

## THE MODELLING OF EARTHQUAKE INDUCED COLLAPSE OF UNREINFORCED MASONRY WALLS COMBINING FORCE AND DISPLACEMENT PRINCIPALS

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### SUMMARY

This paper outlines analytical and experimental research carried out as a collaborative research project at both The University of Adelaide and The University of Melbourne since 1997. The objective of the research has been to develop an understanding of the physical processes governing the collapse behaviour of unreinforced masonry walls subjected to face loading culminating in the development of a simplistic rational analysis procedure for practical applications.

Firstly existing procedures including 'quasi static' acceleration analyses and the dynamic 'equal energy' velocity analysis are critically reviewed. Following this the development of time-history analysis (THA) software by the authors and key outcomes of shaking table tests used for calibration and conformation of the THA are briefly discussed. A displacement based (DB) analysis is then introduced as an alternative procedure to overcome the limitations of the existing procedures. Finally, comparison of existing and DB analyses with THA results are presented concluding with the DB methodology providing the most rational and effective procedure provided that suitable wall natural frequencies are identified.

### INTRODUCTION

In recent years displacement based (DB) design philosophies have gained popularity in the research arena for the seismic design and evaluation of ductile structures although they have not been thought applicable to brittle structures such as unreinforced masonry (URM). Interestingly, extensive non linear time history analysis (THA) and shake table testing of face loaded URM walls have consistently indicated a significant reserve capacity to displace (rock) without collapsing [Kariotis et al 1985, Lam et al 1995]. Typically levels of seismic loading exceeding that predicted by simple static ultimate strength calculations have been reported although this is directly related to the relationship between the dominant ground motion frequency and the natural rocking frequency of the wall. This suggests that DB philosophies could be used to provide a rational means of determining seismic design action in preference to the traditional forced base approach.

The currently available static and dynamic predictive models have not been able to account for the large displacement post cracking behaviour and reserve capacity of URM walls when subjected to the transient characteristics of real earthquake excitations. Traditional quasi-static approaches are restricted to considerations taken at a critical snapshot during the response and hence the actual time-dependent characteristics are not modelled so that the reserve capacity to rock is also not recognised. While these may be appropriate as suitably conservative analysis tools for new structures they may not be for the review of existing URM where an unacceptable economic penalty could be imposed should the reserve capacity not be considered. In recognition of this shortfall a velocity-based approach founded on the equal energy 'observation' was developed by [Priestly 1985] which considers the energy balance of the responding wall. The main disadvantage of this procedure is

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that the energy demand is very sensitive to the selection of elastic natural frequency which is highly variable due to the non-linear post-cracked properties of URM.

## CRITICAL EVALUATION OF CURRENT STATIC AND DYNAMIC PREDICTIVE MODELS

The response of an URM wall to earthquake induced base excitations is a complex dynamic process. Analysis is however often simplified by considering an instantaneous maximum acceleration occurring at a critical snapshot of the response. The critical acceleration is then represented as the associated inertia force to idealise the real dynamic action adopting a 'quasi-static' acceleration philosophy. Earthquake resistant codes and standards typically specify methods based on this philosophy being either the linear elastic or rigid body analysis.

For linear elastic analysis, tensile and compressive stresses at the critical wall cross section where the maximum 'quasi-static' bending moment develops are analyzed. The resistant capacity of the wall is thus proportional to the wall geometry, axial force (self-weight and overburden) at the critical cross-section and the flexural tensile strength ( $f'_t$ ) at the brick-mortar interface. The assumption of an un-cracked wall and elastic stress-strain behaviour is a major limitation of this analysis methodology. Significantly, the formation of flexural tensile cracks will not necessarily result in wall failure but with increasing displacement vertical reactions are rapidly forced in the direction of the compression zone at rotation joints providing a restraint to the deflection.

The post-cracked seismic performance of an URM wall is therefore more realistically analysed by the rigid body equilibrium analysis method, which compares overturning and restoring moments. The overturning moment is obtained again by quasi-static principals, whereas the restoring moment is obtained by the consideration of vertical forces resulting from self-weight and overburden acting on the free body above the crack. Although the location of the top vertical reaction is dependent on the type of top wall connection the base and mid-height vertical reactions are assumed to be at the extreme faces of the wall as would be the case for a rigid object.

