APPROXIMATE STABILITY BOUNDS ON THE SEISMIC FORCE REDUCTION FACTOR

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SUMMARY

In the equivalent lateral force and modal analysis procedures advocated by seismic codes, P-Δ effects are usually taken into account through an amplification factor, which is a function of the stability coefficient θ (= P/(Kh), in which P = acting gravity load, K = lateral stiffness and h = storey height). However, this approach cannot identify situations where dynamic instability governs, resulting in insufficient safety margins. The value of the force reduction factor R is usually based on the structural type, without allowance for stability effects. However, stability can impose an upper bound on R, which for elastic-plastic or bilinear structures can be much lower than the one specified by the codes. Bernal proposed formulas to approximate stability based strength thresholds of one degree-of-freedom structures. They are quite accurate when used with the exact value of the spectral acceleration of a given record, but their use as design tools - when only code spectra are available - has not been tested. Also, they may not be sufficiently simple for design purposes. This paper proposes simple expressions for estimating stability based upper bounds for the seismic force reduction factor R of elastic-plastic and bilinear systems, and compares them with time history results and Bernal's formulas. These expressions, which are a reformulation and a modification of Housner's criterion, are derived from energy considerations. The proposed formulas are functions of θ only, and are very simple to apply. Their accuracy has been tested against exact values of the limiting R, as computed by nonlinear dynamic analysis, for a number of time histories covering relatively wide ranges of frequency distributions and natural periods. The results of the parametric study show that the proposed expressions yield acceptable correlation with computed values. However, they may not be conservative. Therefore, a modified version is proposed for design purposes.

INTRODUCTION

As is known, the gravity load action on the laterally displaced configuration, usually known as the P-Δ effect, can precipitate collapse in yielding structures due to inordinate increase in the lateral deflections, and hence in the ductility demand, when the post-yield stiffness, as modified by gravity, is negative. The design strength of yielding structures, as specified by seismic codes through the force reduction factor R, is based on their expected ductility supply without taking stability effects into consideration. Therefore, when substantial gravity effects are present, the strength of the structure may not be sufficient to limit the deflection to an acceptable level. This fact sets an upper bound on R, which may be lower than specified by the code (e.g. De Stefano & Rutenberg 1999), particularly for elastic-plastic and bilinear systems with a small secondary slope ratio in their force-displacement relation.

Following extensive parametric studies on 1-degree of freedom systems, Bernal (1992) proposed statistical expressions for the minimum base shear coefficient that is required to prevent instability. They were given as functions of several key system and ground motion parameters. These can easily be converted into upper bound R factors. However, albeit being quite accurate, their usefulness depends on prior knowledge of the values of the expected elastic spectral acceleration and record duration, which are not available at the design stage. Their

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applicability as design tools - when only a design spectrum is available - has not been tested. Moreover, they may not be sufficiently simple for use in routine design procedures.

The purpose of this paper is to review these stability bounds on R, and to compare them with a bound that can be gleaned from the work of Housner (1960) and of Sun et al (1973). It is shown that this bound may be quite useful for predicting the limiting value of R for bilinear hysteretic systems. A very simple formula based on this bound, which usually yields satisfactory predictions, is proposed for design purposes.

STATISTICAL STABILITY BOUNDS

Using an ensemble of 24 ground motion time histories recorded on firm soil Bernal (1992) derived expressions for the mean spectral ordinates at the onset of instability, or collapse spectral ordinates, of elastic-plastic and of stiffness-degrading systems. These were given in terms of the stability coefficient $\theta = P/(Kh)$, (P = gravity load, K= elastic stiffness, h = height of pendulum) as shown in Fig.1, T (natural period), PGV (peak ground velocity), PGD (peak ground displacement) and the effective duration of the record t_{eff} (central 90% of record total energy). The following expressions for the collapse spectral ordinates S_{ac} (T) of elastic-plastic systems were presented:

$$S_{ac} = 5\ \mathrm{PGV}\ \theta^{0.75} \sqrt{t_{eff}}\ / T^{1.42} \qquad (1)$$

but need not be taken as larger than:

$$S_{ac} = 36\ \mathrm{PGD}\ \theta^{0.75}\ t_{eff}^{0.2} / T^{1.86} \qquad (2)$$

The limiting value of R, i.e. R_L, can then be obtained from:

$$R_L = S_{ae}(T) / S_{ac}(T) \qquad (3)$$

in which S_{ae} (T) = elastic spectral acceleration of the record.

