MODELING OF REINFORCED CONCRETE SHEAR WALL FOR NONLINEAR ANALYSIS

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SUMMARY

A member model of reinforced concrete shear wall with boundary columns and beams was proposed for nonlinear and dynamic frame analysis. The reinforced concrete shear wall was idealized as axial springs for columns and a panel under plane stress states with rigid beams at top and bottom floor levels. Two methods were compared, in which isoparametric element and incompatible rectangular element with four nodes for the panel element were used. The model was verified through the analysis of T-shaped wall tests conducted at Yokohama National University in 1994. The analytical results obtained by the proposed model, such as envelope of load-displacement, hysteretic loops, shear and flexural displacement components, showed generally good correlation with the experimental results. Shear deformation was overestimated by isoparametric panel element, whereas incompatible rectangular element, which incorporated flexural deformation by using internal displacements, gave better correlation with the experimental results of flexural yielding walls. For the walls in shear failure, the analytical results were basically same either by isoparametric element or incompatible element.

INTRODUCTION

Many analytical models for reinforced concrete wall have been proposed for nonlinear and dynamic frame analysis of structures including shear walls, most of which are line-models. Because several parameters in the model are assumed through personal testing, reliability might be limited. On the other hand, a lot of multi-axial constitutive models of concrete have been developed such as equivalent uniaxial model [Darwin and Pecknold, 1977], endochronic model [Bazant, 1980], based on which significant advances have been made in the microscopic finite element modeling of reinforced concrete members.

Flexural models of RC line members are being developed based on material properties from recent researches, such as fiber model for beam and column, which can correctly simulate the performance of flexural members including bi-axial N-M interaction just by using the dimensional shape and material properties without special parameters. However, there are few macro-models based on material properties for RC walls, which can be used for complete frame-wall structural analysis. In this study, an analytical model of RC wall is proposed for non-linear analysis of complete frame-wall structure by combining FEM panel model and boundary elements.

COMPOSITION OF RC WALL MEMBER MODEL

Three-Vertical-Line-Element-Model (TVLEM)

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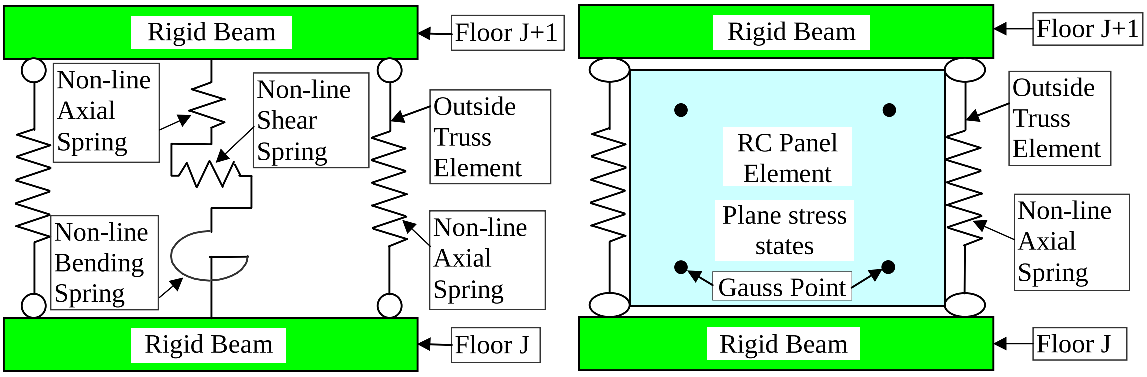
The three-vertical-line-element-model (TVLEM - Figure 1, (a)) was formulated [Kabeyasawa, 1983] to idealize a generic wall member as three vertical line elements with infinitely rigid beams at top and bottom floor levels.

Outside truss elements were represented by the axial stiffness of boundary columns, while the central element was a uniaxial model with vertical (axial), horizontal (shear) and rotational springs. This model has been verified by many test data resulting in a satisfactory correlation between calculated and measured response of structures that use shear walls. However, it was reported [Colotti, 1993] that the response was not adequately described for high shear stresses.