It must be recognised that neither the linear elastic nor the rigid body 'quasi static' analyses take into account the time-dependent nature of the response of the wall since an instantaneous acceleration occurring at a critical snapshot of the response is considered. In an attempt to overcome this shortfall a velocity based approach founded on the 'equal energy observation' was developed, [Priestly 1985]. Here the potential energy (PE) absorptive capacity of a wall undergoing large displacement is first quantified by determining the area under the non-linear force-displacement ( $F-\Delta$ ) curve as the wall is pushed from vertical to a maximum displaced position without overturning. The maximum kinetic energy (KE) demand is then derived from the maximum spectral velocity as obtained from an elastic response spectrum to compare with the PE capacity. The wall is therefore assumed to remain un-cracked and behave linearly up to the time when the maximum KE is reached where it is assumed to crack and rock as the PE is increased to absorb the KE. The abrupt transition from the linear elastic response (in the un-cracked state) to the non-linear rocking response (in the cracked state) is a major assumption of the equal energy procedure. In reality, the dynamic behaviour of the wall prior to the commencement of rocking is not always linearly elastic so that the "elastic" natural period of vibration of the wall, which governs its spectral velocity (and hence its maximum KE) can not be predicted with certainty. A second shortcoming of this procedure is that the accumulated effects of multiple pulses on the wall during rocking are not considered.

Figure [1] provides a comparison of lateral strength predictions for the analysis procedures reviewed. As can be seen for a lightly loaded wall linear elastic analysis provides a less conservative prediction than rigid body while the converse is true for walls with higher overburden stresses levels. Physically this anomaly occurs as the longer internal lever arm assumed by the rigid body analysis with vertical reactions at the leeward faces has a larger beneficial effect on restoring moment than by including the flexural tensile capacity in the linear elastic analysis. In general the equal energy method provides a less conservative prediction again than both of the quasi-static analyses representing the theoretical reserve capacity however as previously discussed the prediction of this is very sensitive to the "elastic" stiffness or modulus selected.

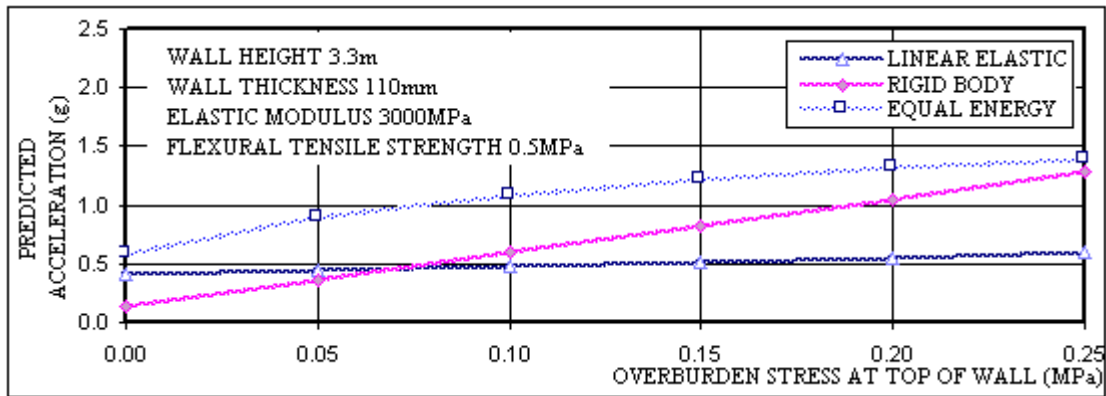


Figure [1] Comparison of Current Static and Dynamic Predictive Models

### DEVELOPMENT OF NON-LINEAR TIME-HISTORY ANALYSIS SOFTWARE

Non-linear time-history analysis (THA) based on time-step integration is the most representative and reliable method of accounting for the time-dependent nature of URM wall response to applied excitations providing the non-linear (F- $\Delta$ ) and damping properties are accurately represented in the analytical model. Although THA has limited application in design it is a valuable research analysis tool used in parametric studies for the assessment of simplified analysis procedures. Specific computer software has been developed by the authors to operate the non-linear THA which will be reported in detail elsewhere however for completeness is briefly described.

