EXPERIMENTAL INVESTIGATION ON OPTIMAL DIRECT OUTPUT FEEDBACK CONTROL OF STRUCTURES

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SUMMARY

The feasibility of optimal direct output feedback control algorithm is verified experimentally. The practical problems of digital formulation and limited measurements are tackled. An active mass damper is implemented on the roof floor of a five-story full-scaled structural model which is 6 m (length) by 4 m (width) by 13 m (height) in dimension. The control forces are simply generated from output measurements multiplied by pre-calculated shift invariant feedback gain. Simple on-line calculation and a small number of sensors make the proposed control algorithm favorable to real implementation. Since both the dynamics of the control device and the structure are carefully calibrated, the correlation between analytical and experimental results is study.

INTRODUCTION

Active structural control is one of the promising alternatives to enhance the safety and serviceability of civil engineering structures against stochastic environmental loads such as ground motions and wind forces (Soong, 1986; Yang and Soong, 1988; Soong et al., 1994). According to modern control theory, the stiffness and damping of the structures are upgraded by state feedback control which requires all states of the structures including displacement and velocity of each degree of freedom to be available (Soong, 1990; Meirovitch, 1990). However, a large number of degrees of freedom is one of the characteristics of civil engineering structures. From the viewpoint of economy, data processing and on-line calculation, it is impossible and impractical to have a complete set of state variables available for the on-line calculation of the control forces. Therefore, output feedback becomes one of the intrinsic properties of active structural control. The control forces have to be determined based on the information of limited measurements. Apart from state reconstruction and compensation, that make the control system complicated by introducing another dynamic observer or compensator system in addition to the structural system (Luenberger, 1966; Fanson and Caughey 1987), direct output feedback is a promising solution to the probem of limited-state feedback (Balas, 1979; Iwan and Hou, 1990).

In real-time control, the flexibility, reliability and speed of digital computer make it superior for the on-line calculation of the control forces (Kuo, 1980). As a consequence, all information for the calculation of the control forces is sampled once every cycle of data acquisition and on-line calculation. The digital control signals are converted into zero-order-hold analog signals in the form of piecewise step functions over sampling intervals. In a word, all information including input control forces and output measurements are basically digital in nature rather than analog functions in the so-called continuous-time control algorithms. Therefore, it is more logical and more realistic to formulate the control system in the digital fashion. In recent years, some digital control algorithms have been developed (Yang et al., 1987; Rodellar et al., 1987) and their feasibility have been successfully verified in laboratory experiments (Lin et al., 1987; Rodellar et al., 1989).

In this paper, the feasibility of optimal direct output feedback control algorithm is verified experimentally. The practical considerations in active structural control, namely direct output feedback and digital formulation, are tackled (Chung and Soong, 1987). An active mass damper is implemented on the roof floor of a five-story full-

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scaled structural model which is 6 m (length) by 4 m (width) by 13 m (height) in dimension. The control system is first formulated in digital form. The constant feedback gain matrix is derived through optimization process such that a certain prescribed quadratic performance index is minimized. The control forces are simply generated from output measurements multiplied by the pre-calculated shift invariant feedback gain matrix. Simple on-line calculation and a small number of sensors make the proposed control algorithm favorable to real-time control implementation. Since both the dynamics of the control device and the structure are carefully calibrated, the correlation between analytical and experimental results is study.

CONTROL ALGORITHM

The equation of motion of an n-DOF discrete-parameter structural system subjected to external loads $\mathbf{w}(t)$ and counteracted by control forces $\mathbf{u}(t)$ is expressed as

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{B}_1\mathbf{u}(t)+\mathbf{E}_1\mathbf{w}(t) \tag{1}$$

where $\mathbf{x}(t)$ is the $n\times 1$ displacement vector, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the $n\times n$ mass, damping and stiffness matrices, respectively, $\mathbf{B}_1$ is the $n\times p$ location matrix of control forces, and $\mathbf{E}_1$ is the $n\times q$ location matrix of external loads.

