ANALYTICAL STUDIES OF SHEAR-YIELDING MOMENT-RESISTANT STEEL FRAMES

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SUMMARY

A novel structural system is described in which yielding of the beam web is relied upon to protect steel moment-resistant connections from fracture. Capacity design principles are invoked to determine the size and spacing of large openings in the web. Initial proportioning criteria are described along with results from detailed nonlinear finite element studies. Results confirm that at least two distinct mechanisms can be obtained, depending on how the web openings are proportioned.

INTRODUCTION

The unexpected fracture of numerous steel beam-column connections in moment-resistant frames in the 1994 Northridge and 1995 Hyogo-Ken Nambu (Kobe) earthquakes created an urgent need to develop improved structural systems, details, and design procedures applicable to both new design and the retrofit of existing "pre-Northridge" steel construction.

Various approaches have been taken to reconfigure these connections to achieve higher deformation capacity, including (1) the "dogbone" or reduced flange connection, in which capacity design principles are applied to protect the connection by shifting the location of flexural hinging away from the column face to a weakened region; (2) various cover-plate, side plate, and haunched beam approaches, which strengthen the beam-column connection; and (3) introducing flexibility into the flanges by cutting slots in the beam web near the column face, thereby relieving the welds of stress and strain concentrations that otherwise could induce fracture.

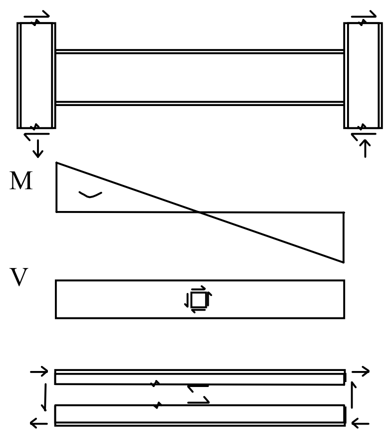

Fig. 1: Actions on a portion of a frame under lateral loading

A different approach is taken in this work. Consider the familiar case of a frame under lateral loading. Shear demands are constant over the beam length, while peak flexural demands occur at the column faces. The free-body diagram in Fig. 1 indicates that limiting the shear carried by the web serves directly to limit the tension-compression couples that can develop at the beam-column connections, and hence can limit the stresses induced in the beam-column welds.

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SHEAR YIELDING CONCEPT

Beams of ordinary dimensions have ample shear strength, and it therefore is necessary to weaken the beam in order to limit the stresses at the beam-column connection. If introducing holes in the web reduces the beam shear strength, it is clear that the mode of deformation obtained depends on the relative strengths in the longitudinal and transverse directions. For example, distortions of the flanges will occur if few holes are introduced (Mode A, Fig. 2), while distortions of the web will occur if many holes are introduced (Mode B). Mode A involves contributions of the floor slab and may "shake down" in the presence of gravity loads. Mode B appears to be preferable, but as shown is kinematically incompatible with the column flange. This incompatibility is discussed in the following.

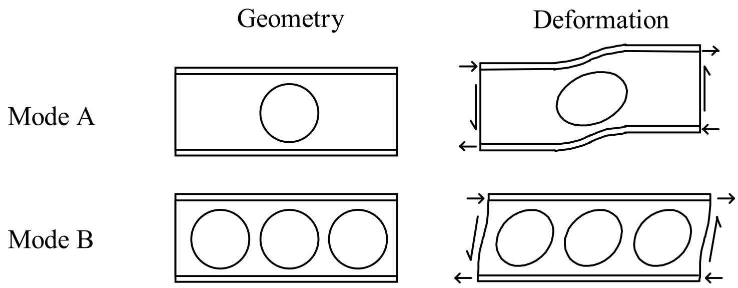


Fig. 2: Deformation Modes

Normal stresses arise from flexure and are highest nearest the columns. Interaction between shear and normal stresses can be employed to induce yielding at the opening nearest the column. One resolution to the kinematics of Mode B is as shown to the right (Fig. 3). The web posts (material between the openings) are yielding as per Mode B, and Section 1 is yielding under shear-moment interaction. The web posts are proportioned to deliver any desired level of stress to the beam-column connection welds, typically aiming to keep the region of the beam-column connection nominally elastic.