Consider now the design situation where S_{ae} is given by a typical code formula, say S_{ae} = PGA/T (T > 0.4sec), and assume t_{eff} = 25 sec. Substituting these values in Eqns. 1 and 2 leads to:

$$R_L = 0.4\ (a/v)\ T^{0.42}\ /\ \theta^{0.75} \qquad (T > 0.4\ \mathrm{sec}) \qquad (4)$$

but need not be smaller than:

$$R_L = 0.15\ (a/v)\ (v/d) T^{0.86}\ /\ \theta^{0.75} \qquad (T > 0.4\ \mathrm{sec}) \qquad (5)$$

In which a = PGA, v = PGV and d =PGD. According to Mohraz (1976) the mean value of v/d (in 1/sec) varies from 3 for rock sites to 2 for deep alluvial sites. Note that Eqn. 5 may govern for a substantial part of the range.

For design purposes it may be more appropriate to use the mean plus one standard deviation (STD) prediction for S_{ac}, i.e., to divide Eqns. 4 and 5 through 1.4 (Bernal 1992).

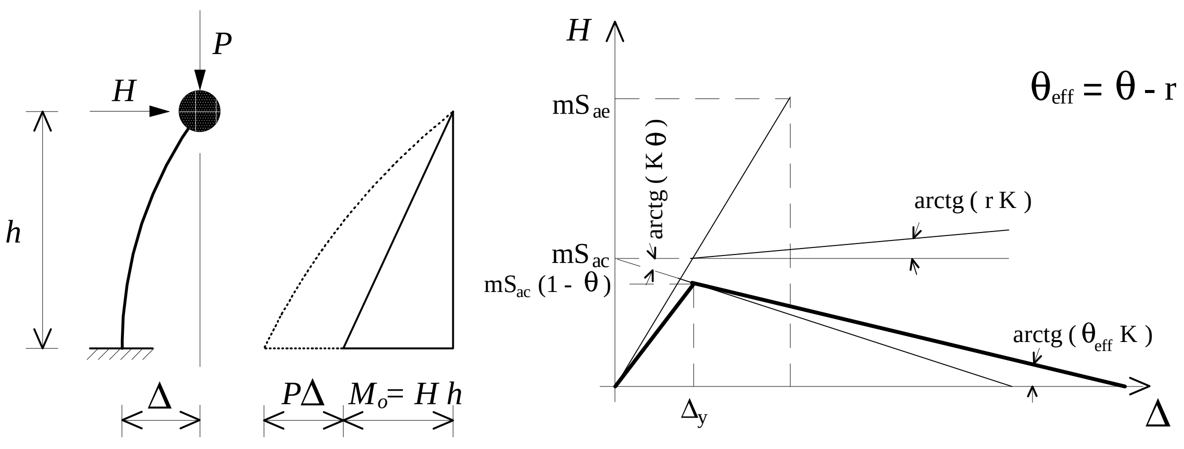

Figure 1: P-Δ effect and its influence on bending moment and force-displacement relation

ENERGY APPROXIMATIONS

Housner (1960) derived a very simple expression for the minimum value of the yield moment at the onset of instability M_0 as function of the spectral velocity of the record S_v and the height h (Fig. 1). His expression takes the form:

$$M_0 = S_V \; P \sqrt{h/g} \tag{6a}$$

in which g = acceleration of gravity. Since M_0 = Hh, where H = base shear, it follows that:

$$H = S_V \; P / \sqrt{gh} \tag{6b}$$

Noting that P = mg (m = mass of pendulum), H = $mS_{ac}(T)$, $\omega = \sqrt{(K/m)}$ = circular frequency (without gravity), and letting $S_{ae} = \omega S_v$, it can easily be shown that:

$$R_L = 1/\sqrt{\theta} \tag{7}$$

This result can also be obtained by comparing the two areas shown in Fig.2a.