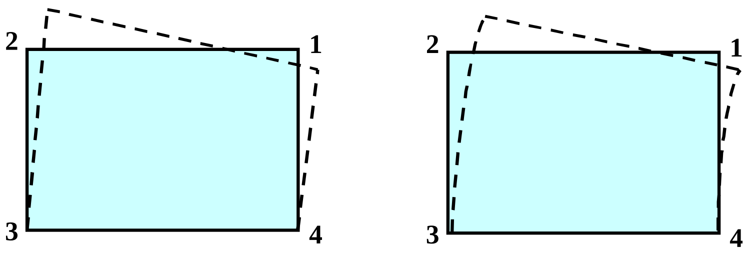


(a) TVLEM Model **(b) RC Panel Model**

Figure 1: Macro-Element Model for RC Wall Member

FEM Panel Model

In order to improve the prediction of the overall (shear and flexural) behavior of RC structural walls for nonlinear earthquake response analysis, this study proposes a 4-node panel under biaxial loading to represent the wall effect (Figure 1(b)). The boundary column uses the same axial spring proposed in TVLEM. Using just one panel to constitute the wall, shear strain will be overestimated by the isoparametric element (Figure 2(a)), satisfactory result can not be obtained for the flexural problem. Flexural deformation must be considered in the element. In this study, incompatible rectangular element is used, in which flexural deformation can be introduced (Figure 2(b)).



(a) Isoparameteric Element **(b) Incompatible Element**

Figure 2: Panel Element – Isoparametric Element and Incompatible Element

Stiffness Matrix of Panel Element

In the incompatible element, the interior displacements of element are assumed as the following formula (1).

$$\begin{cases} u = \sum_{i}^{4} N_i u_i + N_5 u_5 \\ v = \sum_{i}^{4} N_i v_i + N_5 v_5 \end{cases} \tag{1}$$

where, N_i is the same shape function as that of the isoparametric element, which can be calculated by

$$N_i = \tfrac{1}{4}(1+\xi_i \xi)(1+\eta_i \eta) \tag{2}$$

(•$_i$,•$_i$) has the values of (1,1), (-1,1), (-1,-1), (1,-1) for the nodes 1,2,3,4, respectively. • and• are in the local coordinates. (u_i, v_i) is the displacement of the nodes(i=1,2,3,4). N_5 is the shape function for interior deformation of element, called incompatible mode, and is given by the following formula (3),.

$$N_5 = 1 - \eta^2 \tag{3}$$

Displacement (u_5, v_5) represents the flexural deformation inside the element. The displacement along the vertical coordinate has nonlinear distribution, while linear distribution along horizontal coordinate. Thus compatibility of displacement can be maintained between shear walls in case of multi-story walls. From formula (1), strains of element can be given by formula (4). The first term is relative to node displacements, and the second term is produced by the interior displacements.

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \partial[N]/\partial x & 0 \\ 0 & \partial[N]/\partial y \\ \partial[N]/\partial y & \partial[N]/\partial x \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{v\} \end{Bmatrix} + \begin{bmatrix} \partial N_5/\partial x & 0 \\ 0 & \partial N_5/\partial y \\ \partial N_5/\partial y & \partial N_5/\partial x \end{bmatrix} \begin{Bmatrix} u_5 \\ v_5 \end{Bmatrix} = [B]\{\delta\} + [\overline{B}]\{\overline{\delta}\} \tag{4}$$

Where,

$[N] = [N_1 \quad N_2 \quad N_3 \quad N_4]$

$\{u\}^T = [u_1 \quad u_2 \quad u_3 \quad u_4]$

$\{v\}^T = [v_1 \quad v_2 \quad v_3 \quad v_4]$

Equilibrium equation can be derived as the following formula (5).

$$\begin{bmatrix} [K_{uu}] & [K_{u\bar{u}}] \\ [K_{\bar{u}u}] & [K_{\bar{u}\bar{u}}] \end{bmatrix} \begin{Bmatrix} \{\delta\} \\ \{\overline{\delta}\} \end{Bmatrix} = \begin{Bmatrix} \{f\} \\ 0 \end{Bmatrix} \tag{5}$$

Where,

$[K_{uu}] = \iint [B]^T [D][B] t dx dy \qquad [K_{u\bar{u}}] = \iint [B]^T [D][\overline{B}] t dx dy$

$[K_{\bar{u}u}] = \iint [\overline{B}]^T [D][B] t dx dy \qquad [K_{\bar{u}\bar{u}}] = \iint [\overline{B}]^T [D][\overline{B}] t dx dy$

and, {f} is the vector of node forces. Interior displacements (u_5, v_5) can be calculated by formula (6).