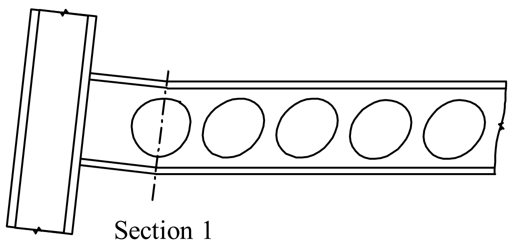


Fig. 3: Preferred Mechanism (Tentative)

INITIAL DESIGN APPROACH

At this early stage, three alternate design objectives are being considered. In the first, the web posts are proportioned such that when they yield, they simultaneously deliver sufficient force to Sections 1 to cause these sections to yield under combined shear and moment. In the second, the web posts are stronger relatively, and hence Sections 1 yield under combined shear and moment (Mode A). In the third, the web posts are weak, and additional deformation is required to induce yielding at Sections 1. These objectives are being explored in the experimental and analytical portions of the research. Initial analysis results are discussed below.

Various stress distributions have been suggested by other investigators (e.g. Redwood, 1983, AISC, 1990) for the design of beams with web openings. In most cases, elastic response is desired and various kinematically inadmissible distributions have been shown to work well. In the present case, plasticity is intended, and thus the usual lower and upper bound theorems should apply. Normal and shear stress distributions for solid (unperforated) I-beams were put forward by Hodge in 1957. These stress distributions satisfy plane stress equilibrium and simultaneously satisfy the Tresca yield criterion. Hodge's stress distributions, modified to satisfy the Von Mises yield criteria are given by:

$$\sigma_x = \frac{\sqrt{3}\,\lambda\,F_y\,y}{\sqrt{(1+3\,\lambda^2 y^2)}}$$

$$\tau = \frac{F_y}{\sqrt{3}\,\sqrt{(1+3\,\lambda^2 y^2)}}$$

where F_y is the yield strength of the material and y is the distance from the neutral axis.

These stresses are a function of an Euler multiplier, λ, which is the ratio of curvature to shear strain on an unperforated section. These stresses are plotted for a W21x68 wide flange beam in Fig. 5 for $\lambda = .070$ and $F_y = 50$ ksi (345 MPa). Variation of λ varies the shear and normal stress distributions as well as the corresponding shear and moment stress resultants, associated with full plasticity of the web.

It is conceivable that the strength of a perforated cross section can be represented approximately by applying Hodge's stress distribution over the remaining material. This is approximate because tractions necessarily are zero at the boundary of the opening and some stress and strain concentrations are present. Nevertheless, this approximation serves as a starting point for understanding the behavior and design of beams having web openings. A curve representing the interaction of shear and moment for a W21x68 section having a 15-in. (38.1-cm) diameter circular web opening determined using this approximation is presented in Fig. 5. Values shown are normalized by the nominal shear strength of the web ($\tau_y t_w d$) and yield moment ($f_y S_x$) of the unperforated section. Fig. 5 indicates that the web openings cause large reductions in shear strength and much smaller reductions in flexural strength, as would be expected.

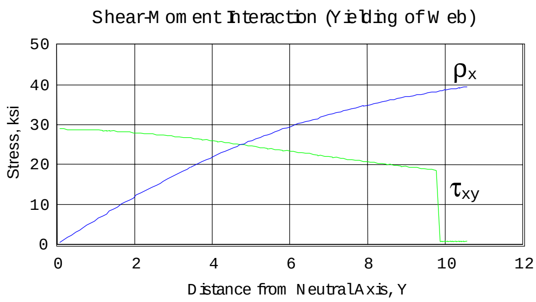


Fig. 4: Hodge's solution for shear-moment interaction in an unperforated W21x68 section

