PROBABILISTIC SEISMIC HAZARD ANALYSIS: IMRPOVING CONSISTENCY WITH PRECARIOUS ROCK OBSERVATIONS BY REMOVING THE ERGODIC ASSUMPTION

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SUMMARY

An ergodic process is a random process in which the distribution of a random variable in space is the same as the distribution of that same random process at a single point when sampled as a function of time. An ergodic assumption is commonly made in probabilistic seismic hazard analysis (PSHA). Regression analysis derives a mean curve to predict ground motions, as a function of magnitude and distance (and other parameters some of the time). The standard deviation of the regression is determined mainly by the misfit between observations and the prediction at multiple stations for a small number of well-recorded earthquakes. Thus the standard deviation is dominantly related to the statistics of the spatial variability of the ground motions. The basic model used for probabilistic seismic hazard analysis makes an ergodic assumption when it uses this estimate of the standard deviation to describe the temporal distribution of ground motion at a single site over multiple earthquakes. To the extent that path and site response play a major role in controlling ground motions, this assumption cannot be correct.

More general PSHA models distinguish between epistemic uncertainty (due to lack of knowledge) and aleatory uncertainty (due to truely random effects). A thought experiment involving a site where hazard is dominated by repetition of identical characteristic earthquakes on a single fault demonstrates that the correct separation of aleatory and epistemic uncertainty can have a large impact on the results of PSHA. We propose that the distinction between aleatory and epistemic uncertainty in the attenuation relationships depends on an absolute standard rather than a model-dependent standard. The aleatory uncertainty should only include uncertainty that arises from temporal dependence in the Earth's behavior, such as variability in the source processes on a fault that change from one earthquake to the next. In contrast, epistemic uncertainty treats the repeatable, but presently unknown, behavior caused by path and site response.

The optimum distribution of uncertainty between aleatory uncertainty and epistemic uncertainty must be determined from data, not assumed. Evidence from the distribution of precarious rocks near the San Andreas fault suggests that the ergodic assumption causes the aleatory uncertainty to be overestimated and the epistemic uncertainty to be underestimated. The distinction is important for any part of a seismic hazard analysis where the exposure time is large compared to the repeat time of the earthquakes, as may happen in the magnitude 5-6 range, or for larger events on very active faults.

INTRODUCTION

An ergodic process is a random process in which the distribution of a random variable in space is the same as the distribution of that same random variable at a single point when sampled as a function of time. An ergodic assumption is commonly made in probabilistic seismic hazard analysis (PSHA). A regression analysis is used to obtain a mean curve to predict ground motion as a function of magnitude and distance (and sometimes other

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parameters). The standard deviation of this ground motion regression is determined mainly by the misfit between observations and the corresponding predicted ground motions at multiple stations for a small number of well-recorded earthquakes. Thus, the standard deviation of the ground motion regression is dominantly related to the statistics of the spatial variability of the ground motions. An ergodic assumption is made when PSHA treats that spatial uncertainty of ground motions as an uncertainty over time at a single point (Anderson and Brune, 1998, 1999a). The paper by Anderson and Brune (1999a) goes into some detail on the effect that this ergodic assumption might have on probabilistic seismic hazard analysis. This contribution summarizes those and other results.

Some recent PSHA maps that make the ergodic assumption predict what seem to us to be high values of maximum ground motion for long repeat times. For example, recent PSHA maps for the United States by the US Geological Survey (Frankel et al., 1996) give near-fault ground motions of 1.2g to 1.6g, and occasionally higher, for long return periods (2% probability in 50 years recurrence time, corresponding to an average repeat time of once in 2475 years). Two other recent PSHA maps for southern California (Working Group on California Earthquake Probabilities (WGCEP), 1995; Ward, 1995) give similar values of ground motion. Of course, we do not have accelerograph evidence to prove that these values are incorrect, but this paper suggests some reasons why they might be overestimated.

Brune (1996) found that the distribution of precarious rocks in Southern California is not consistent with the large values of ground motion predicted by these PSHA studies. Similarly, Anderson and Brune (1999b) concluded that most of the known precarious rocks in Nevada are inconsistent with PSHA maps. A subsequent study by Brune (1999) illustrates this point. Brune (1999) estimates the peak accelerations that would be sufficient to topple precarious rocks on a profile perpendicular to the San Andreas fault near Palmdale in the Mojave Desert. For instance, Figure 1 shows a rock which Brune estimates could be toppled by horizontal accelerations in excess of about 0.4g.

