

## **ANALYSIS OF HIGHRISE BUILDING STRUCTURE WITH SETBACK SUBJECT TO EARTHQUAKE GROUND MOTIONS**

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### **SUMMARY**

The earthquake response behaviour of unframed highrise buildings with setbacks is still not fully understood. Many highrise buildings with setback damage were observed during recent earthquakes. The purpose of this study is to investigate the effect of setback on the dynamic behaviour of highrise buildings, as well as the development of the general analytical methods for highrise buildings.

Analysis of highrise building structures subjected to earthquake acceleration is often a complex task. Many methods have been developed for either simplifying the problem or making the analysis procedure more effective and efficient. This study presents several innovative procedures for improving the efficiency of dynamic analysis of highrise buildings. These procedures focus on the application of substructure and Rayleigh-Ritz techniques. An analysis model for evaluating the seismic response of highrise buildings with setbacks is developed.

### **INTRODUCTION**

In current practice, earthquake analysis has been required by many local authorities in the design of highrise building structures. However, engineers for sometimes have been very concerned about the response of highrise buildings under earthquake ground motions. Earthquake analysis of highrise building structures has received increasing attention in recent years by the building industry. Thus, it is important to produce economical means for such analysis.

A highrise building structure contains a large number of degrees of freedom. A detailed analysis of such a structure is expensive and difficult. In order to reduce the size of problems involving analysis of response of structural systems, two techniques have been utilised. Firstly, the use of the substructuring technique is developed. Thus, a linear elastic three dimensional analysis can be performed economically by this technique for a building of arbitrary plan. Reference [2]. Because the structural form is repeated through the height of the building, considerable economies in data preparation and in subsequent analysis can be achieved by using substructures which reflect the form of the building.

Secondly, a transformation based on appropriately selecting the Rayleigh-Ritz mode shapes is employed. The lower mode shapes may at times serve as effective Rayleigh-Ritz shapes. The success of the Rayleigh-Ritz method depends directly on how closely the mode shapes resemble the actual eigen mode of the structure. In this study, good approximations to these modes are available from those of a cantilever beam with uniformly distributed mass and stiffness [1]. From the Rayleigh-Ritz mode shapes, an elastic analysis firstly provides better approximations to real mode shapes of the building without the need for a complete eigenvalue analysis. The Rayleigh-Ritz mode shapes and technique are believed to be efficient [3], and at least as effective the mode shapes that take considerable time to compute. The use of these techniques is illustrated by examples.

A procedure and a finite element computer program have been developed using eight-node isoparametric elements to analyse unframed highrise building structures. However, for the framed highrise building structure, isoparametric element can be replaced by a frame member element, using the same processing program.

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The present study is restricted to thirty storey buildings having setbacks at two, five and seven levels along their height. The horizontal earthquake motions of the structure are considered. Further, the structure is assumed to be linearly elastic and viscously damped. The model of a building is used as a “shear wall building”. The effect of the setback levels on the modal quantities and the responses to the El-Centro and the Hanshin recorded earthquakes record are examined.

## 2. FINITE ELEMENT METHOD

In the planar substructure, the local stiffness matrix is obtained from the assemblage of eight-node isoarametric plane elasticity elements. The transformation from global to local coordinates of a membrane plane elasticity element is given by the equation

$$\mathbf{r}' = \mathbf{Lr} \quad (1)$$

where  $\mathbf{r}'$  is the local coordinate displacement vector,  $\mathbf{L}$  is the transformation matrix and  $\mathbf{r}$  is the global coordinate displacement vector. For the plane element, the local force-displacement equation can be written as

$$\mathbf{F}' = \mathbf{K}'\mathbf{r}' \quad (2)$$

where  $\mathbf{K}'$  is the (2n x 2n) corresponding stiffness matrix for the substructure,  $\mathbf{F}'$ ,  $\mathbf{r}'$  are the local axes force and displacement vectors.

For the same element the relationship between global force  $\mathbf{F}$  and displacement  $\mathbf{r}$  can be written as

$$\mathbf{F} = \mathbf{K}\mathbf{r} \quad (3)$$

In which  $\mathbf{K}$  is the global stiffness matrix obtained from  $\mathbf{K}'$  using the transformation matrix  $\mathbf{a}$  by

$$\mathbf{K} = \mathbf{a}^T \mathbf{K}' \mathbf{a} \quad (4)$$

where  $\mathbf{a}$  is a matrix composed of  $\mathbf{L}$  matrix as a diagonal submatrix blocks.