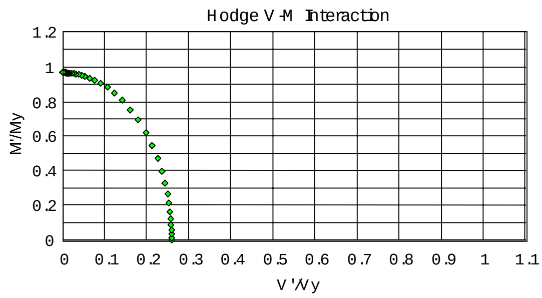


Fig. 5: Approximate interaction for a 15-in. (381-mm) diameter opening

Conservation of shear flow between the web and flange necessarily reduces the shear stresses in the flange. Given the Tresca or Von Mises yield criterion, this implies that the flange remains elastic in the vicinity of an opening. An elastic flange may have significant implications for the stability of the system. Results from the finite element studies have shown the flange remains substantially elastic, with just the top surface of the flange yielding due to local curvatures. A yielding web and elastic flange may seem incompatible, but the kinematics of Section 1 require that the inelastic web translate relative to the flange to be compatible with the elastic region near the column.

To proportion the web openings, a target stress in the beam-column welds is established. If web post yielding is to occur, the web openings must be proportioned such that the yielding web posts produce just enough normal stress at Section 1 to cause this section to plastify under the corresponding shear and moment. A solution for the opening diameter, number of openings, and spacing for a wide flange beam can be obtained by iteration on a spreadsheet, even though the Hodge approximation does not model the stresses at Section 1 accurately, based on the results of the finite element studies. The web post may yield in shear or may develop plastic hinges above and below the beam centerline, depending on its slenderness. Analysis suggests that shear yielding will occur if the spacing, *s*, of circular holes is greater than about 1.26 *d'*, where *d'* is the hole diameter.

The approximate Hodge criteria were applied to design several beams to illustrate various potential behaviors, summarized in Table 1. All the beams were designed to develop 0.75 F_y in the flanges at the beam column connection, based on simple beam theory. Table 1 illustrates that different behaviors can be obtained from relatively small changes in opening geometry. Analysis results for the first two of these beams are described below.

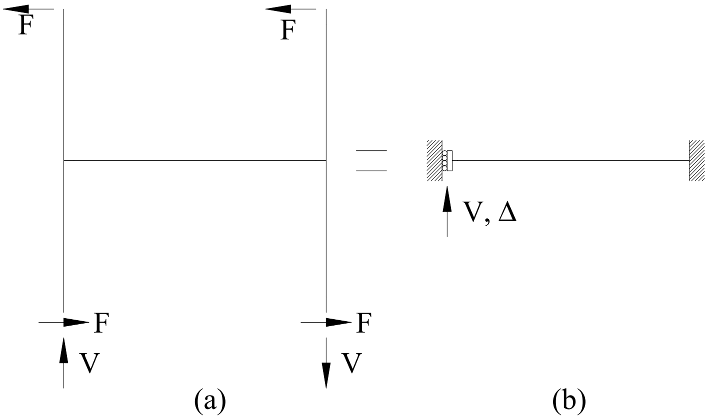


Fig. 6: (a) Beam Frame and (b) Beam Model

Table 1: Preliminary Beams Proportioned Using Hodge Criteria

	Beam Section	Number of Openings, n	Opening Diameter, d', in. (mm)	Spacing of Openings, s, in. (mm)	Clear Span, L, in. (m)	Distance Between Sections 1, L', in. (m)	s/d'	Comments
1.	W21x68	2	15.00 (381)	103 (2620)	160 (4.06)	103 (2.62)	n.a.	Strong web
2.	W21x68	6	15.00 (381)	20.60 (523)	160 (4.06)	103 (2.62)	1.37	Web posts yield in shear
3.	W21x50	6	16.00 (406)	20.40 (518)	160 (4.06)	102 (2.59)	1.28	Web posts yield in flexure
4.	W12x106	10	8.25 (210)	12.56 (319)	160 (4.06)	113 (2.87)	1.52	Web posts yield in shear
5.	W12x106	10	8.50 (216)	11.44 (291)	160 (4.06)	103 (2.62)	1.35	Web posts yield early in shear

FINITE ELEMENT MODELING

Beams were assumed to be part of frames under pure lateral loading (Fig. 6(a)). For these conditions, actions on the beam can be represented using the centerline model of Fig. 6. A detailed finite element mesh was used to model half the beam, consisting of a channel-shaped portion representing one half the web and half of each flange. The ends of the beam were assumed perfectly-connected to the supports.