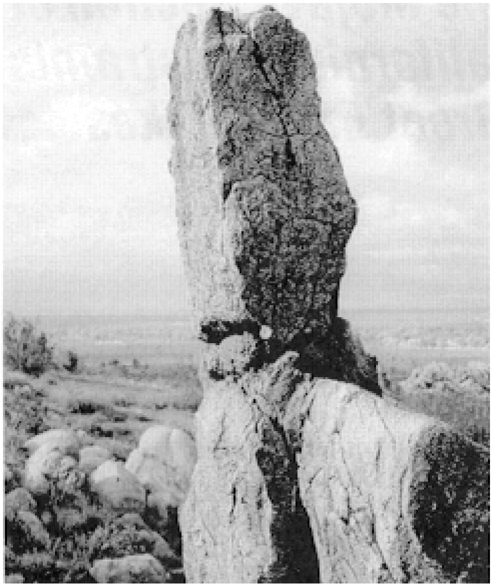

Figure 1. Photograph, from Brune (1999), of a "semiprecarious" rock located at Lovejoy Buttes, about 15 km from the San Andreas fault. The San Andreas fault is at the base of the hills in the background. Brune (1999) defines a semiprecarious rock as a rock that could be toppled by a horizontal acceleration of 0.3 to 0.5 g.

These estimated accelerations are close to the mean predictions of the peak accelerations from a magnitude 8 earthquake on the San Andreas fault based on some current regression equations, as shown in Figure 2. These precarious rocks have ages that are on the order of 10,000 years (e.g. Bell et al, 1998) and thus have experienced numerous earthquakes on the San Andreas fault. Furthermore, their ages are large compared to the exposure times that are used for the probabilistic curves that are obtained from the maps of Frankel et al. (1996), WGCEP (1995), and Ward (1995). Figure 2 shows PSHA estimates for peak accelerations with exposure times of approximately 2500 years. Deaggregation of the hazard at Lovejoy Butte (Fig. 1) is dominated by earthquakes on the San Andreas fault. This implies that the likely source of the inconsistency is in the predicted ground motions and their uncertainties from great earthquakes on the San Andreas fault. The recent result is consistent with the earlier study of precarious rocks by Brune (1996). A reasonable explanation is that at the sites of these precarious rocks, the maximum accelerations have been near the mean of the regression curves over thousands of years. The distribution of the precarious rocks in southern California is reasonably consistent with hazard maps given by Wesnousky (1986).

These observations motivated us to look in more detail at the way the probabilistic seismic hazard is estimated (Anderson and Brune, 1999a). For the sites in the Mojave Desert, the controlling difference between the PSHA maps and the maps of Wesnousky (1986) was that Wesnousky (1986) used only the mean value for attenuation of peak ground acceleration with distance, whereas the PSHA maps added a statistical (Gaussian) uncertainty to the ground motion (Stirling et al., 1997). This statistical variation was estimated primarily from the spatial variation of strong motion but was applied to estimate the repeatability of ground motions at single sites over time, thus utilizing the ergodic assumption. Combined with the reasonable assumption that the occurrence of earthquakes are Poissonian over time, this resulted in the probable maximum ground motion at a given site increasing indefinitely as the time window of the PSHA increases, due to the increasing influence of the tails of the Gaussian distribution on the probabilistic values. We show in this study that the inappropriate use of the ergodic assumption can, at least in specific idealized cases, result in overestimates of ground motion when exposure times are longer than earthquake return times.