Sun et al (1973) studied the free vibration stability of frames with tension-only cross bracing having elastic-plastic force-displacement relationship. They concluded that the system would oscillate about the zero displacement position when the spectral displacement S_d does not exceed $\Delta_y/\sqrt{\theta}$ (Δ_y = yield displacement, Fig.2). In other words, they arrived at the same ratio for R_L as in Eqn. 7, for limiting residual displacements rather than for avoiding possible collapse.

A somewhat more conservative result is reached when the work demand of a fully developed collapse mechanism, or the area under the bilinear force-displacement curve, is compared with the area under the elastic curve (Fig. 2b), namely:

$$R_L = \sqrt{(1-\theta)/\theta} \tag{8}$$

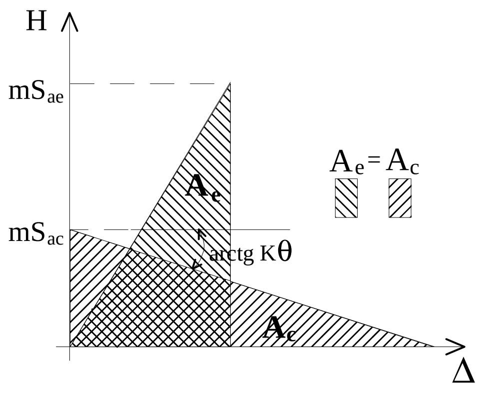

Figure 2a: Derivation of Eqn. 7

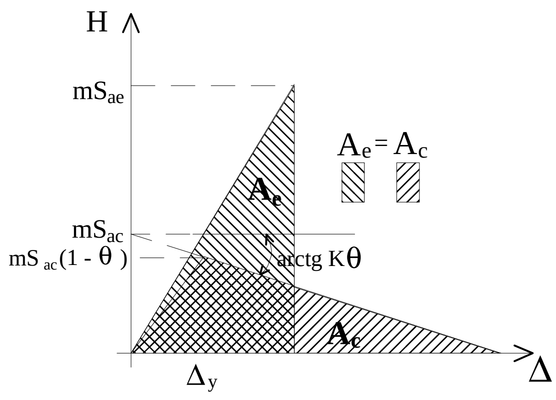

Figure 2a: Derivation of Eqn. 8

For a bilinear system with a resulting effective, or net, secondary slope ratio θ_{eff} (Fig. 3) a more lengthy expression is obtained:

$$R_L = (1-\theta)\sqrt{1+\theta_{eff}(1-\theta)} \; / \sqrt{\theta_{eff}} \tag{9}$$

which, for small values of θ and θ_{eff}, can conservatively be approximated as:

$$R_L = (1-\theta) / \sqrt{\theta_{eff}} \tag{10}$$

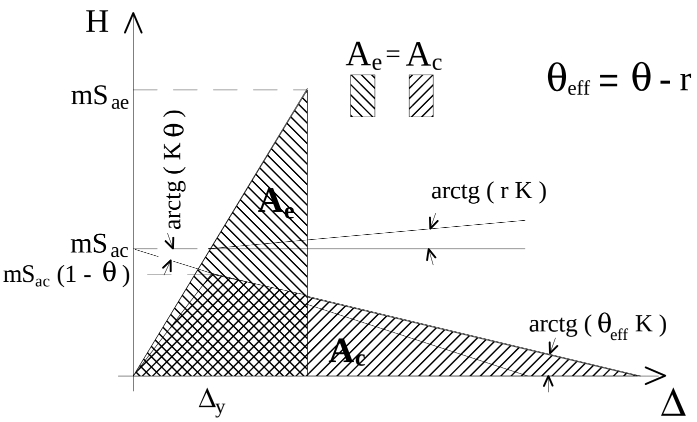

Figure 3: Derivation of Eqn. 9 for bilinear systems

Finally, for design purposes and for comparison with the design curve based on Eqns. 4 and 5, a more conservative approximation based on Eqn. 7 is proposed, namely:

$$R_L = (1-\sqrt{\theta}) / \sqrt{\theta} \tag{11}$$

COMPARING FORMULAS WITH DYNAMIC RESULTS

In order to obtain meaningful comparisons it is necessary to choose input ground motions that cover a relatively wide range of earthquakes. A simple and useful parameter for characterising the frequency content of earthquake ground motions is the ratio a/v, in which a = PGA and v = PGV of the earthquake record. Motions with high a/v generate significant response in short period structures, whereas those with low a/v generate significant response in long period ones. If a is expressed in units of g and v in m/sec, then a/v ratios for actual earthquake records can range from about 0.2 to over 3. Typical intermediate values (which characterise ground motions in the west coast of Canada and the U.S.) are in the neighbourhood of 1. For this study six ground motion records have been chosen, two records in each of the following a/v categories: High, Intermediate and Very low. These records are listed in Table 1.