## 3. RAYLEIGH-RITZ TECHNIQUE

The Rayleigh-Ritz technique is used to reduce the degrees of freedom of the structure. For the shear wall building, the method assumes that the deflection can be written as a sum of the independent functions,

$$\mathbf{r} = \mathbf{A}\mathbf{q} \quad (5)$$

where columns of  $\mathbf{A}$  are the assumed mode shapes. Each of vectors in  $\mathbf{A}$  should be an approximation to the corresponding true vibration mode shape and  $\mathbf{q}$  is the vector of generalized coordinate amplitudes.

Then for free harmonic oscillations, deflections are given by,

$$\mathbf{x} = \mathbf{r} \cos(\boldsymbol{\omega}t + \boldsymbol{\phi}) \quad (6)$$

and accelerations by,

$$\ddot{\mathbf{x}} = -\boldsymbol{\omega}^2 \mathbf{r} \cos(\boldsymbol{\omega}t + \boldsymbol{\phi}) \quad (7)$$

The nodal forces are proportional to the maximum values of the inertia force,

$$\mathbf{X} = -\mathbf{M} \ddot{\mathbf{x}} = \boldsymbol{\omega}^2 \mathbf{M} \mathbf{A} \mathbf{q} \cos(\boldsymbol{\omega}t + \boldsymbol{\phi}) \quad (8)$$

Since the structure is in a state of free vibration, the only disturbing influence on the elastic system is the set of inertia forces. The structure deflections of the nodes may be calculated by applying the inertia forces. With  $\mathbf{F} = \mathbf{K}^{-1}$ ,  $\mathbf{F}$  being the structure flexibility matrix,

$$\mathbf{r} = \mathbf{F} \mathbf{X} = \boldsymbol{\omega}^2 \mathbf{F} \mathbf{M} \mathbf{A} \mathbf{q} \quad (9)$$

Use the substitution,

$$\boldsymbol{\phi} = \mathbf{F} \mathbf{M} \mathbf{A} \quad (10)$$

Instead of the original shapes  $\mathbf{A}$ , the newly calculated values  $\boldsymbol{\phi}$  are used as the approximations to the mode shapes. The vibrations in matrix form for free undamped oscillations are written

$$\mathbf{K} \mathbf{r} = \boldsymbol{\omega}^2 \mathbf{M} \mathbf{r} \quad (11)$$

Substituting for the vector  $\mathbf{r}$  of Equations (9) and (10) in terms of  $\mathbf{q}$

$$\mathbf{K} \boldsymbol{\phi} \mathbf{q} = \boldsymbol{\omega}^2 \mathbf{M} \boldsymbol{\phi} \mathbf{q} \quad (12)$$

Pre-multiply both sides of Equation (12) by  $\boldsymbol{\varphi}^T$ , then

$$\boldsymbol{\varphi}^T \mathbf{K} \boldsymbol{\varphi} q = \omega^2 \boldsymbol{\varphi}^T \mathbf{M} \boldsymbol{\varphi} q \quad (13)$$

$$\boldsymbol{\varphi}^T \mathbf{K} \boldsymbol{\varphi} = \mathbf{A}^T \mathbf{M} \mathbf{F} \mathbf{K} \boldsymbol{\varphi} = \mathbf{A}^T \mathbf{M} \boldsymbol{\varphi} \quad (14)$$

From Equation (13) and (14), let

$$\mathbf{K}^{**} = \mathbf{A}^T \mathbf{M} \boldsymbol{\varphi} \quad (15)$$

$$\mathbf{M}^{**} = \boldsymbol{\varphi}^T \mathbf{M} \boldsymbol{\varphi} \quad (16)$$

The matrices  $\mathbf{K}^{**}$  and  $\mathbf{M}^{**}$  may be considered to be generalized stiffness and mass matrices. Then Equation (13) can be written as

$$\mathbf{K}^{**} q - \omega^2 \mathbf{M}^{**} q = 0 \quad (17)$$

Solve for eigenvalues  $\boldsymbol{\omega}$  and eigenvector  $\boldsymbol{\phi}_q$ . Then the mode shapes are

$$\boldsymbol{\phi}_r = \boldsymbol{\varphi} \boldsymbol{\phi}_q \quad (18)$$

The coordinates  $q$  may now be expressed in terms of the orthogonal (uncoupled) principal coordinates. That is,

$$q = \boldsymbol{\phi}_q Y \quad (19)$$

where  $Y$  are the modal amplitudes.