For the purpose of calibration and confirmation of analytical results extensive out-of-plane static and shaking table tests on URM wall panels have been undertaken. While these have been reported elsewhere [Lam 1995, Doherty 1998] key outcomes will be presented here in relation to the development of the non-linear THA. Freestanding parapet, non-loadbearing simply supported (cornice support at top) and loadbearing simply supported walls (representative rigid slab above) were investigated all having damp proof course (DPC) connections at their base. For simply supported (SS) walls a rigid frame transferred the base excitation in phase and without amplification to the top wall support. In Australia the majority of URM construction is limited to 2-3 story 'walk up flats' with a typical overburden stress range of 0.1MPa at roof level to 0.5MPa at ground level. Considering overburden stress improves wall behaviour and the amplification of accelerations into the higher levels of a structure, 0.075MPa and 0.15MPa where selected for the loadbearing walls tested. Walls constructed of both standard 3 hole extruded clay brick 76x110x230mm and half scale clay paver 35x50x230mm where tested at aspect ratios of 15 and 30 using a mortar based on a 1 Cement: 1 Lime: 6 Sand mix. Instrumentation was located to fully account for the rocking displacement and acceleration response of the test walls.

#### Non-Linear Force Displacement (F- $\Delta$ ) relationship

The non-linear (F- $\Delta$ ) relationship of a face loaded URM wall results from the complicated interaction of gravity restoring moments, the movement of vertical reactions with increasing displacement and if loadbearing P- $\Delta$  overturning moments. The wall properties that therefore dictate the shape of the curve are wall geometry, support conditions, overburden stress, material modulus and importantly the condition of the mortar joint rotation points. While theories have been postulated to define this complex behaviour these have been found to not correlate well with experimental results.

For each of the SS test walls static-push tests were undertaken to provide the static non-linear (F- $\Delta$ ) curve for each of the wall configurations. Free vibration tests were then used to confirm the assumption of a triangular rigid acceleration response profile to be later used in the development of the non-linear THA algorithm. Figure [2a] shows the good correlation of the (F- $\Delta$ ) curves both determined by static push test and dynamic test in accordance with the triangular response acceleration assumption. The bi-linear rigid (F- $\Delta$ ) curve is also shown which is approached by the real non-linear semi rigid curve at larger displacements for relatively small levels of overburden stress.

As a result of the experimental investigation into the non-linear ( $F-\Delta$ ) relationship, critical parameters were determined to be the initial stiffness, maximum resistance force and rigid negative stiffness. For this reason a tri-linear approximation of the ( $F-\Delta$ ) relationship is introduced, refer figure [2b]. This simplification allows the calculation of pseudo-static stiffness for each of the three linear portions as required by the Newmark constant-acceleration method. With the rigid resistance force  $R_e$  and negative stiffness  $K_e$  determined from wall geometry, overburden and mass the two displacement points,  $u_y(1)$  and  $u_y(2)$ , will define the tri-linear curve.

As mentioned earlier the shape of the ( $F-\Delta$ ) curve is not only dependent on constant parameters such as wall geometry and overburden stress but by the condition of the mortar joint at rotation points. Degradation of these joints occurs quickly during rocking with both the initial stiffness defined by  $u_y(1)$  and the semi rigid force resistance plateau defined by  $u_y(2)$  decreasing resulting in a much flatter curve with lower frequency characteristics than for a relatively newer wall. To simplify this complicated behaviour the following very broad definitions have been determined empirically for SS URM walls defining  $u_y(1)$  and  $u_y(2)$  as a percentage of the wall thickness for various levels of wall condition: (1) New wall { $u_y(1)=6\%$ ,  $u_y(2)=28\%$ }, (2) Moderately degraded wall, { $u_y(1)=13\%$ ,  $u_y(2)=40\%$ } and (3) Severely degraded wall, { $u_y(1)=20\%$ ,  $u_y(2)=50\%$ }.