Represented in state-space form, Equation (1) is written as

$$\dot{\mathbf{z}}(t)=\mathbf{A}_c\mathbf{z}(t)+\mathbf{B}_c\mathbf{u}(t)+\mathbf{E}_c\mathbf{w}(t) \tag{2}$$

where $\mathbf{z}(t)=\begin{bmatrix}\mathbf{x}(t)\\ \dot{\mathbf{x}}(t)\end{bmatrix}$ is the $2n\times 1$ state vector, $\mathbf{A}_c=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}$ is the $2n\times 2n$ system matrix, $\mathbf{B}_c=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{B}_1\end{bmatrix}$ is the $2n\times p$ control matrix, and $\mathbf{E}_c=\begin{bmatrix}0\\ \mathbf{M}^{-1}\mathbf{E}_1\end{bmatrix}$ is the $2n\times q$ load matrix.

The complete set of state variables $\mathbf{z}(t)$, which includes the displacement and velocity of each degree of freedom of the structure, is too large to be measured for the on-line calculation of control forces. Only output measurements from a limited number of sensors, say r and $r<<2n$ are available for the on-line calculation. The output measurements $\mathbf{y}(t)$ are just a part of or a combination of the state variables $\mathbf{z}(t)$ as

$$\mathbf{y}(t)=\mathbf{D}_c\mathbf{z}(t) \tag{3}$$

where $\mathbf{D_c}$ is the $r\times 2n$ output matrix.

The control system described by Equation (2) is linear and time-invariant so that its solution takes the form

$$\mathbf{z}(t_2)=\mathrm{e}^{\mathbf{A}_c(t_2-t_1)}\mathbf{z}(t_1)+\int_{t_1}^{t_2}\mathrm{e}^{\mathbf{A}_c(t_2-\tau)}[\mathbf{B}_c\mathbf{u}(\tau)+\mathbf{E}_c\mathbf{w}(\tau)]\,\mathrm{d}\tau \tag{4}$$

During real-time control, suppose all information for the on-line calculation of control forces is sampled with period Δt and the control forces are calculated once every sampling period. Then the discrete-time control signals are converted into zero-order-hold continuous-time signals which are piece-wise step functions over sampling intervals. They are used as command signals for the controllers and the control forces generated by the controllers are applied to the structure. The control forces are thus constant between two consecutive sampling instants. Between two consecutive sampling instants, $k\Delta t$ and $(k+1)\Delta t$, the best available information about the loads is $\mathbf{w}(k\Delta t)$. Therefore, external loads are also considered to be sampled as zero-order hold and are thus assumed to be constant between two consecutive sampling instants. As a whole, it is more logical and more realistic for the structural control system to be modeled in a digital fashion as

$$\mathbf{z}[k+1]=\mathbf{A}\mathbf{z}[k]+\mathbf{B}\mathbf{u}[k]+\mathbf{E}\mathbf{w}[k] \tag{5}$$

where $\mathbf{A}=\mathrm{e}^{\mathbf{A}_c\Delta t}$ is the $2n\times 2n$ discrete-time system matrix, $\mathbf{B}=\mathbf{A}_c^{-1}(\mathbf{A}-\mathbf{I})\mathbf{B}_c$ is the $2n\times p$ discrete-time control matrix, and $\mathbf{E}=\mathbf{A}_c^{-1}(\mathbf{A}-\mathbf{I})\mathbf{E}_c$ is the $2n\times q$ discrete-time load matrix.

The corresponding discrete-time output equation is

$$\mathbf{y}[k] = \mathbf{D}\mathbf{z}[k] \tag{6}$$

where $\mathbf{D} = \mathbf{D}_c$ is the $r \times 2n$ discrete-time output matrix.

Under classical quadratic performance criterion, the active control force is found such that the summation

$$J = \sum_{k=0}^{\infty} \{\mathbf{z}^T[k]\mathbf{Q}\mathbf{z}[k] + \mathbf{u}^T[k]\mathbf{R}\mathbf{u}[k]\} \tag{7}$$

is minimized. In the above equation, $\mathbf{Q}$ is the $2n \times 2n$ symmetric positive semi-definite weighting matrix for the responses, $\mathbf{R}$ is the $p \times p$ symmetric positive definite weighting matrix for the input control forces, and superscript T denotes transpose of a matrix.