The computer program WARP3D (1999) was used for the finite element analyses, with FEMAP (1998) providing an interface for visualization and modeling; PATWARP (1999) was used as a translator between these programs. The meshes consist of 8-node brick elements, with 2 elements through the height of the flange, 1 element through the thickness of the web, and greater nodal density around the holes. The mesh was refined to minimize shear locking of the elements. Assumed material properties are as follows: Yield Strength = 50 ksi, Elastic Modulus = 29,000 ksi, and Post-Yield Modulus = 200 ksi (see Fig. 7). Displacement increments were applied monotonically at one end of the beam, corresponding to Fig. 6(b).

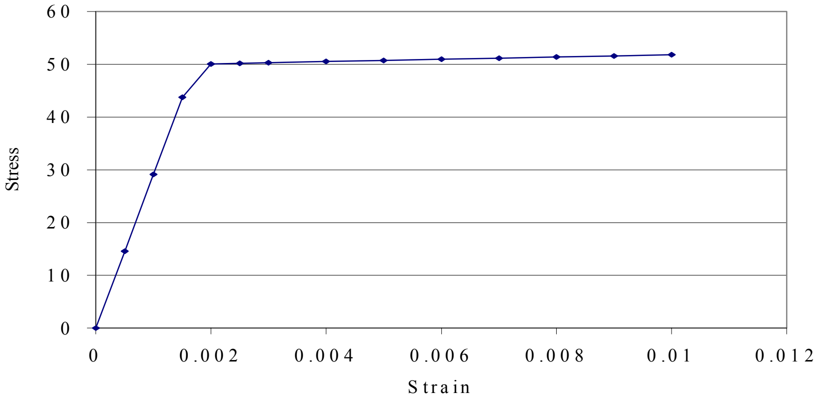


Fig. 7: Uniaxial Material Stress-Strain Relation

ANALYTICAL RESULTS

This section describes analytical results determined from the finite element models: model behavior (yield mechanisms), focusing on response, load-deflection behavior, strain concentrations, and a review of the applicability of the Hodge stress assumption.

MODEL BEHAVIOR:

Analysis of the beams in Table 1 is in progress; results are reported below for the first two beams. These W21x68 beams were designed using the Hodge approximation to yield at 66 kips by different failure mechanisms. Results for a control beam (having no perforations) are also reported.

Figures 8 - 12 present Von Mises stress contours determined at a monotonic tip displacement of 5 inches, corresponding to a drift of 3.125% in the beams alone. The grayscale stress distributions range from 0 to 75 ksi as shown in Fig. 8, applicable to Figures 8 – 12.

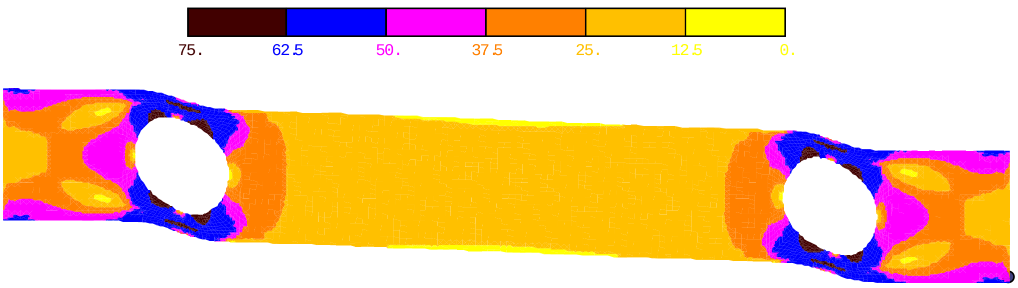


Fig. 8: Von Mises Stress Distribution at 5” Tip Displacement for Beam

Fig. 8 shows Mode A yielding for the two-hole beam. Peak stresses approach 74 ksi at this displacement. Maximum stresses at the beam-column connection were limited to 50 ksi. The modeled deformation was anticipated correctly using the Hodge criteria.