Finally, the global structure displacements can be calculated for each active node.

$$r = \boldsymbol{\varphi} \boldsymbol{\phi}_q Y \quad (20)$$

The eigenvalue problem can be uncoupled because of the orthogonality conditions.

### 3. NUMERICAL EXAMPLES OF THIRTY STOREY BUILDINGS WITH SETBACK

In principle, the method and technique can be applied to all the problems for which reduced models are required for earthquake analysis. Here, an uniformed building and three buildings with setback incorporated at 2, 5 and 7 storey levels in thirty stories are demonstrated, shown in Figure 1. These buildings, 100 m height and idealized as reinforced concrete shear walls and connected by the floor of the structure, are assumed to be supported on rigid ground and have the same properties throughout. The modal damping ratio for all four natural vibration modes was assumed to be 5%.

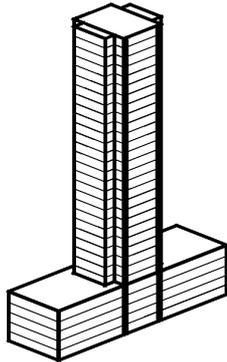


Figure 1: Layout of a 30 storey setback building

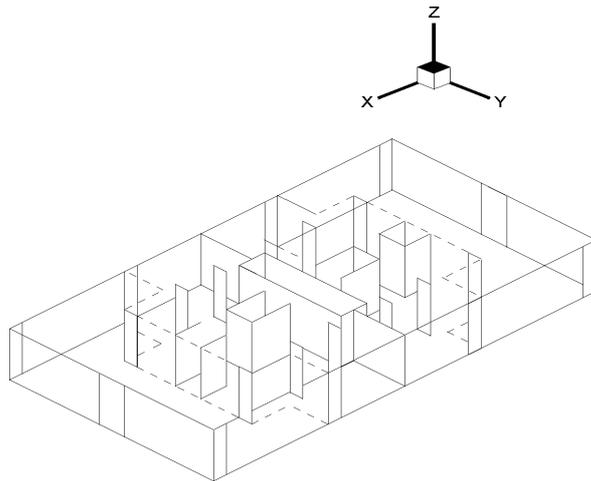
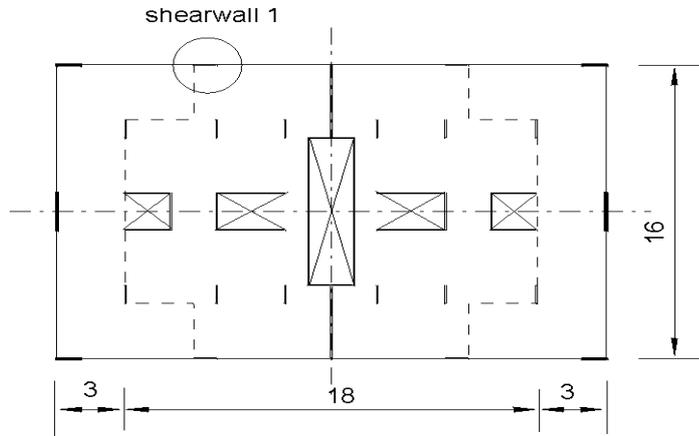


Figure 2: Structure idealisation



**Figure 3: Floor plan of setback building**

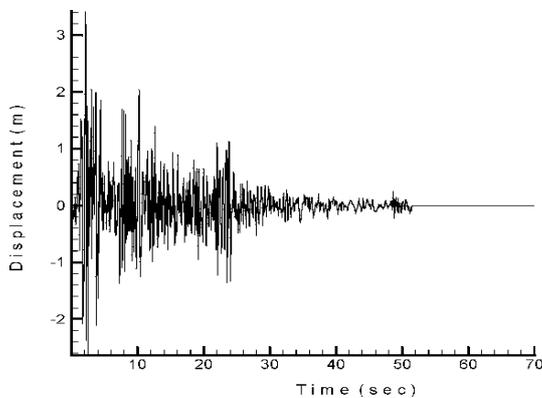
### Structure idealisation

Each storey type of the setback building of the tower portion is same as the uniform building, it is firstly modelled with an assemblage of properly selected substructures. Typical floor and shear wall substructure is shown in Figures 2. The between floor shear walls are composed of 41 elements shown in Figure 3. The floor substructure has 72 primary element for setback buildings.