$$\{\overline{\delta}\} = \begin{Bmatrix} u_5 \\ v_5 \end{Bmatrix} = -[K_{\bar{u}\bar{u}}]^{-1}[K_{\bar{u}u}]\{\delta\} \tag{6}$$

From formula (5) and (6), the following equation (7) can be derived.

$$[K]\{\delta\} = \{f\} \tag{7}$$

Where, [K] is the stiffness matrix of element, given by formula (8).

$$[K] = [K_{uu}] - [K_{u\bar{u}}][K_{\bar{u}\bar{u}}]^{-1}[K_{\bar{u}u}] \tag{8}$$

The stiffness for isoparametric element takes the first term [K_{uu}]. The tangent matrix [D] can be combined from those of concrete [D_C] and reinforcement [D_S], given by the following formula (9).

$$[D] = [D_c] + [D_s] \tag{9}$$

CONSTITUTIVE MODEL FOR REINFORCED CONCRETE PANEL

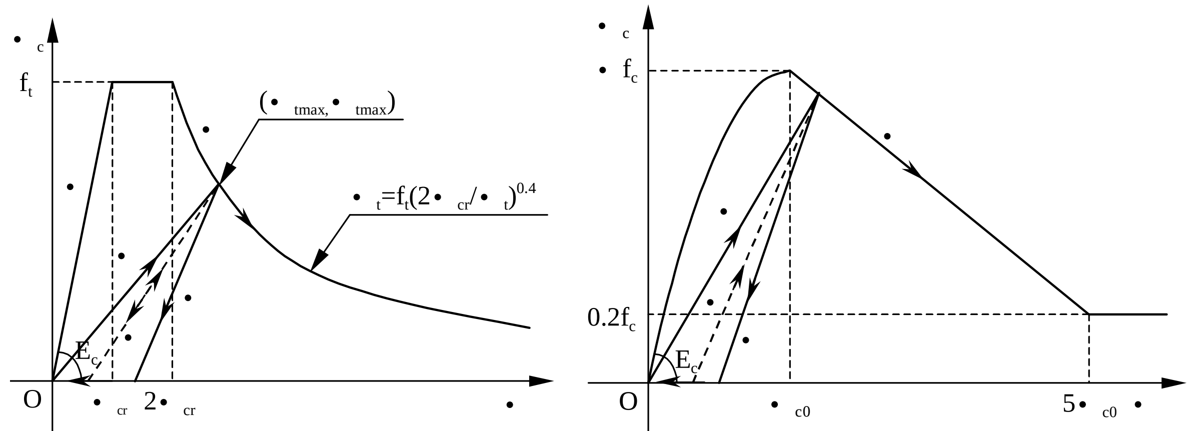
Reinforced concrete wall panel is usually subjected to in-plane shear, moment and axial force. As a plane stress states problem, the following assumptions are made in this study:

- Constitutive equations are given as the relationships between the average stress and average strain;
- The reinforcing bars have orthogonal and uniform distribution within the element, and are subjected to the same strain as concrete;
- The steel bars can resist only axial stresses;
- The principal strain axes of the concrete coincide with its principal stress axes;
- The cracks are always normal to the direction of the principal average tensile strain.

Using these assumptions, the reinforced concrete panel element model can be constructed by combining the constitutive law for concrete and that for steel bars.

Constitutive Model for Concrete

In concrete constitutive model, tension stiffening model for tension envelope [Izumo, Shima and Okamura, 1989] and stress softening model for compression envelope [Vecchio and Collins, 1982] are adopted. The hysteresis rules for tension are shown in Figure 3(a), and those for compression are shown in Figure 3(b).



(a) Tension hysteresis rules **(b) Compression hysteresis rules**

Figure 3: Hysteresis Rules for Concrete

where, f_t: tensile strength of concrete, f_c: compressive strength of concrete, ε_{cr}: strain at tensile strength, ε_{c0}: strain at compressive strength. ζ is reduction factor of compressive strength by tensile strain in orthogonal direction, given by formula (10):

$$\zeta = 1.0/(0.8 - 0.34\varepsilon_t/\varepsilon_{c0}) \tag{10}$$

where, ε_t: tensile strain in orthogonal direction. On the assumption that the principal strain axes of the concrete coincide with its principal stress axes, shear modulus G can be derived as formula (11) from Mohr's circles of stress and strain.

$$G = (\sigma_1 - \sigma_2)/(2(\varepsilon_1 - \varepsilon_2)) \tag{11}$$

where, σ_1, ε_1: stress and strain in the first principal direction, σ_2, ε_2: stress and strain in the second principal direction. Tangent matrix $[D_C]$ [Darwin and Pecknold, 1977] is adopted before cracking, while incremental constitutive relations [Aoyagi and Yamada, 1984] are adopted after cracking.