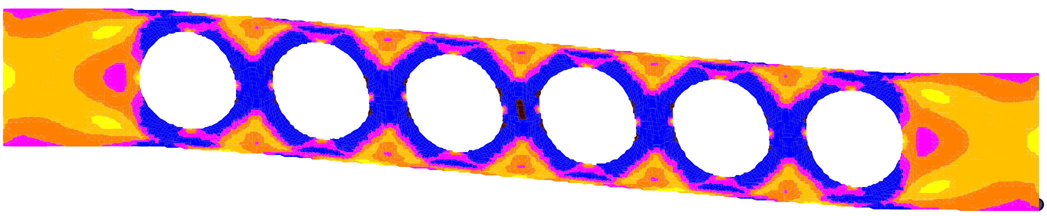


Fig. 9: Von Mises Stress Distribution at 5” Tip Displacement for Beam No. 2

Fig. 9 shows the tentatively preferred mode of yielding, developed in the 6-hole beam. Peak stresses approach 66 ksi at this displacement. Maximum stresses at the beam-column welds were 43 ksi. It is interesting to note that the high stress areas around the holes can be viewed as acting like a truss, with diagonal members either in compression or tension. This mode of deformation also was anticipated correctly using the Hodge criteria.

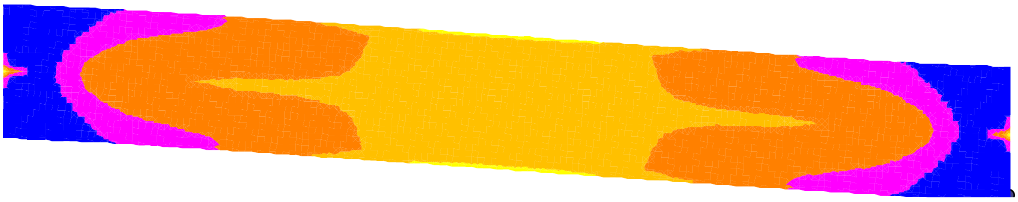


Fig. 10: Von Mises Stress Distribution at 5" Tip Displacement for the Control Beam

Fig. 10 shows the control beam (with no holes), for which plastic hinges developed at the ends. Peak stresses of 57 ksi occur at the beam-column welds. The web openings were effective at reducing the stress at the beam-column welds.

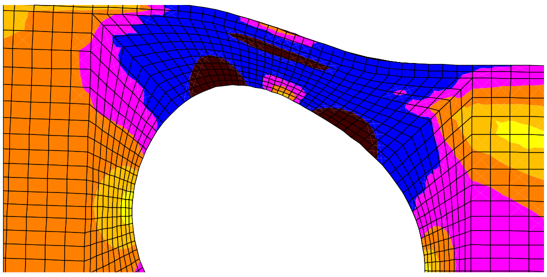


Fig. 11: Von Mises Stress Distribution at 5" Tip Displacement around hole in 2-hole beam

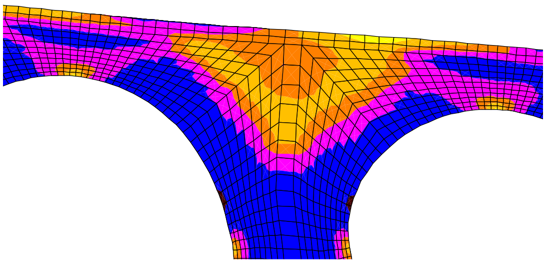


Fig. 12: Von Mises Stress Distribution at 5" Tip Displacement around 2 holes in 6-hole beam

Fig. 11 shows a close-up of one of the holes in the 2-hole beam. The highest stressed areas are not directly above the hole (Section 1), but instead are to either side of Section 1, due to local deformations imposed on the "Tee" section above the hole. Nearly the entire area above the hole yielded, while the zones between the holes remained elastic.