Table 1. Characteristics of the selected earthquake records

Earthquake	Station	Component	PGA (g)	a/v
Loma Prieta (1989)	Santa Cruz	37 N - 122 W	0.409	1.93
Northridge (1994)	Santa Monica	34 N - 119 W	0.883	2.11
Imperial Valley (1940)	El Centro	S00E	0.348	1.04
Kern County (1952)	Taft	S69E	0.179	1.01
Bucharest (1977)	Bucharest	S00E	0.205	0.27
Mexico (1985)	SCT	N90W	0.171	0.28

The results for five percent damping are presented as R_L spectra, i.e. R_L vs. the natural period T (computed without P-Δ) of the system. Only elastic-plastic models have been studied, since it was shown (e.g. De Stefano & Rutenberg 1999) that it is the effective secondary slope that affects the response (Fig. 3), and the small change in the primary slope due to the P-Δ effect has little influence on R_L.

Figure 4 compares R_L values (Eqn. 3) based on Eqns. 1 and 2 with their exact counterparts for the El Centro record with θ = 0.05 and 0.2. The straight lines representing Eqn. 11 are also shown. It can be seen that, given S_{ae} of the record, the agreement of Eqn. 3 with the nonlinear dynamic results is excellent, whereas Eqn. 11 gives only a rough estimate.

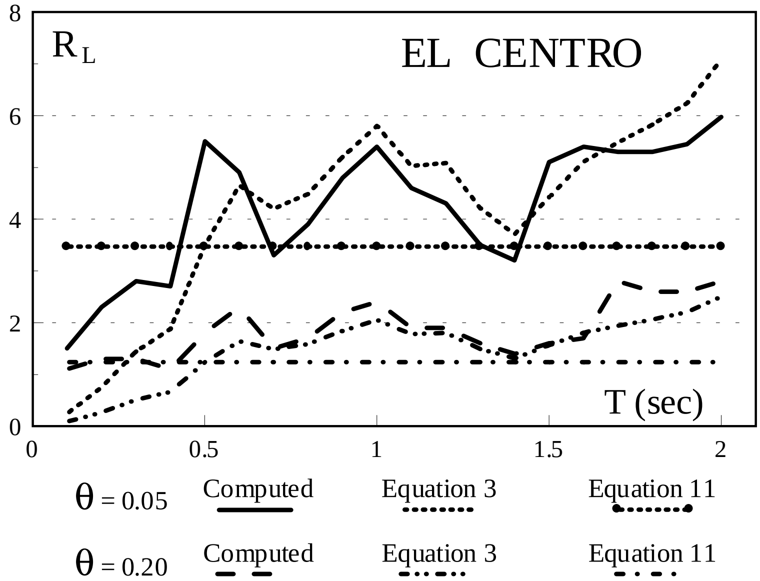

Figure 4: Comparison of R_L spectra from Eqn. 3 with exact results and Eqn. 11: El Centro; θ=0.05, 0.20

Figures 5 and 6 compare Eqns. 4 and 5 (assuming v/d =3) and 11 with the dynamic values for Loma Prieta, and Northridge (a/v = 2), and El Centro and Taft (a/v = 1) respectively, for θ = 0.02, 0.05, 0.1 and 0.2. The design (i.e. mean /1.4) curves for these equations are also shown. Note that design curve based on Eqns. 4 and 5 leading to $R_L < 1.0$ have no practical meaning. Equation 11 is the horizontal straight line. It can be seen that for these records the prediction of Eqn. 11 is usually close to the lower bound of R_L provided the natural period is not very low. However, Eqn. 11 cannot predict the increase in R_L with T for Loma Prieta and Northridge at low values of θ. The predictions of Eqns. 4 and 5 are not very satisfactory either, and they tend to exaggerate the period dependence of R_L.