With direct output feedback, the control forces $\mathbf{u}[k]$ are generated from the output measurements $\mathbf{y}[k]$ multiplied by constant feedback gains as

$$\mathbf{u}[k] = \mathbf{G}\mathbf{y}[k] \tag{8}$$

where $\mathbf{G}$ is the $p \times r$ shift-invariant output feedback gain matrix.

The optimization problem is to search an optimal feedback gain matrix $\mathbf{G}$ that minimizes the performance index J in Equation (7) subject to the constraints of the dynamic equation (5), the output equation (6) and the control law equation (8). By substituting the output equation (6) into the control law equation (8), the control forces $\mathbf{u}[k]$ are shown to be linearly related with the state variables $\mathbf{z}[k]$. The result is further substituted into the state equation (5). After rearrangement, the state equation (5) is expressed as

$$\mathbf{z}[k+1] = (\mathbf{A} + \mathbf{BGD})\mathbf{z}[k] + \mathbf{E}\mathbf{w}[k] \tag{9}$$

where $(\mathbf{A} + \mathbf{BGD})$ is defined to be the effective system matrix. It is observed that the effect of digital output feedback control is essentially to upgrade the system matrix from $\mathbf{A}$ to $(\mathbf{A} + \mathbf{BGD})$. If the output feedback gain matrix $\mathbf{G}$ is chosen properly, the system parameters and thus the system performance can be enhanced to a favorable situation.

The effective digital control system described by the state equation (9) is linear and shift-invariant so that its solution takes the form

$$\mathbf{z}[k] = (\mathbf{A} + \mathbf{BGD})^k \mathbf{z}[0] + \sum_{l=0}^{k-1} (\mathbf{A} + \mathbf{BGD})^{k-1-l} \mathbf{E}\mathbf{w}[l] \tag{10}$$

The first term of the right hand side of the above equation is the structural response induced by initial disturbance $\mathbf{z}[0]$ while the second term is the response induced by external loads $\mathbf{w}[k]$.

After the system matrix is upgraded from $\mathbf{A}$ to $(\mathbf{A} + \mathbf{BGD})$ by direct digital output feedback control, it is seen that if the control system works well for the suppression of vibration induced by arbitrary initial disturbance $\mathbf{z}[0]$, it will also work well for arbitrary external disturbance $\mathbf{w}[k]$, and vice versa. Therefore, in order to simplify the derivation, only arbitrary initial disturbance is considered here. The arbitrary initial disturbance is further generalized to be random. Under random initial disturbance $\mathbf{z}[0] = \mathbf{z}_0$, the solution to the effective state equation (9) is simplified as

$$\mathbf{z}[k] = (\mathbf{A} + \mathbf{BGD})^k \mathbf{z}[0] = (\mathbf{A} + \mathbf{BGD})^k \mathbf{z}_0 \tag{11}$$

After Equations (8), (6) and (11) are sequentially substituted into the performance index equation (7), it gives

$$J = \sum_{k=0}^{\infty} \mathbf{z}_0^T [(\mathbf{A} + \mathbf{BGD})^k]^T (\mathbf{Q} + \mathbf{D}^T\mathbf{G}^T\mathbf{RGD})[(\mathbf{A} + \mathbf{BGD})^k]\mathbf{z}_0 = \mathbf{z}_0^T \mathbf{H} \mathbf{z}_0 \tag{12}$$

where the $2n \times 2n$ constant matrix $\mathbf{H}$ is defined as

$$\mathbf{H} = \sum_{k=0}^{\infty} [(\mathbf{A}+\mathbf{BGD})^k]^T (\mathbf{Q}+\mathbf{D}^T\mathbf{G}^T\mathbf{RGD})[(\mathbf{A}+\mathbf{BGD})^k] \tag{13}$$