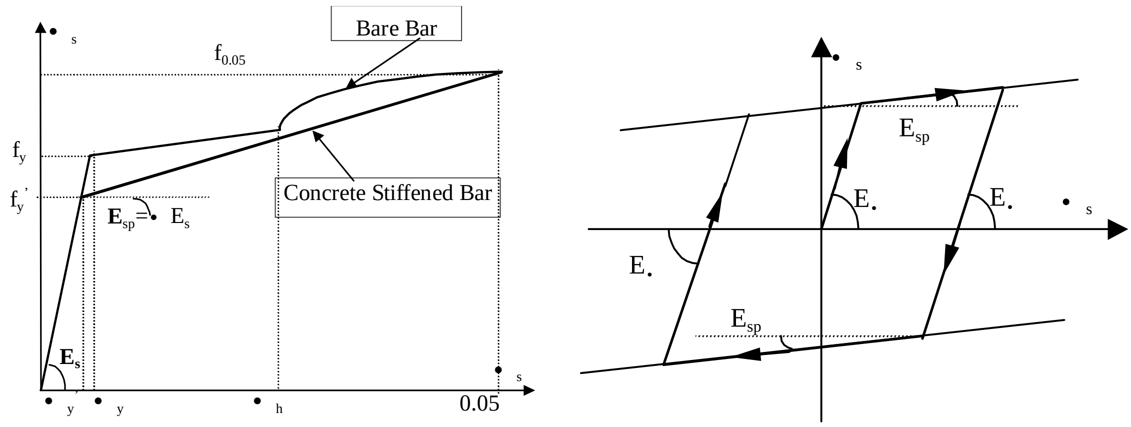
Constitutive Model for Reinforcing Steel

The constitutive model for reinforcing steel has to be numerically modeled based on the properties of bars including the effect of the bond to concrete. The bilinear model proposed by [Hsu, 1993] is adopted, as shown in Figure 4.

Average yield strength of steel bars is given by the following formula (12).

$$f_y' = \left(1 - \frac{2 - 4\alpha/\pi}{1000\rho}\right)\left[0.93 - \frac{2}{\rho}\left(\frac{f_{cr}}{f_y}\right)^{1.5}\right] f_y \tag{12}$$

where, f_y: the yield stress of bare bar, f_{cr}: cracking stress of concrete, •: reinforcement ratio, α: angle between opened crack and reinforcement.

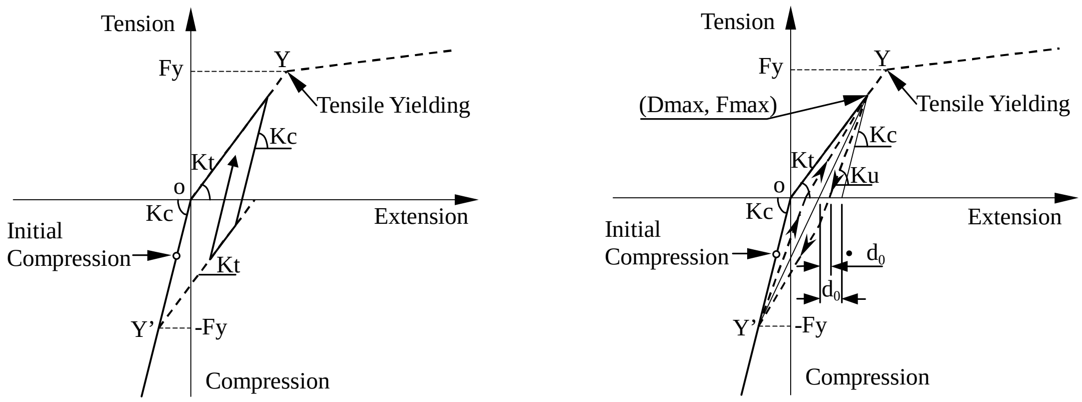


Envelope of Reinforcing Steel **Hysteresis Rule of Reinforcing Steel**

Figure 4: Reinforcing Steel Model in Cracked Concrete

AXIAL SPRING MODEL FOR BOUNDARY COLUMN

In this study, the boundary column model is based on that of TVLEM, but unloading and reloading stiffness is modified. In TVLEM model, the axial model of boundary column is assumed as shown in Figure 5(a), where unloading stiffness is equal to the initial stiffness before yielding point.