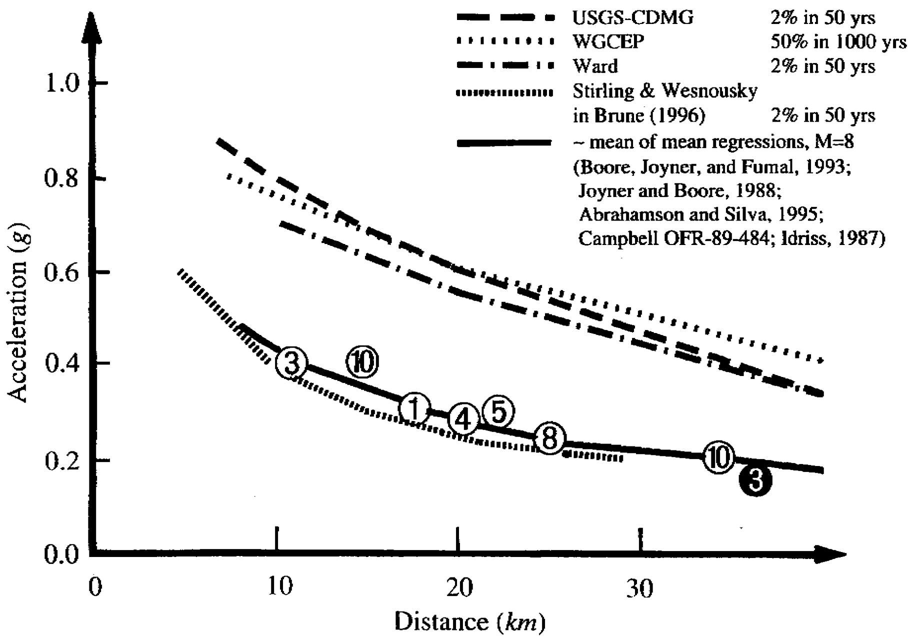

Figure 2. Comparison of estimates of toppling accelerations of precarious rocks in the Mojave Desert with PSHA estimates (from published PSHA maps) and with the approximate mean of mean attenuation curves derived by various authors. Numbers in the circles are lower bound on the number of precarious rocks at each locality. From Brune (1999).

FUNDAMENTALS OF PSHA

The basic elements of probabilistic seismic hazard analysis (PSHA) were formulated by Cornell (1968) and reviewed briefly by Anderson and Brune (1999a). Figure 3 shows the results of a basic PSHA. PSHA presumes the existence of some mean occurrence rate curve r(Y) that is measurable in a thought experiment, which we designate as Thought Experiment 1. Suppose that an instrument were operated at a certain location for a long time, say 10^5 years. Let the peak amplitude of y during the ith year be designated as yi. After sorting yi by increasing value, it is straightforward to find the mean rate at which Y is exceeded and the probability P(Y,T)=P(y>Y,T) that Y is exceeded in a randomly selected interval of duration T. Assuming that the earthquakes are random, uncorrelated events, then the Poisson model should hold, and the probability of exceedance in a time interval of duration T is P(Y,T)=1-exp(-r(Y)T). The curve P(Y,T) is a hazard curve. Hazard curves that correspond to the mean occurrence rate curve given in Figure 3 are shown in Figure 4.

PSHA estimates r(Y) through the synthesis of observed features of the seismicity and ground motion relations. The ground motion relation is used to estimate the mean ground motion as a function of the magnitude and distance (and sometimes additional parameters); its standard deviation σ_T is also developed by regression.

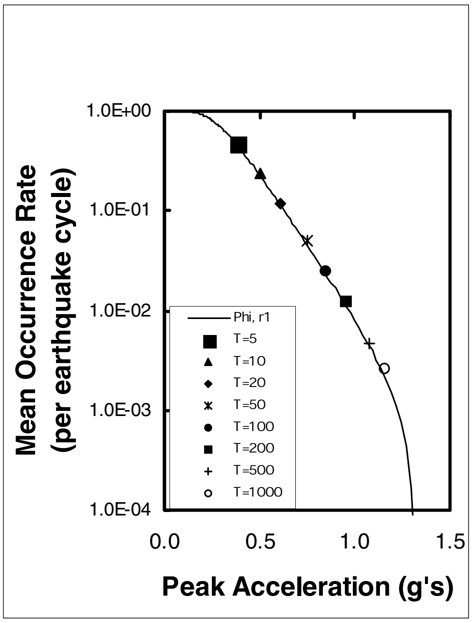

Figure 3. A basic PSHA for a site dominated by characteristic earthquakes on a fault that produces only magnitude 8.0 earthquakes, 15 km from the site. The PSHA uses the regression of Abrahamson and Silva (1997) (mean=. 0.36g, standard deviation of σ_T =0.430 in natural logarithm units). r1(Y) = the mean occurrence rate curve. This is equal to Φ, the cumulative probability that a single realization of the earthquake causing ground motion in excess of the abscissa. The points give the peak accelerations that would be exceeded with 90% probability for the given number of earthquake cycles.