Since initial disturbance $\mathbf{z}_0$ is random, the quadratic performance index J in Equation (12) is also random. Therefore, instead of the performance index J itself, its expected performance index

$$\bar{J} = \mathrm{E}\{J\} = \mathrm{E}\{\mathbf{z}_0^T \mathbf{H} \mathbf{z}_0\} = tr(\mathbf{HZ}_0) \tag{14}$$

is minimized, where $\mathrm{E}\{\bullet\}$ denotes expectation, $\mathrm{tr}(\bullet)$ denotes trace of a square matrix, and $\mathbf{Z}_0 = \mathrm{E}\{\mathbf{z}_0\mathbf{z}_0^T\}$ is the second statistical moment of the initial disturbance $\mathbf{z}_0$.

If the constant matrix **H** is first pre-multiplied and post-multiplied by $(\mathbf{A}+\mathbf{BGD})^T$ and $(\mathbf{A}+\mathbf{BGD})$, respectively, and the constant matrix **H** itself is then subtracted from the product, it gives

$$(\mathbf{A}+\mathbf{BGD})^T \mathbf{H}(\mathbf{A}+\mathbf{BGD}) - \mathbf{H} = \lim_{k\to\infty} [(\mathbf{A}+\mathbf{BGD})^k]^T (\mathbf{Q}+\mathbf{D}^T\mathbf{G}^T\mathbf{RGD})[(\mathbf{A}+\mathbf{BGD})^k] - (\mathbf{Q}+\mathbf{D}^T\mathbf{G}^T\mathbf{RGD}) \tag{15}$$

Because structural system is intrinsically stable, the system matrix $\mathbf{A}$ is a stability matrix. Moreover, the feedback gain matrix **G** should not destabilize the system so that the effective system matrix $(\mathbf{A}+\mathbf{BGD})$ is also a stability matrix. Since the eigenvalues of the stability matrix of the discrete-time system are located within the unit circle of the complex plane, the limit given in the first term of the right hand side of Equation (15) exists and vanishes (Bellman, 1970). Equation (15) is thus reduced as

$$(\mathbf{A}+\mathbf{BGD})^T \mathbf{H}(\mathbf{A}+\mathbf{BGD}) - \mathbf{H} + (\mathbf{Q}+\mathbf{D}^T\mathbf{G}^T\mathbf{RGD}) = \mathbf{0} \tag{16}$$

The optimization problem is now converted to the one that minimizes the expected performance index $\bar{J}$ in Equation (14) subject to the constraint of Equation (16). Incorporated with the constraint equation (16), the Lagrangian $\bar{J}'$ can be introduced as

$$\bar{J}' = \mathrm{tr}(\mathbf{HZ}_0) + \mathrm{tr}\{\mathbf{L}[(\mathbf{A}+\mathbf{BGD})^T \mathbf{H}(\mathbf{A}+\mathbf{BGD}) - \mathbf{H} + (\mathbf{Q}+\mathbf{D}^T\mathbf{G}^T\mathbf{RGD})]\} \tag{17}$$

where **L** is the $2n \times 2n$ Lagrangian multiplier matrix. Because the performance index is quadratic, and $\mathbf{Q}$ and **R** are positive semi-definite and positive definite matrices, respectively, the necessary and sufficient conditions for minimization of the Lagrangian $\bar{J}'$ are

$$\bar{J}'_{\mathbf{L}} = \frac{\partial \bar{J}'}{\partial \mathbf{L}} = (\mathbf{A}+\mathbf{BGD})^T \mathbf{H}(\mathbf{A}+\mathbf{BGD}) - \mathbf{H} + (\mathbf{Q}+\mathbf{D}^T\mathbf{G}^T\mathbf{RGD}) = \mathbf{0} \tag{18}$$

$$\bar{J}'_{\mathbf{H}} = \frac{\partial \bar{J}'}{\partial \mathbf{H}} = (\mathbf{A}+\mathbf{BGD})\mathbf{L}(\mathbf{A}+\mathbf{BGD})^T - \mathbf{L} + \mathbf{Z}_0 = \mathbf{0} \tag{19}$$

$$\bar{J}'_{\mathbf{G}} = \frac{\partial \bar{J}'}{\partial \mathbf{G}} = 2\mathbf{B}^T\mathbf{H}(\mathbf{A}+\mathbf{BGD})\mathbf{LD}^T + 2\mathbf{RGDLD}^T = \mathbf{0} \tag{20}$$