(a) Original Model of Column Axial Spring **(b) Modified Model of Column Axial Spring**

Figure 5: Axial Spring Model for Boundary Column

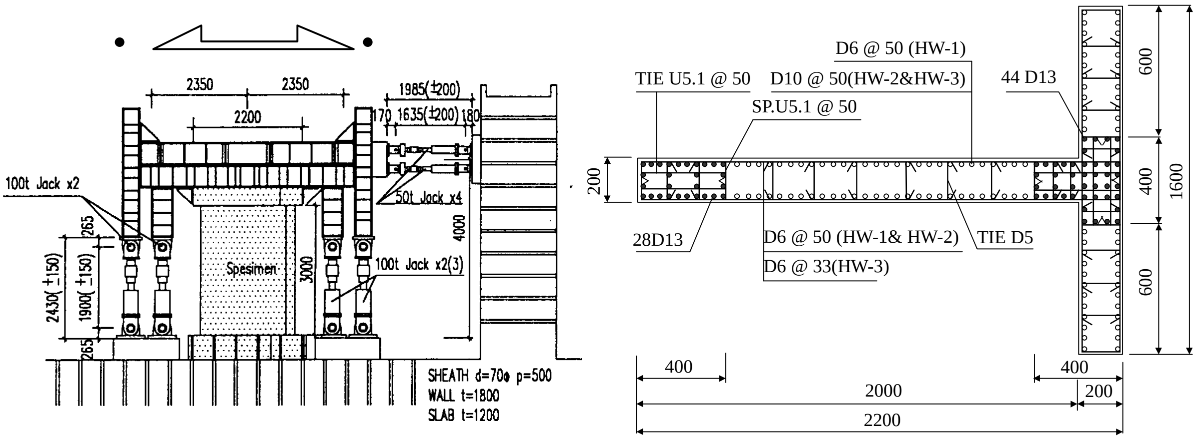
The unloading stiffness is modified as shown in Figure 5(b) and is given by formula (13) with parameter • (•=0.1) which is a factor of residual displacement.

$$K_u = \frac{1}{(1-\gamma)/K_0 + \gamma/K_C}$$
$$K_0 = \frac{F_{max} + F_y}{D_{max} + F_y/K_C} \qquad (13)$$

where, K_0 is the stiffness from point (D_{max}, F_{max}) to point (Y', $-F_y$), and K_C is the initial stiffness.

VERIFICATION OF RC PANEL MODEL

The reinforced concrete wall model is verified through the analysis of T-shaped wall tests conducted at Yokohama National University in 1994, as a part of US-Japan cooperative research program on hybrid wall system [Kabeyasawa, et al, 1995]. Three test specimens HW1, HW2 and HW3 having the same dimension were subjected to lateral and axial force, and bending moment by using four horizontal and six vertical oil jacks. Test set-up and dimension of specimens are shown in the Figure 6. The lateral shear and varying axial load simulate the earthquake response of the structure because the specimens represent the lower parts of the reinforced concrete coupled core walls where the wall is subjected to the large amplitude of varying axial loads from the boundary beams. Specimens HW1 and HW3 were planned to collapse by flexural yielding, whereas HW2 was planned to collapse by shear failure. The properties of materials and analytical model are shown in Figure 7.

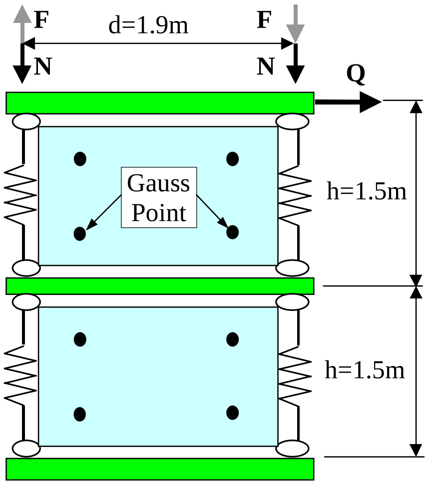


Loading Set-up **Dimension of Specimen**

Figure 6: T-shaped Wall Test

Properties of Materials

Steel Bar		
Diameter (mm)	Yield Strength (MPa)	Tensile Strength (MPa)
D6	402	548
D10	366	517
D13	366	537
U5.1	1374	1445
Concrete		
Age (day)	Compressive Strength (MPa)	Elastic Modulus(MPa)
99	35.5	27200