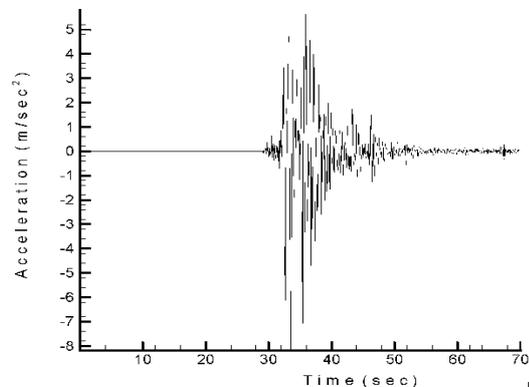
**The procedure of the analysis is as follows:**

1. The static analysis of a three dimensional structure using the finite element method and substructure technique.
2. Characteristic eigenvalues from assumed mode shapes (four modes were used) by the Rayleigh-Ritz method, obtaining generalized stiffness and mass matrices.
3. Computation of the generalized displacement time histories for generalized forces obtained from earthquake acceleration records. Back-substitution into the mode shapes to obtain the time histories of the global displacements.
4. Back-substitution into local Cartesian coordinates to obtain deflection of internal nodes, and calculation of element stresses.

The earthquake accelerations used for the study were those of those for the South-East component of 1940 El Centro earthquake, and North component of January 17, 1995 Hanshin earthquake in Japan, shown in Figures 4 and 5. The time interval in the numerical analysis of the earthquake chosen for study was 0.02 seconds. The duration for both ground motions is taken to be 70 seconds.



**Figure 4: 1940 El Centro (S00E)**



**Figure 5: 1995 Hanshin (N-S)**

The structural mode shapes are shown in Figure 6. Mode shapes of the building give some indication of the dynamic deflections of the structure. All four mode shapes appear in same plot of the uniform and setback (5 levels) buildings.

In the first and second mode shapes, one consistency can be seen in the way the amplitudes of displacement of the mode shapes of the setback building are almost the same as the uniform building. In the third and fourth mode shapes in the way of setback buildings is greater than the uniform building. In general, the effect of setback on the mode shapes is to increase the amplitude of displacements. This observation is born out with the third, fourth mode shapes.

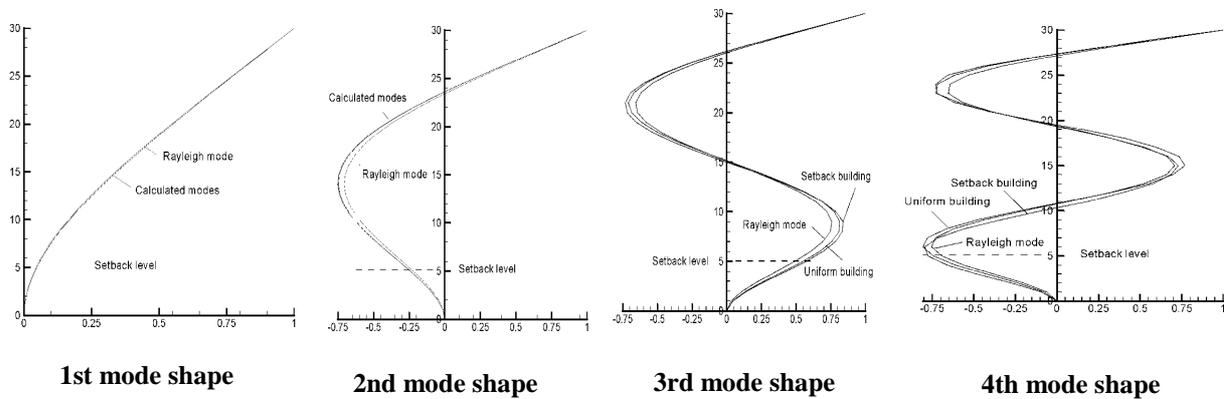


Figure 5: Mode shapes –X-axis

The earthquake response of the structure is obtained by combining the modal response in all four modes of vibration. Figures 6 and 7 show the response of buildings in total modes with five levels of setback subjected to the El Centro and Hanshin earthquake accelerations at selected time.