Since the transpose of the constant matrix **H** and the Lagrangian multiplier matrix **L** also satisfy Equations (18) and (19), respectively, and the effective system matrix $(\mathbf{A}+\mathbf{BGD})$ is a stability matrix, the matrices **H** and **L** are symmetric and positive definite. By solving the non-linear equations (18), (19) and (20) numerically, the output feedback gain matrix **G** is obtained.

EXPERIMENTAL SETUP

A standardized structural model is erected at I-Lan Experimental Park of the National Center for Research on Earthquake Engineering to study the performance of various active, passive and hybrid control devices. The model, which is a full-scale five-story steel structure, is 6 m in length, 4 m in width and 13 m in height. Concrete-blocks are mounted on the floors of the structure to simulate the weight and the total weight of the structure is about 100 ton. An active mass damper is implemented on the roof floor of the structure and the mass is about 1.36 ton in weight. The active mass is mounted on the linear guides and actuated to slide on the linear guides freely by means of a ball screw which is in turn driven by an AC servo motor.

Structural dynamics. The structural model can be simplified as a six-degree-of-freedom system. Under earthquake excitation $w(t)$, the equation of motion of the structure can be expressed as

$$\mathbf{M}_{\mathrm{s}}\ddot{\mathbf{x}}_{\mathrm{s}}(t)+\mathbf{C}_{\mathrm{s}}\dot{\mathbf{x}}_{\mathrm{s}}(t)+\mathbf{K}_{\mathrm{s}}\mathbf{x}_{\mathrm{s}}(t)=\mathbf{e}_{\mathrm{s}}w(t) \tag{21}$$

Actuator dynamics. The dynamics of the actuator can be modeled as a second-order system

$$(m_{\mathrm{d}}+m_{\mathrm{b}})\ddot{x}_{\mathrm{d}}(t)+c_{\mathrm{d}}\dot{x}_{\mathrm{d}}(t)+k_{\mathrm{d}}x_{\mathrm{d}}(t)=bu(t) \tag{22}$$

where $x_{\mathrm{d}}(t)$ is the displacement of the active mass relative to the supporting floor, m_{d} is the mass of the moving parts including the active mass, m_{b} is the effective linear mass of the rotating parts, c_{d} is the damping coefficient of the actuator, k_{d} is the stiffness of the actuator, $u(t)$ is the driving voltage of the servo motor and b is the force coefficient of the actuator. The effective linear mass m_{b} is derived from the rotational mass I_{b} of the ball screw, coupler and servo motor.

$$m_{\mathrm{b}}=\frac{2\pi I_{\mathrm{b}}}{l} \tag{23}$$

where l is the pitch of the ball screw. The damping coefficient c_{d}, stiffness k_{d} and force coefficient b are extracted by system identification. Relevant parameters of the actuator are listed in Table 2.

Structural control system. When the control device is mounted on the roof of the structural model, the system becomes five-degree-of-freedom structure and its equation of motion is expressed as

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{b}u(t)+\mathbf{e}w(t) \tag{24}$$

Controller. The controller used in the experiment is a PC-586 associated with the AD/DSP2101 cpu and facilitated with A/D and D/A boards. The controller would be able to handle the multiple input and output, which would be required in response control of actual structures. To achieve this target, we select the DSP, digital signal processor, to develop the control unit of the mass damper, which could have a high-speed capacity for real time computation and control. The detail construction of schematic diagram is illustrated in Figure. In the response control experiment , the sampling period was set at 0.001 sec.