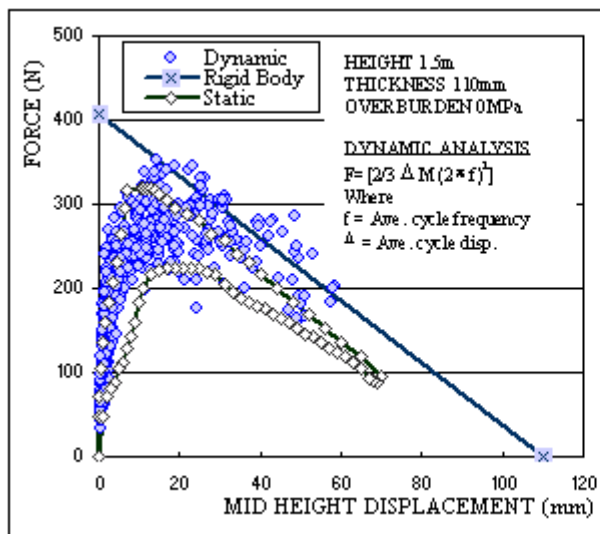
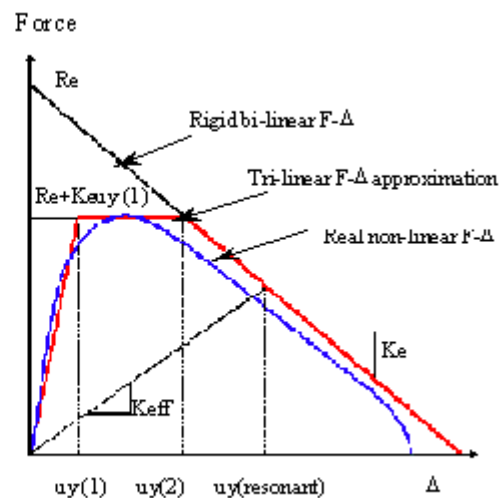


Figure [2a] ( $F-\Delta$ ) Dynamic V's Static



[2b] Tri-linear ( $F-\Delta$ ) Simplification

### Non-Linear Damping-Displacement Relationship

Due to the complex physical nature of damping it is usually represented in a highly idealized fashion as an equivalent viscous (velocity proportional) damping force with a similar decay rate under free vibration conditions to that of the real system being modeled. Thus for a rocking URM wall the proportional damping coefficient,  $C$ , is selected so that the vibrational energy dissipated is equivalent to the rocking system energy losses including elastic, friction, impact and joint rotation. As such, free vibration tests were used to determine the real non-linear damping properties of the rocking system. Rayleigh damping, being a combination of both mass and stiffness proportional damping was found to best represent the real system with increases in percent critical damping ( $\xi$ ) at both large and small frequency responses. As shown in figure [3a] a lower bound to  $\xi$  ranges from 13% at large displacement, low frequency responses to an average of 4.5% for the majority of the remaining frequency response.

### Non-Linear Frequency-Displacement Relationship

As has been well documented in the past, the frequency-displacement ( $f-\Delta$ ) relationship of rocking bodies is highly non-linear with higher frequencies typically observed at small vibration amplitudes decaying exponentially with increasing vibration amplitude to a lower frequency asymptote prior to failure, refer figure [3b]. The non-linearity of the ( $f-\Delta$ ) relationship has been shown to be a direct result of the shape of the non-linear ( $F-\Delta$ ) curve with a relatively smaller influence from damping as discussed in the next section.

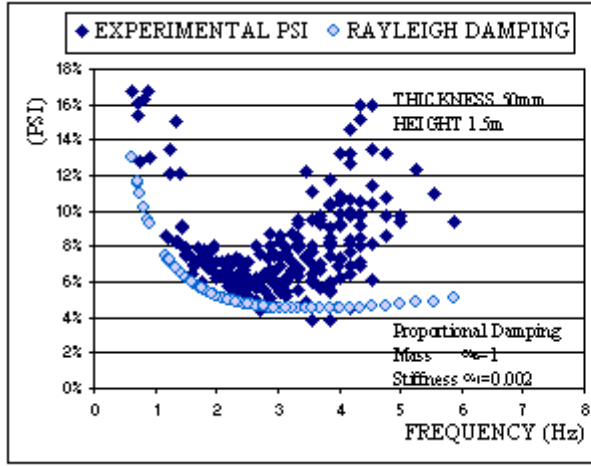
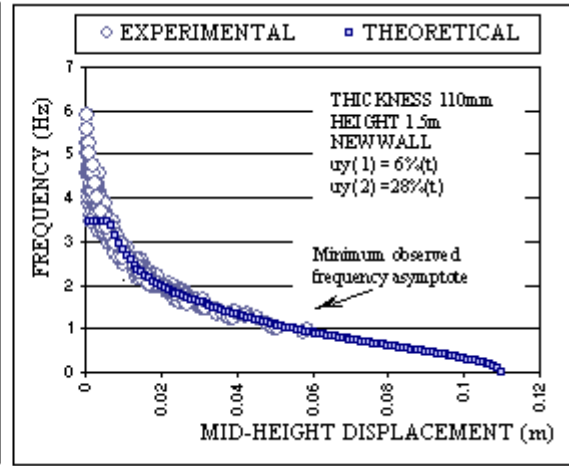