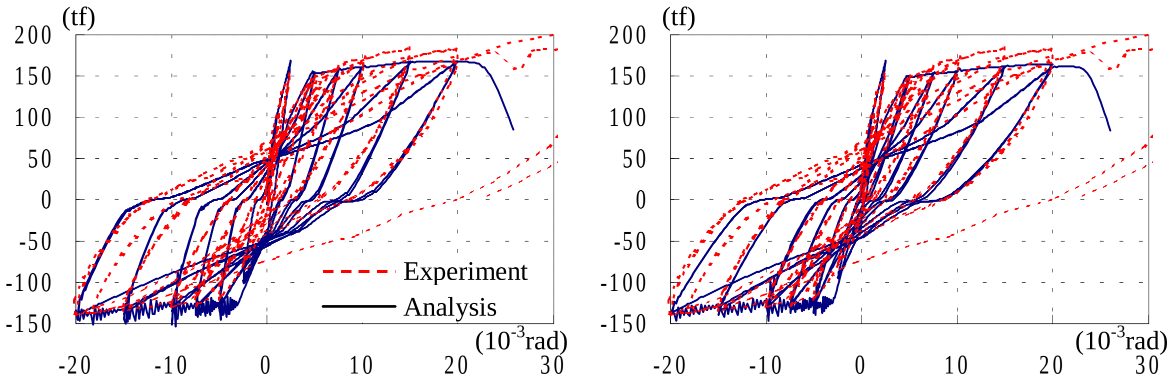
Analytical Model

Figure 7: Properties of Materials and Analytical Model

To investigate the influence of unloading stiffness of axial spring for column, the relations between shear force and drift angle at the top are compared as shown in Figure 8(a) and (b). Figure 8(a) is the result using the original model for axial spring, while Figure 8(b) is the result when the unloading stiffness was modified. The original model of column overestimates unloading stiffness slightly which was improved by the modified model.

In case of specimen HW1, which was planned to yield in flexural mode, isoparametric element method and incompatible element method were compared as shown in Figure 9. The shear and flexural displacement components at the top of specimen are normalized to the ratios to the overall displacement. In the decomposition of displacement in the analysis, the curvature is assumed to linearly distribute along the height of loading point. The difference between isoparametric element and incompatible element is small within the displacement drift angle of 0.01rad. When the displacement drift angle excess 0.01rad, shear displacement by isoparametric element was overestimated, but the incompatible element had good correlation with experimental result until concrete compressive fracture of wall at the displacement drift angle of 0.02rad. The ratios of shear deformation were 0.37 from experiment, and 0.42 from incompatible element model, whereas 0.50 from isoparametric element model.

In case of specimen HW2, which was planned to collapse by shear failure, the two methods have little difference with the test result as shown in Figure 10. The ratios of shear deformation were 0.50 from experiment, and 0.48 from incompatible element model, whereas 0.53 from isoparametric element model. The analytical results have a good correlation with the experimental results until concrete compressive fracture of wall at the displacement drift angle of 0.015rad.



(a) Original Axial Model **(b) Modified Axial Model**

Figure 8: Influence of Unloading Stiffness of Axial Spring for Boundary Column (Specimen HW1)

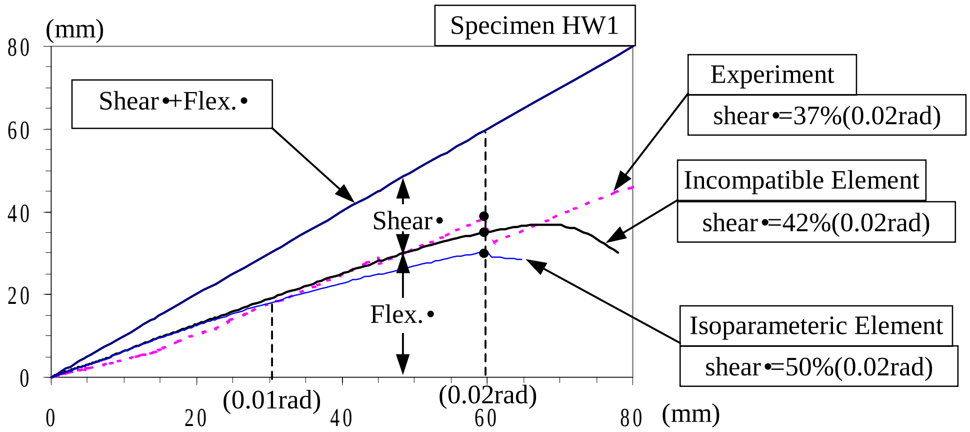


Figure 9: Comparison Shear and Flexural displacement components (Specimen HW1)