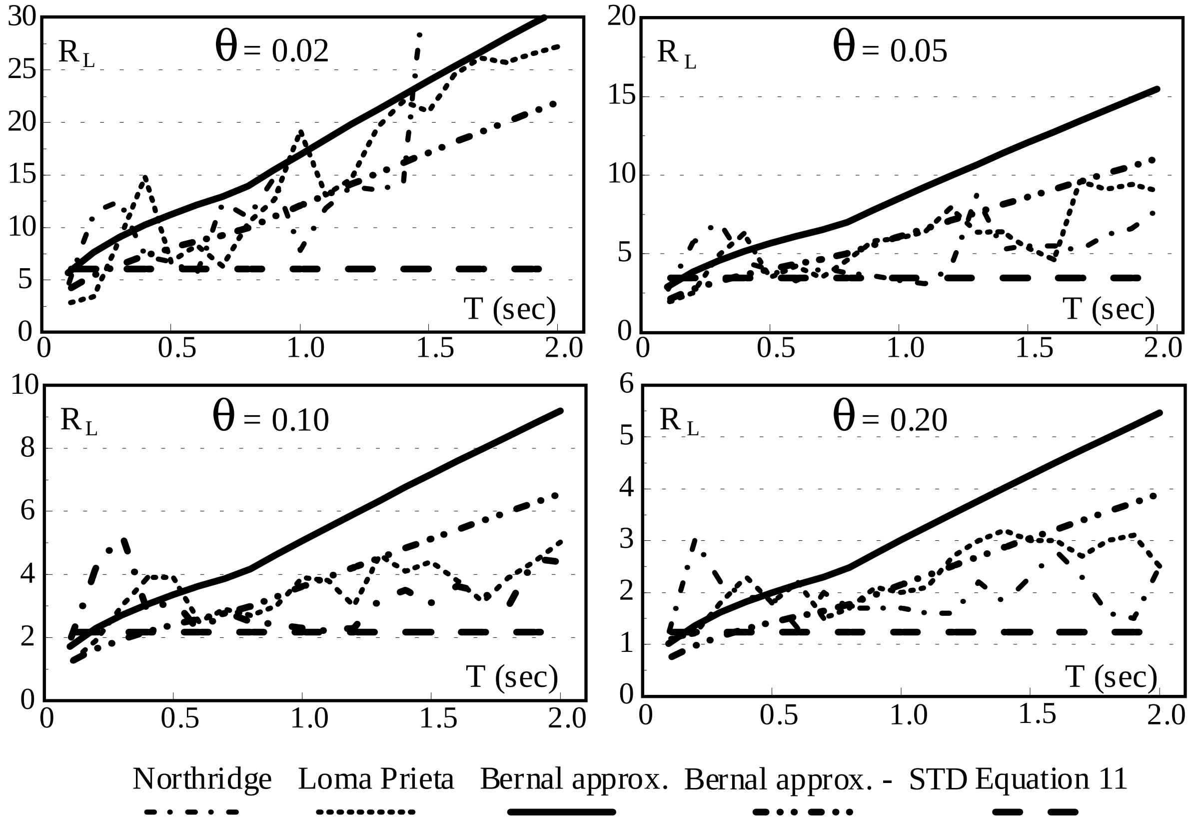

Figure 5: Comparison of R_L spectra from Eqns. 4 & 5, Eqns. (4 & 5) / 1.4 and Eqn. 11 with exact results: Loma Prieta & Northridge