Figure [3a] Non-linear Damping



[3b] Non-linear ( $f-\Delta$ ) Relationship

### Time History Analysis Algorithm Development

To apply THA for the prediction of the semi-rigid rocking response of an URM wall the real highly non-linear system must first be modelled as a basic linear single degree of freedom (SDOF) oscillator. In doing this it permits the use of time-stepping integration procedures such as the Newmark constant-acceleration approximation. The modelling conversion is achieved by comparison of the respective system dynamic equations of motion. Equation [1] represents the widely accepted dynamic equation of motion for a basic linear SDOF oscillator subjected to base excitation  $\ddot{a}_g$  where  $C$  is the proportional damping coefficient,  $M$  the system mass,  $v(t)$  the displacement response and  $\omega$  the system natural angular frequency. Since for the SDOF oscillator the system frequency ( $f = \omega/2\pi$ ) is constant the single equation can describe the dynamic behaviour. For semi rigid URM walls with the tri-linear ( $F-\Delta$ ) simplification used to model the real non-linear curve three equations are required to describe the dynamic behaviour with changing stiffness at each of the three linear portions. Equations [2-4] therefore represent the dynamic equation of motion where  $v(t)$  is the displacement response at either the mid-height of an SS wall or at the wall top of a free standing parapet wall. Significantly the  $3/2$  factor is directly related to the assumption of a triangular acceleration response and will be important later when discussing the 'substitute structure' in relation to displacement based methodologies in section 4.

$$\ddot{v}(t) + \left[ \frac{C}{M} \right]_{SDOF} \dot{v}(t) + [\omega^2]_{SDOF} v(t) = -[\ddot{a}_g]_{SDOF} \quad [1]$$

$$\ddot{v}(t) + \frac{3}{2} \left[ \frac{C}{M} \right]_{EXP} \dot{v}(t) + \frac{3}{2} \left[ \left( \frac{Re(1) + Ke(1)uy(2)}{M uy(1)} \right) \right]_{EXP} v(t) = -\frac{3}{2} [\ddot{a}_g]_{EXP} \quad \text{for } v(t) < uy(1) \quad [2]$$

$$\ddot{v}(t) + \frac{3}{2} \left[ \frac{C}{M} \right]_{EXP} \dot{v}(t) + \frac{3}{2} \left[ \left( \frac{Re(1) + Ke(1)uy(2)}{M v(t)} \right) \right]_{EXP} v(t) = -\frac{3}{2} [\ddot{a}_g]_{EXP} \quad \text{for } uy(1) < v(t) < uy(2) \quad [3]$$

$$\ddot{v}(t) + \frac{3}{2} \left[ \frac{C}{M} \right]_{EXP} \dot{v}(t) + \frac{3}{2} \left[ \left( \frac{Re(1) + Ke(1)v(t)}{M v(t)} \right) \right]_{EXP} v(t) = -\frac{3}{2} [\ddot{a}_g]_{EXP} \quad \text{for } v(t) > uy(2) \quad [4]$$

$$f_{SRR} = \frac{\sqrt{\frac{3}{2} \left[ \left( \frac{Re(1) + Ke(1)uy(2)}{M v(t)} \right) \right]_{SRR}}}{2\pi} \quad \text{for } uy(1) < v(t) < uy(2) \quad [5]$$

With the exception of the damping term to be treated later, comparison of the above equations show that simple substitutions need only be made to permit the real non-linear semi rigid rocking system to be modelled as a basic linear SDOF oscillator. In fact this can be achieved by applying a  $3/2$  conversion factor to the applied base excitation and real acceleration-displacement relationship. Interestingly, the first substitution also implies the importance that the ( $F-\Delta$ ) curve shape has on the non-linear ( $f-\Delta$ ) response. To illustrate further, equation [5] represents the non-linear ( $f-\Delta$ ) relationship for response displacement values between  $uy(1)$  and  $uy(2)$ . A similar

calculation can be performed to determine the linear ( $f\Delta$ ) relationship for other response displacement. Figure [3b] shows the good correlation between the analytical and experimental results.