Fig. 12 shows a close-up of two adjacent holes in the 6-hole beam. The initial design calculations predicted web-yielding simultaneously across the top of the hole and in the web posts. The opening geometry determined using the spreadsheet calculations achieved the intended behavior, even though it predicted the entire section centered directly over the hole would yield completely.

LOAD DEFLECTION RESULTS:

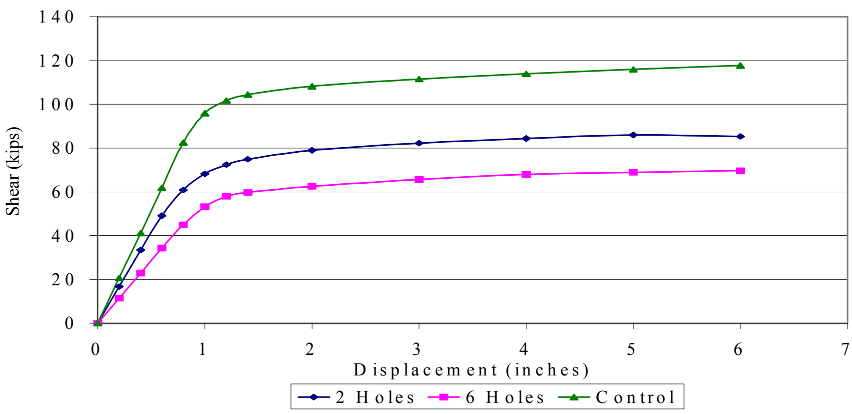


Fig. 13: Displacement vs. Shear Graph

Fig. 13 plots the computed load-displacement response of the two perforated beams and the control beam. The beams with holes are less stiff than the control beam. It is clear from Fig. 13 that the 6-hole beam yields at a smaller shear force and is not as stiff as the 2-hole beam. Both beams were designed to yield at the same time, yet the 6-hole beam is weaker.

STRAIN CONCENTRATIONS:

The different modes of deformation imply different strain concentrations; in particular, the localization of plastic strains in Mode A implies the development of large material strains for a given beam displacement. While the largest strains computed in the finite element mesh depend on the mesh descretization, these strain values still provide qualitative information on the relative behavior of the beams.

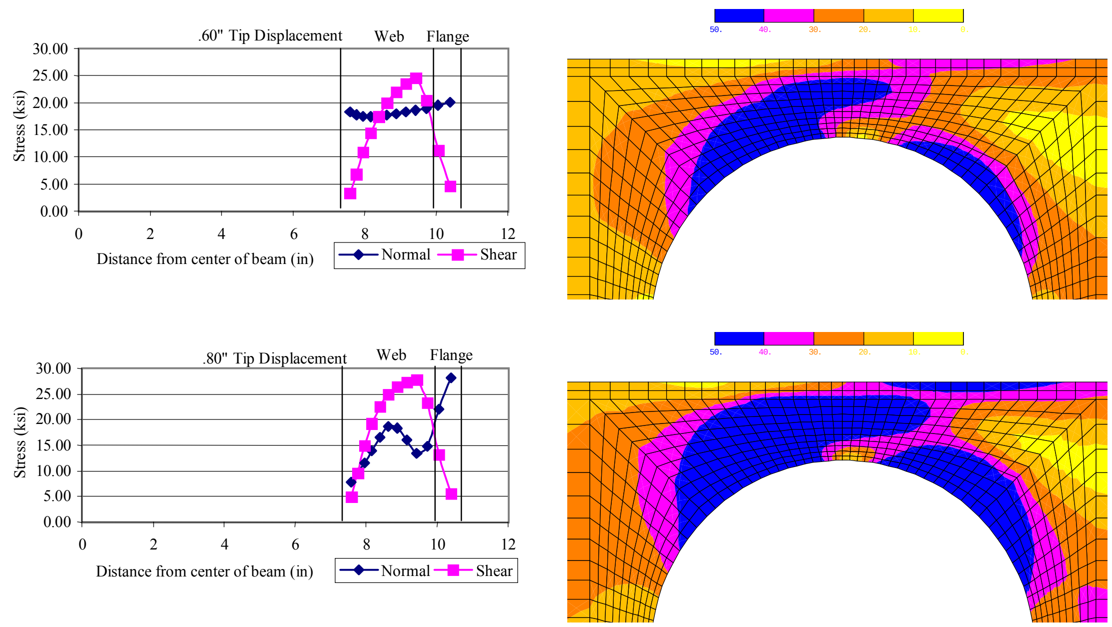
Peak Von Mises strains in the mesh are reported in Table 1 as a function of beam tip displacement for the two perforated beams and the control beam. The 2-hole beam, with Mode A behavior, has strains roughly 2 to 4 times larger than the control beam, while the 6-hole beam's peak strains were about 60% larger than the control beam.