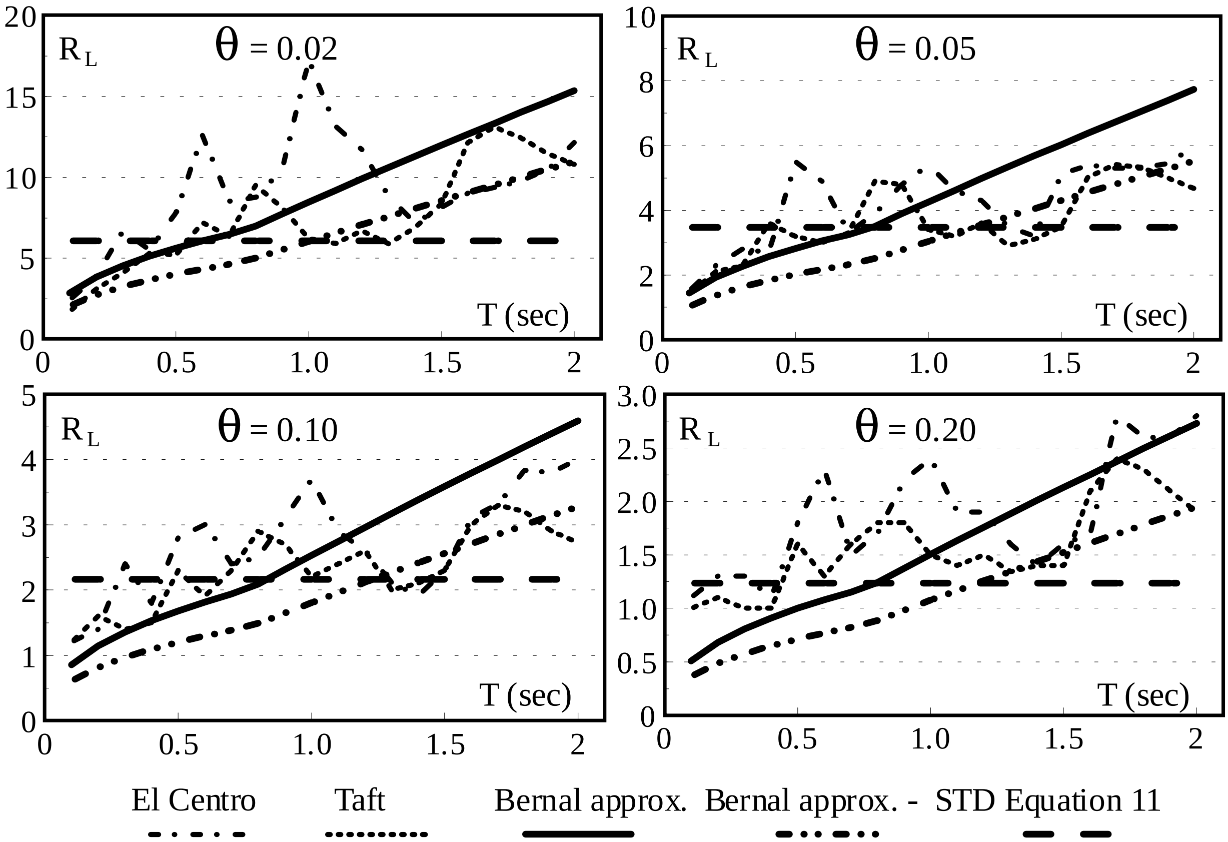

Figure 6: Comparison of R_L spectra from Eqns. 4 & 5, Eqns. (4 & 5) / 1.4 and Eqn. 11 with exact results: El Centro & Taft

Figure 7 makes the same comparisons for the Bucharest and Mexico records. It is seen that Eqn.4 (assuming a/v = 0.5, which is high for these two records) can be used as a lower bound prediction for these records provided θ is not large, but the mean – STD curves underestimate R_L appreciably. Note, however, that Bernal's formulas (Eqns. 4 & 5) have not been calibrated for low frequency records, so they should not be expected to predict the response of such records. It is also seen that Eqn. 11 is not useful for these earthquakes. Note also that when θ ≥ 0.1, $R_L \approx 1$ is a realistic lower bound approximation for the Mexico record for most of the period range shown herein.

It may be recalled that the effective duration of record t_{eff} has been arbitrarily set at 25 sec for all the records used. Changing this value to be more compatible with the mean expected duration of each a/v group is not likely to improve the agreement of Eqns. 4 and 5 with the time history results. For a/v =2 records t_{eff} =25 sec is usually too long, therefore setting a smaller value would result in much lager R_L values for the longer period range and a smaller increase for the shorter period range. However, t_{eff} =25 sec is quite realistic for the a/v =1 records. Similarly, no improvement in the agreement is expected from using a larger value of v/d for the large a/v records.