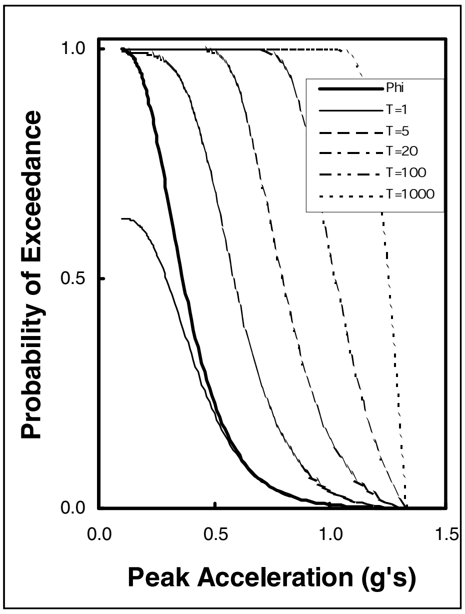

Figure 4. Hazard curves giving the probability of a peak acceleration exceeding Y for various time intervals, corresponding to . the PSHA from Figure 1. The heavy solid line, again, gives Φ, the probability of exceeding a peak acceleration (Y) conditional on the occurrence of one earthquake. The unit of time is the mean recurrence time of the characteristic earthquake.

THOUGHT EXPERIMENT: CHARACTERISTIC GROUND MOTION EARTHQUAKE

Anderson and Brune (1999a) proposed the following thought experiment, which we designate as Thought Experiment 2, in which the basic PSHA model does not work. Suppose, as a rough approximation for situations where the hazard is dominated by large earthquakes on one fault (e.g. the San Andreas fault for some locations in California), that there is only one fault source for the earthquakes in an area. We further suppose that when the fault ruptures, it fails with a Characteristic Earthquake (e.g. Schwartz and Coppersmith, 1984) that repeats previous earthquakes not only in the static slip but also in the dynamics of rupture. We call this a "Characteristic Ground Motion Earthquake", in which the ground motion at the site will be identical in every earthquake. We designate the ground motion at the site as yCE.

Prior to the first earthquake, we do not know what yCE will be, and thus the ground motion relation correctly describes our probability distribution on the peak motion conditional on the earthquake occurrence. However, after the earthquake has occurred the conditions of the thought experiment require that yCE is known and in all future earthquakes y will equal yCE. Thus, the outcome of Thought Experiment 2 should be that the true mean occurrence rate curve is a step function (rTE2(Y), say). Before yCE is measured, the best estimate for the hazard curve PTE2(Y,T) for Thought Experiment 2 obviously cannot be obtained by substituting rTE2(Y) into Equation 1. Rather, conditional on either one or several earthquakes happening, the best estimate for the probability of obtaining ground motions $y \geq Y$ is the same as the prediction from the ground motion relation, no matter how many earthquakes occur. For small T there is a finite chance that no earthquakes will occur when using the Poissonian assumption (Equation 1), but PTE2(Y,T) should converge to Φ as T increases, and not to anything larger. The basic PSHA, as illustrated in Figure 2, does not behove in this way. Rather, for every peak below the cutoff, the probability of exceeding Y increases towards certainty as the number of earthquakes increases. In summary, in this thought experiment the ground motion at the site is always the same in characteristic ground motion earthquakes, and its most likely value is in the vicinity of the mean value predicted by the regression equations. In contrast, a basic PSHA predicts that with the occurrence of several earthquakes, peak accelerations approaching a value well above the mean are likely to occur. Although we do not expect that the characteristic ground motion hypothesis will be achieved exactly for repeating major earthquakes on a single fault, it still seems reasonable that an acceleration so much above the mean of the prediction equation overestimates the hazard.