To determine the level of damping required for the THA a comparison of the damping terms of the dynamic equations of motion must be made. Since the real experimental damping values are determined with the assumption of the system being a SDOF oscillator a conversion factor of 2/3 must be applied to that value before application to the THA. As discussed earlier the value of  $\xi$  is non-linear being displacement dependent such that to correctly model this damping an iterative approach has been in-built into the THA software.

### Comparison of Non-Linear THA and Experimental Results

Having calibrated the THA software comparisons of analytical response and shaking table tests were found to give very good correlation. Figure [4a] shows the experimental mid-height displacement response of SS non-loadbearing wall with theoretical THA results. As can be seen peak displacement and both the forced and free vibration phases are modelled well. Figure [4b] demonstrates that the hysteretic behaviour is also modelled well highlighting the shape resulting from the non-linear ( $F\Delta$ ).

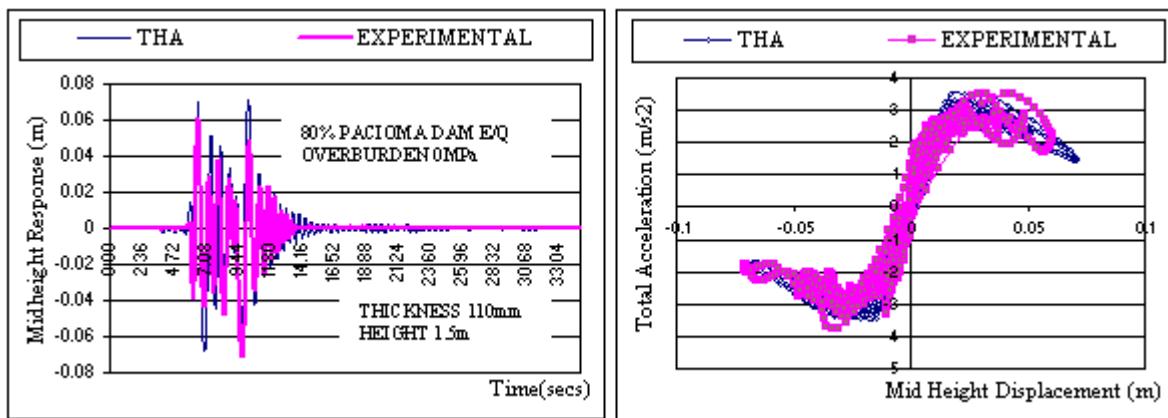


Figure [4a] Displacement Response Comparison

[4b] Hysteretic Behaviour Comparison

### DISPLACEMENT BASED ANALYSIS FOR FACE LOADED URM WALLS

The displacement-based methodology provides a rational means for determining seismic design actions as an alternative to the more traditional force-based approach. The procedure is based on the comparison of the structure's displacement capacity and predicted displacement demand. For simplification the displacement demand and capacity are defined using the 'substitute structure' methodology. Here dynamic behaviour is represented by an elastic SDOF oscillator characterized by the real structures secant stiffness at maximum in-elastic displacement and a level of equivalent viscous damping appropriate during the non-linear response.

For SS URM walls subjected to face loads the instability or ultimate mid-height displacement limit is considered to have been reached when the resultant vertical force due to self weight and overburden above the mid-height crack is displaced outside of the rocking edge of the wall. The mid-height displacement at which this occurs is dependent on the top wall connection. For example, a non-loadbearing or wall with the top vertical reaction at the leeward face (concrete slab above) the ultimate mid-height displacement can be shown to be the thickness of the wall ( $t$ ). If the top vertical reaction is closer to the wall centreline (narrow timber top plate connection) the ultimate displacement is reduced to half of the wall thickness. For illustrative purposes further discussion will be limited to support conditions with ultimate mid-height displacement equal to the wall thickness. As discussed in the development of the THA algorithm to represent the rocking wall as the SDOF oscillator 'substitute structure' a triangular acceleration response is appropriate so the two effective masses are located at the one-third wall heights. The effective displacement capacity of the 'substitute structure' is therefore 2/3 of the wall thickness.