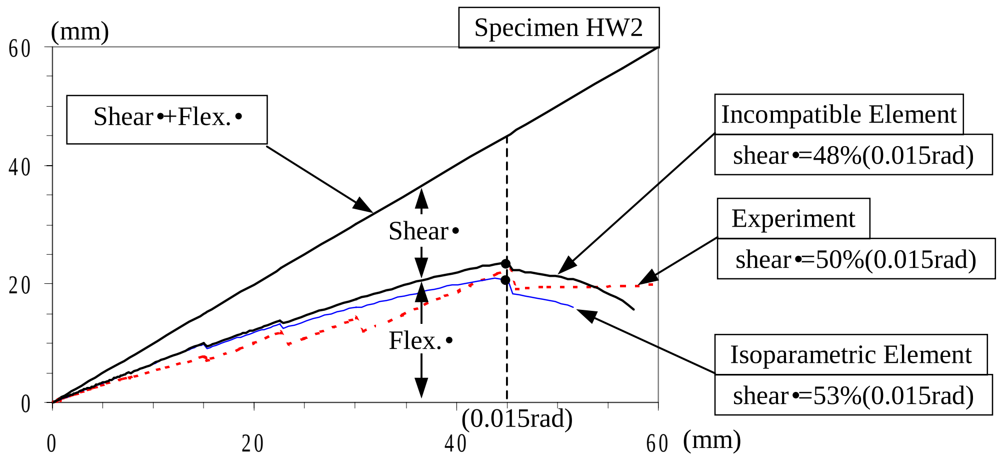


Figure 10: Comparison of Shear and Flexural displacement components (Specimen HW2)

Thus considering the interior displacements by incompatible element, good prediction of flexural displacement and shear displacement can be made for shear walls in flexural yielding. The same degree of accuracy can be obtained by both models for the wall in shear failure as well. The overall hysteresis relations of the specimens HW2 and HW3 are shown in Figure 11 simulated by the incompatible element method.

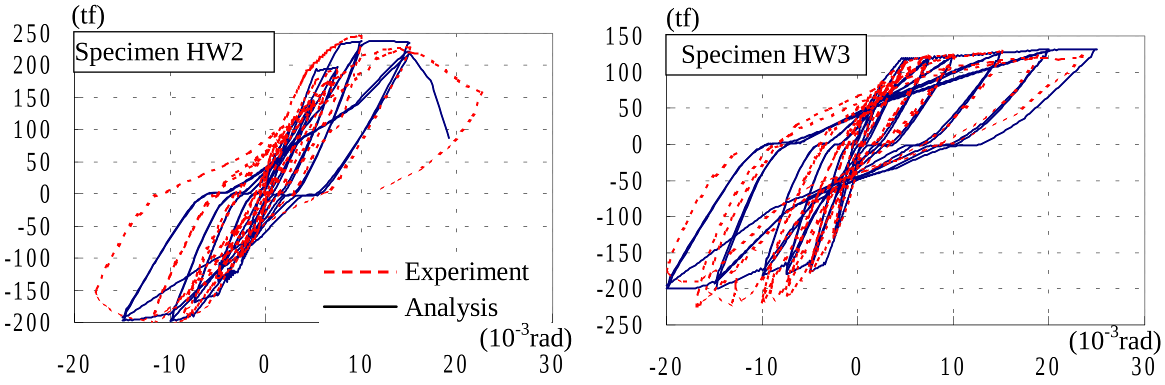


Figure 11: Hysteresis relationships of Specimen HW2, HW3

CONCLUSIONS

A new model for reinforced concrete shear wall was proposed, which combined a FEM panel element and boundary line elements. Two methods using isoparametric element and incompatible element for a panel were compared with the test results. The model with isoparametric element slightly overestimates shear deformation for flexural yield shear walls. The model with incompatible rectangular element is better than the model isoparametric element for prediction of shear and flexural displacement components. Hysteresis shapes of inner loops were improved by modifying the axial stiffness model of column. The model gave a good correlation between analytical and experimental results.

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