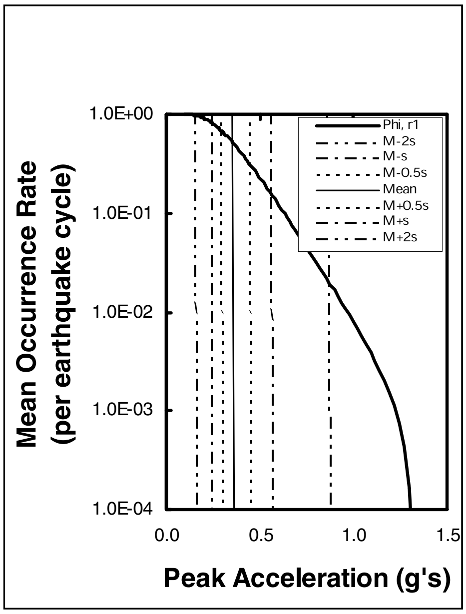

Figure 5. Alternative extreme model of the PSHA for the situation in Figures 3 and 4. For this analysis, the ergodic assumption is dropped. Two kinds of uncertainty are recognized: aleatory and epistemic. The aleatory uncertainty (σ_A) is assumed to be zero, meaning that the ground motion from the characteristic earthquake is assumed to be the same for every earthquake. The total uncertainty is assumed to be the same as in Figures 3

and 4, meaning that the epistemic uncertainty equals the total uncertainty ($\sigma_E = \sigma_T$). With $\sigma_A = 0$, each possible mean occurrence rate curve, calculated using Equation 7, is a step function like those shown. The epistemic uncertainty means that we do not know which step function to choose, so the figure shows several to illustrate the range of choices. (In the legend, M+2s, for instance, denotes the mean occurrence rate curve if the characteristic ground motion at the site is two standard deviations above the mean, etc.) The curve phi is again shown for reference.

The failure, as shown in Figures 3 and 4, of the basic PSHA model to match the expected results of Thought Experiment 2 can be understood by considering the effects of two types of uncertainties, aleatory and epistemic (e.g. SSHAC, 1997; Toro et al, 1997a). Toro et al (1997a) define epistemic uncertainty (σ_E) as: "Uncertainty that is due to incomplete knowledge and data about the physics of the earthquake process. In principle, epistemic uncertainty can be reduced by the collection of additional information." On the other hand, Toro et al. (1997a) define aleatory uncertainty (σ_A) as "uncertainty that is inherent to the unpredictable nature of future events." They go on to state that "Aleatory uncertainty cannot be reduced by collection of additional information." The total uncertainty (σ_T) is related to σ_A and σ_E by:

$$\sigma_T^2 = \sigma_A^2 + \sigma_E^2$$

In the basic PSHA, there is no distinction between these two different types of uncertainty. In Thought Experiment 2, all of the uncertainty is epistemic, as the collection of additional information (i.e., ground motion in the next of the series of characteristic ground motion earthquakes) reduces the uncertainty for ground motion in future earthquakes to zero, implying that the true value of r(Y) is a step function as described above. Anderson and Brune (1999a) show that a more general formulation of PSHA, in which aleatory and epistemic uncertainty are handled separately, correctly models Thought Experiment 2.

To be specific, probabilistic assessments that use logic trees for multiple models of the input (e.g. SSHAC, 1997) treat aleatory and epistemic uncertainty in different ways. Aleatory uncertainty is treated as in the basic PSHA. Epistemic uncertainty is treated by development of multiple models of the PSHA analysis which are subsequently weighted according to some estimate of the probability of each. The critical feature of this approach is that the effect of time is put in before weighting in the effect of uncertainty in the ground motion. For time intervals in which the mean occurrence rate is small, the result is the same as in the basic approach. However, as pointed out by SSHAC (1997), the correct procedure is to average the hazard curves, and when exposure times are large compared to the earthquake return times, the difference is significant.

By this procedure, the hazard curve is developed for time intervals of duration T. To find the curve for a different time interval, it is necessary to go to the original mean occurrence rate curves, find the corresponding set of possible hazard curves for the new value of exposure time, and then find the new average. This contrasts with the procedure one would use when the mean occurrence rate curve is known exactly and the earthquake occurrences are assumed to be a Poisson process (Equation 1). For instance, if the hazard curve is found using Equation 1 with a known mean occurrence rate curve, the ground motion with a probability of 10% in 50 years is the same as the motion with a probability of 1/475 in one year. However, when the hazard curve is uncertain and different possible hazard curves are averages, those two ground motions are not the same.