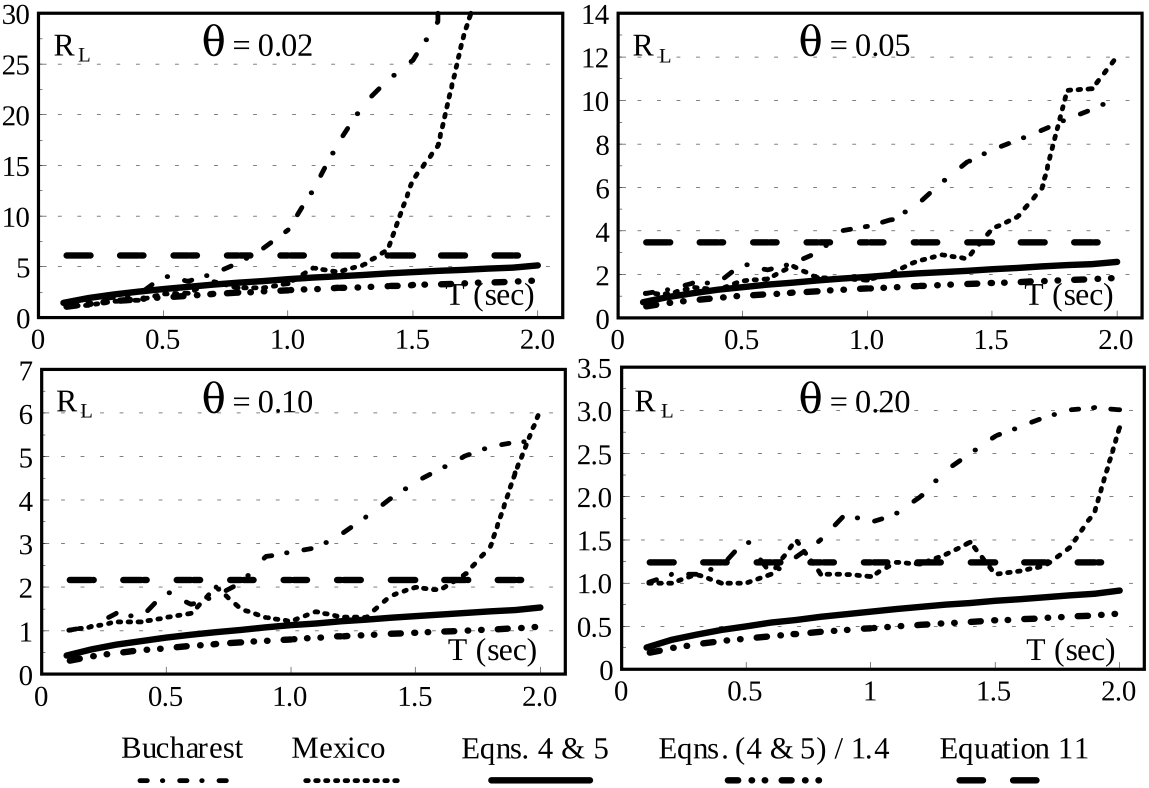

Figure 7: Comparison of R_L spectra from Eqns. 4 & 5, Eqns. (4 & 5) / 1.4 and Eqn. 11 with exact results: Bucharest & Mexico

SUMMARY AND CONCLUSIONS

Two approximate expressions for estimating the stability based limiting force reduction factor R_L for bilinear one-degree of freedom structures have been checked against time history results for six earthquake records covering relatively wide ranges of frequency distributions and natural periods. One expression was derived statistically from an ensemble of 24 time-histories (Bernal 1992), the other is a modification of an energy–based expression.

Neither of these formulas can adequately predict R_L for the realistic ranges of the stability factor θ and natural period T, as well as for different frequency contents of the input earthquake time-histories. Because of its simplicity the authors would nevertheless recommend the use of Eqn. 11 for design purposes, provided it is not used for sites with very low a/v, and for sites with intermediate a/v (=1), provided it is used only when T > 0.5sec. As can be seen from Fig. 5, this recommendation may result in very conservative values of R_L for a/v = 2 sites with $\theta < 0.05$ and $T > 1.0$sec.

Finally, it should not be overlooked that the sample size used for this study has been very limited, and therefore the conclusions drawn herein should be treated accordingly.

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