EXPERIMENTAL RESULTS

The five-story building model is considered as a six-degree-of-freedom structure. The natural frequencies of the structure are 0.886, 2.833, 5.198, 7.707, 9.445 and 26.269 Hz, and the corresponding damping ratios are 0.389, 0.365, 1.577, 1, 1 and 1 %. With the implementation of the active mass damper, the structure becomes seven degrees of freedom. Only the displacement and velocity of the roof floor, and the displacement and velocity of the active mass damper relative to the roof floor are measured as feedback data. Under control, the first four effective natural frequencies of the structures are 0.739, 0.879, 2.835, and 5.330 Hz, and the corresponding damping ratios are 16.54, 18.49, 1.81 and 2.07 %. The energy dissipation capacity is enhanced dramatically for the first two modes. The damping ratios of the higher modes are only increased a little bit but the remains stable because the controller and the sensors are collocated. Firstly, the active mass damper is considered as exciter and the structure is excited under its natural frequency. Afterwards, the active mass damper stops for the uncontrolled case while the active mass damper shifts to control mode for the control case. For the uncontrolled case the response of the structure attenuates slowly and for the control case, the response dies down in three cycles (Figure 1).

CONCLUSIONS

The feasibility of optimal direct output feedback control algorithm is successfully verified. The control force is generated simply from output measurements multiplied by the pre-calculated shift invariant feedback gain matrix. Simple on-line calculation and a small number of sensors make the proposed control algorithm favorable to real-time control implementation.

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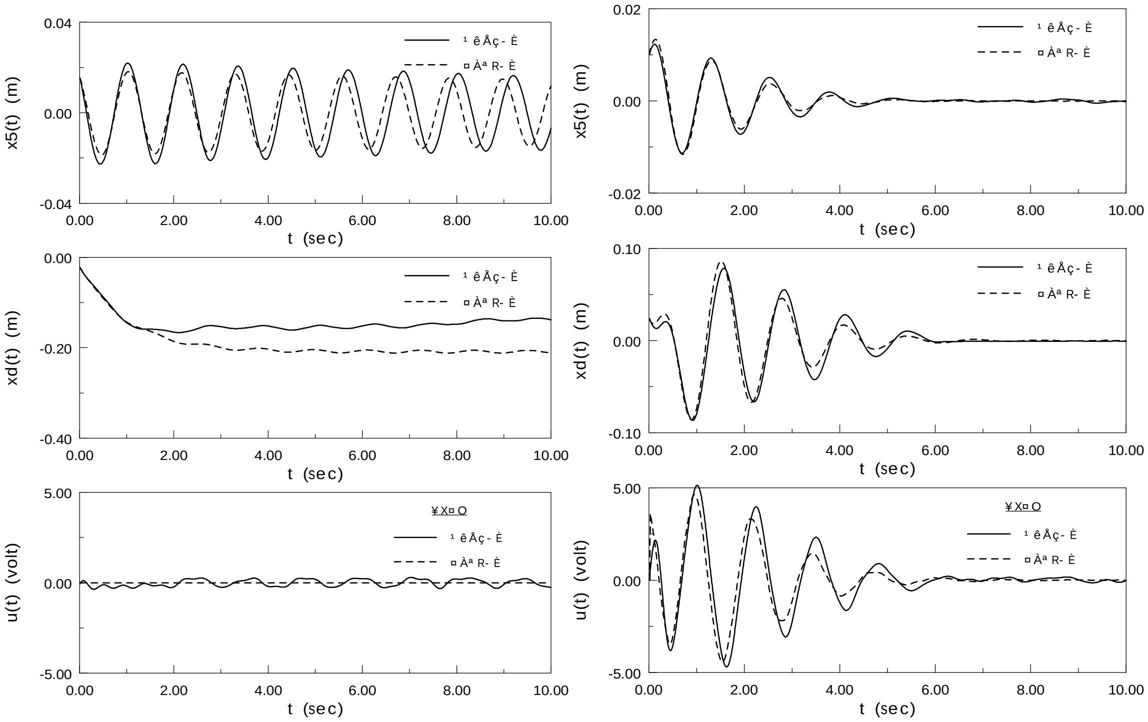

uncontrolled case **control case**

___ experimental **---- analytical**

upper: roof floor displacement

middle: relative displacement of mass damper

lower: control force

Figure 1: Control effectiveness of five-story structure with and without active mass damper