Table 2: Maximum Von Mises Strain in Beam for a given Tip Displacement, %

	Tip Displacement (inches)					
Beam	0.40	0.80	1.00	2.00	4.00	6.00
2-Hole	0.17	0.81	1.36	4.43	9.90	14.24
6-Hole	0.10	0.26	0.40	2.83	6.84	9.58
Control (No Holes)	0.08	0.17	0.37	1.70	4.07	5.94

STRESS DISTRIBUTIONS VS. HODGE CRITERIA

Fig. 14 presents normal and shear stress distributions at Section 1 determined from the finite element analyses as a function of beam shear versus displacement for the 2-hole beam. A large shift in the normal stress distributions is apparent as the tip displacement increases. While the shear stress distributions increase in magnitude with increasing displacement, their distributions remain nearly parabolic in the web.



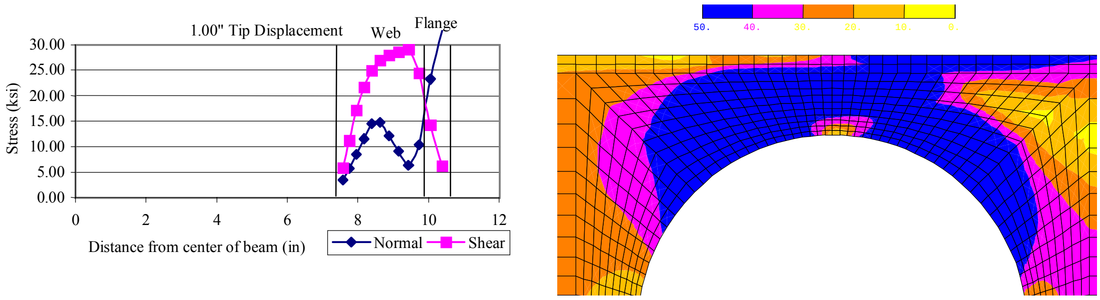


Fig. 14: Stress Distributions at Section 1 of the 2-Hole Beam

RESEARCH DIRECTIONS

The ongoing research involves analytical and experimental studies. Laboratory tests will be conducted in two phases. The first phase focuses on fundamental modes of behavior and will include the beams discussed above with minor modifications. Testing will commence in October. The second phase will focus on the modes that seem preferable for building applications, and will consider the presence of simultaneous lateral and gravity loads.

PRELIMINARY CONCLUSIONS

The finite element analyses have indicated that the initial design approach was effective in achieving the desired modes of behavior, even though some discrepancies were found between the computed and assumed stress distributions and peak weld stresses.

The data gathered up to now shows a promising system may be developed by using web holes in steel beams. While the initial proportioning criteria did not locally model the zones around the holes exactly, the criteria demonstrated an ability to model the beam actions on a global scale. Experimental tests will shed more light on the behavior of these systems and their applicability to earthquake resistant structures.

ACKNOWLEDGEMENTS

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