Figure 5 shows occurrence rate curves and Figure 6 shows hazard curves for the procedure outlined by Anderson and Brune (1999a). The dashed lines in Figure 5 show a series of alternative possible mean occurrence rate curves. Each mean occurrence rate curve is a step function showing that every time there is an earthquake the peak acceleration is exactly the same value. Thus, for each of these curves $\sigma_A = 0$. The epistemic uncertainty with $\sigma_E = \sigma_T$ is illustrated by the range of values of the steps in the mean occurrence rate curves. We do not know which of these curves to choose, but after the characteristic ground motion earthquake occurs, we will be able to select the correct curve from among these alternatives. The final estimate for the hazard curve, PN(Y,T) (Figure 6) depends on the duration T for small values of T, but it converges quickly towards $\Phi(y \geq Y|\hat{Y}, \sigma_T)$. For this case, the peak acceleration which occurs at a rate of 1/475 is about 1.6 g, while the peak acceleration that occurs with probability of 10% in 5 or more earthquake cycles is about 0.87g.

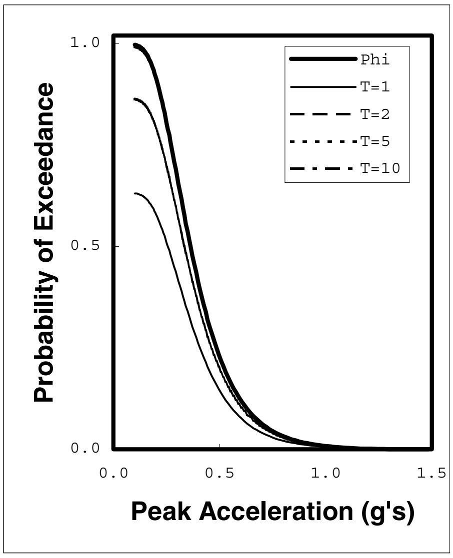

Figure 6. The mean hazard curves for four time intervals, T, when the ergodic assumption is dropped. The hazard curves are obtained by averaging individual hazard curves corresponding to each of the possible individual mean occurrence rate curves in Figure 5. For large T the mean hazard curve converges to the curve phi, rather than exceeding it as in Figure 4.

DISCUSSION

The definition and separation of aleatory and epistemic uncertainty has been articulated most extensively by SSHAC (1997). SSHAC and the Panel on Seismic Hazard Evaluation (1997) saw the division between aleatory and epistemic uncertainty is model dependent, somewhat arbitrary, and ambiguous. In contrast, Thought Experiment 2 finds a critical importance of distinguishing between aleatory and epistemic uncertainties for at least one special case. An absolute standard for the definition of the aleatory uncertaintyis that it should describe the variability of the ground motion parameter over time. It should only come from changes in the source when there are repeated realizations of the same event on the same fault. Since the path is identical in repeated realizations, its effect is knowable and the uncertainty is epistemic. When a PSHA treats total uncertainty as an aleatory uncertainty, that analysis makes an ergodic assumption. To the extent that path and site response play a major role in controlling ground motions, this assumption cannot be correct. If path and site factors play a major roll, a major fraction of the uncertainty is epistemic. There is conflicting evidence for how the total uncertainty should be partitioned between aleatory and epistemic uncertainty. Anderson and Brune (1999a) hypothesize that the best estimate for the aleatory uncertainty, at least for large magnitudes, is less than 0.15, but recognize the necessity for experiments to resolve the question.

ALTERNATIVE EXPLANATIONS FOR PRECARIOUS ROCKS

Anderson and Brune (1999a) identified four alternative explanations for the inconsistency between the presence of precarious rocks and the PSHA models. One is that the physics of toppling precarious rocks is poorly calibrated to peak acceleration. A second alternative is that the peak accelerations from a magnitude 8 earthquake on the San Andreas fault are significantly smaller than the mean of the regressions. A third possible, but unlikely, explanation is that perhaps some precarious rocks, due to some luck in their locations, survive the strong earthquakes in spite of the general radiation field around them being more than strong enough to cause them to topple. The last explanation is the one developed in this paper: that the majority of the uncertainties in ground motions are epistemic uncertainties and that ground motions at a single site in the presence of repeated earthquakes are approximately repeatable.

CONCLUSIONS

This paper has identified a thought experiment, suggested by evidence from the distribution of precarious rocks, for which the usual PSHA approach fails. The approach to correct PSHA is to distinguish between aleatory and epistemic uncertainties. It is of the utmost importance to resolve the relative roles of epistemic and aleatory uncertainty in regression analysis.

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