In accordance with displacement based design, displacement demand is determined as the response spectral displacement (RSD) as obtained from the elastic response spectrum and representative frequency of the 'substitute structure'. Plotted against system period, displacement spectrum are typified by a gradual increase in RSD to a peak level of maximum displacement amplification after which the RSD decreases and gradually

converges to the peak ground displacement (PGD). The frequency level at which the peak and convergent points are reached are dependent on the dominant frequency of the ground motion. If the dominant ground motion period is small (high frequency) compared with the wall at large displacements the RSD will have already converged to the PGD at the walls natural period. This suggests that during high frequency ground motion the wall will be safe from overturning provided that the PGD does not exceed 2/3 of the wall thickness. On the other hand should the dominant ground motion frequency be near that of the walls natural resonant frequency the RSD will be significantly amplified above that of the PGD. THA and shaking table tests have shown this maximum displacement amplification of 1.5 to 3.3 which correspond to dynamic amplification of 1 to 2.2 for 'substitute structure'. This suggests that during ground motion with dominant frequencies in the vicinity of the walls resonant frequency the wall will be safe from overturning provided that the PGD does not exceed 0.3 of the wall thickness (0.66/2.2). Of course for practical applications safety factors would need to be applied.

It appears from the foregoing that the displacement demand for rocking URM walls can similarly be predicted in accordance with the elastic displacement response spectrum provided that a suitable effective natural frequency or stiffness,  $K_{eff}$ , refer figure [2b] is identified. The wall effective natural frequency can be related to the resonant frequency at which maximum displacement amplification occurs. Some care must be applied in using this methodology however as 'period lag' can occur in the averaging out of the non-linear response. To test the effectiveness of the displacement criterion for face loaded walls an extensive two phase parametric study using the non-linear THA software developed by the authors has been undertaken.

Phase (1) of the study was conducted to determine the resonant frequency for various wall configurations. This was achieved by running a series of 500 gaussian pulse THA at frequencies ranging from 0.25Hz to 10Hz and amplitudes from 2.5mm to 110mm for the wall configurations under consideration. As expected the parameters found to affect the resonant frequency included joint condition, height and overburden stress as these affect the shape of the (F-Δ) curve. Interestingly, wall thickness had little effect, as although it increases the size of the (F-Δ) curve it does not affect the shape. Figure [5a] represents the RSD V's PGD for a single 500 gaussian point THA run for a non-loadbearing SS wall in new condition, 3.3m tall and 110mm thick. Of the 2 lines shown the left most represents the envelope of maximum mid-height displacement amplification and the second zero displacement amplification where the input frequency is relatively large such that  $RSD \cong 2/3 PGD$ . The variation in resonant frequency for the 3.3m high wall at various levels of overburden stress and joint degradation is shown by Figure [5b]. Significantly the resonant frequencies determined analytically matched those found experimentally as the minimum observed asymptotic frequency very well.

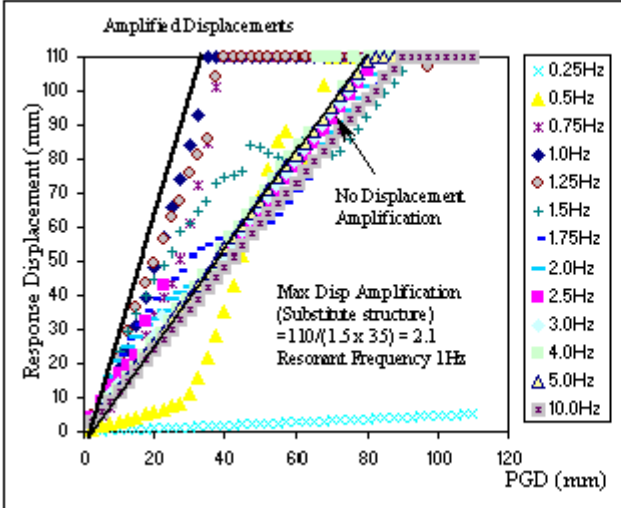
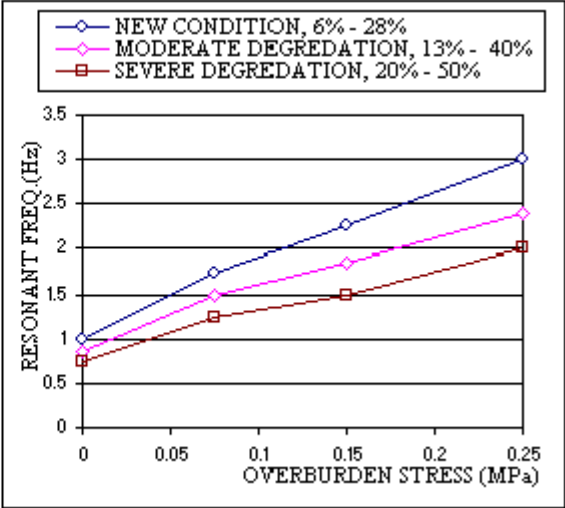


