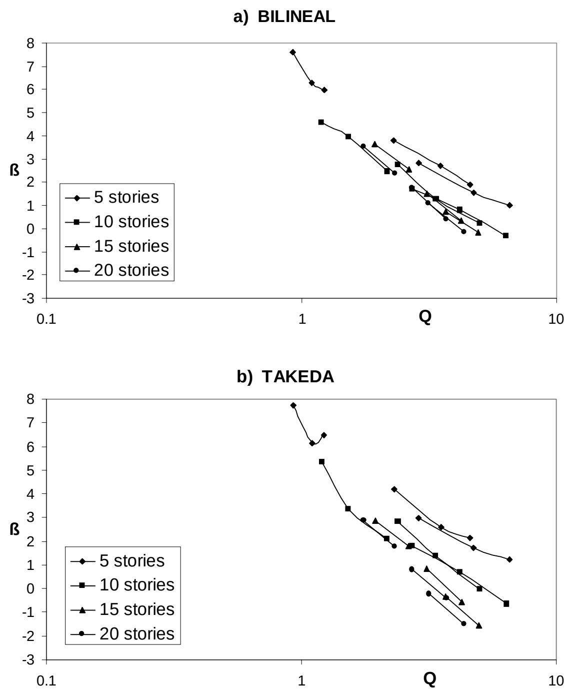


Fig. 6 β vs. Q for frames of Group 2

The linearity expressed by Eq.5 is confirmed by the Monte Carlo simulation results shown in Figs. 5 and 6 for frames included in Group 1 and Group 2, respectively. The upper parts of these figures show that for systems with plastic hinges with bilinear behavior the relation between β and Q is not sensitive to the number of stories (which is strongly correlated with the fundamental period of vibration) or to the beam/column stiffness ratio. A comparison of the upper and lower parts of the same figures show that, as expected, for systems with plastic hinges behaving in accordance with Takeda's constitutive function, the resulting values of β are in general smaller than those obtained for the corresponding systems with bilinear behavior. The figures also show that the differences are larger for the larger values of Q and, also, that they grow with the number of stories. Another important observation from the lower part of Figs. 5 and 6 is that the values of β corresponding to fifteen- and twenty-story buildings are practically equal, in particular for buildings belonging to Group 1. This may imply that the differences between the values of β associated with systems with different numbers of stories but equal values of Q are mainly due to the differences in natural periods and, to a lesser extent, to the differences in the numbers of stories. The latter are related to the spatial variation in ductility demands and to the multiplicity of potential failure modes that characterize multistory frame structures.

CONCLUSIONS

1. A reliability function $\beta(Q)$ has been defined that links the values of the safety index to the peak ductility demand of a simplified nonlinear reference system. Its applicability to multistory building frames has been presented and discussed.

2. The values of β are sensitive to the methods used to estimate the story deformation capacities.

3. For the models used, β is a linearly decreasing function of Q. For systems whose nonlinear behavior is concentrated at plastic hinges with bilinear constitutive functions, β is not sensitive to the fundamental period of the system. For other types of constitutive functions, β decreases with that period, at least in the range of values explored, which corresponds to structural periods shorter than the dominant period of the ground motion pseudo-acceleration response spectrum.

4. Due to the limitations of the methods used to estimate story deformation capacities, the values of the reliability indexes obtained are meaningful only for the purpose of comparing relative values of the reliability levels attained by different structural arrangements or design criteria. However, they can not be interpreted in absolute terms.

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