Figure [5a] Gaussian Pulse Output

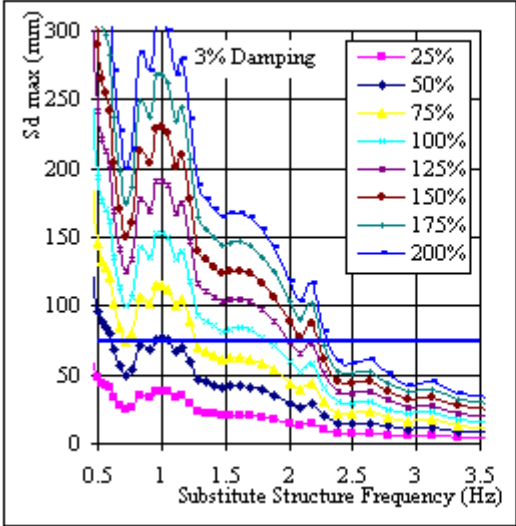


[5b] 3.3m High Wall, Resonant Frequencies

Phase (2) was conducted similarly to phase (1) however real earthquake excitations including Elcentro, Pacoima Dam, Nahanni and Taft were used as representative of transient excitations with various dominant frequencies being gradually increased until failure. Results from both phase (1) and (2) were then used to test the effectiveness of the DB criterion. In accordance with the DB design methodology the resonant frequency determined in phase (1) and the elastic displacement response spectrum at 3% damping (2/3x4.5%) were used to determine the displacement demand on each of the wall configurations. For illustrative purposes the DB procedure for the 3.3m wall above is described. Figure [6] gives the 3% damping displacement response spectrum for the normalized Elcentro earthquake gradually increased from 25% to 200% with frequency rather

than period used for convenience. The horizontal line at 73mm (110x2/3) displacement represents the displacement capacity of the 110mm thick wall. For each of the wall configuration resonant frequency, refer figure [5b] the level earthquake for displacement demand to equal capacity representing failure can be determined. A brief comparison of the THA and DB results for Elcentro are shown in Table [1] with very good correlation indicating the effectiveness of the DB analysis.

**Table [1] THA and DB Analysis Comparison**



Overburden (MPa)	Wall Condition	Resonant Frequency	THA % Elcentro	PGA (g)	DB % Elcentro
0	NEW	1	25-50%	0.18	50%
	MOD.	0.85	25-50%	0.18	50%
	SEVERE	0.75	50-75%	0.26	75%
0.075	NEW	1.75	75-100%	0.35	100%
	MOD.	1.5	75-100%	0.35	90%
	SEVERE	1.25	50-75%	0.26	75%
0.15	NEW	2.25	No Fail	>0.7	200%
	MOD.	1.75	125-150%	0.53	100%
	SEVERE	1.5	100-125%	0.44	90%
0.25	NEW	3	No Fail	>0.7	No Fail
	MOD.	2.35	150-175%	0.61	200%
	SEVERE	2	125-150%	0.53	125%

**Figure [6] Elcentro Displacement Spectrum**

**COMPARISON OF PREDICTIVE MODEL RESULTS**

As expected for such a rigorous procedure the THA was consistently found to provide the closest agreement with experimental results regardless of the transient excitation properties. The DB analysis was also found to give good predictions indicating the potential usefulness of this procedure for the analysis and design although consideration does need to be given to 'period lag'. For ground motions with dominant frequencies in the same vicinity as the wall resonant frequency failure accelerations were generally found to be slightly lower than that predicted by rigid body analysis. Alarmingly for walls with somewhat degraded rotation joints, failure accelerations were often found to be much lower than those determined by rigid body analysis. This can be seen by comparison of the Table (1) THA predicted accelerations with Figure (1). On the other hand for input ground motion with dominant frequencies much higher than the walls resonant frequency failure accelerations tended to be greater than predicted by rigid body analysis. In most cases the 'equal energy' procedure overestimated the walls lateral strength with the exception of high frequency excitations.

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