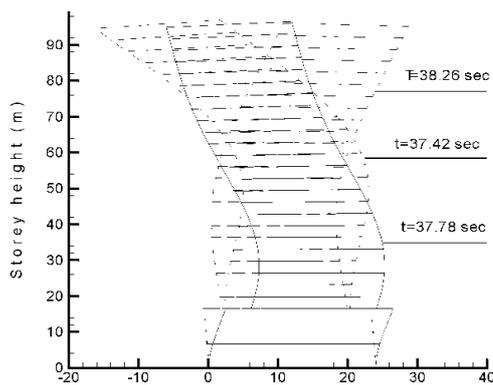


Figure 6: Structure deflected shapes subjected to Hanshin earthquake

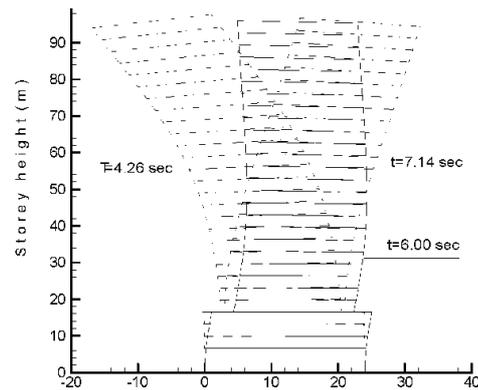


Figure 7: Structure deflected shapes subjected to El Centro earthquake

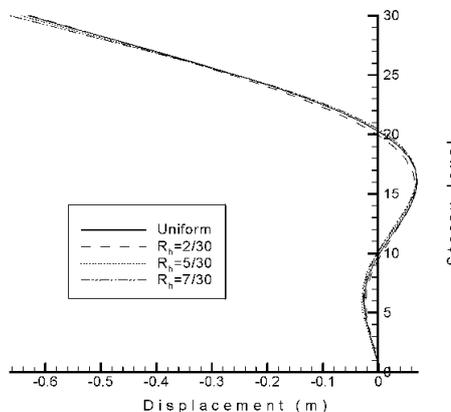


Figure 8: Maximum displacement of buildings subjected to Hanshin earthquake at t=37.42 sec

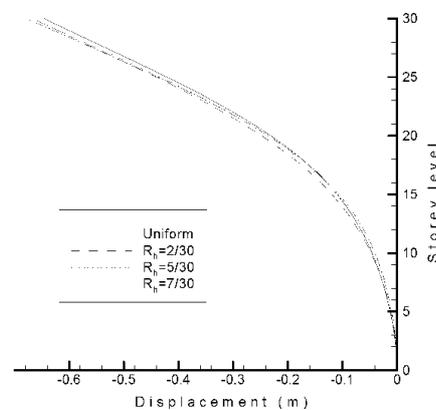


Figure 9: Maximum displacement of buildings subjected to El Centro earthquake at t=4.26 sec

It can be seen from Figures 8 and 9 that maximum displacement of the building is close to the first mode in the x-axis direction when the building is subjected to El Centro earthquake; that the building vibrates close to third mode of vibration when it subjected to Hanshin earthquake. The Hanshin earthquake is unusual in this respect in the response in the modes higher than the first. This probably explains some of failure experienced in buildings. This implies Resonance in the third mode. Horizontal shears will thus be much larger in the response to the Hanshin earthquake than to El Centro.

The distribution of stresses can be calculated for each shear wall. Tables 1 and 2 show maximum compressive stresses of shear wall 1 only. Comparing the maximum stresses of both buildings at the same point the setback building stresses are greater than those in the uniform building. For a setback structure identified as being irregular, and for which stresses tower damage is deemed likely, the design should impose increased strength on the tower relative to the base. From this analysis it is also possible to identify the location in the building shear walls where significant cracking of concrete can be expected. However, the difference in base stresses between different building depends very much on the nature of the earthquake.

**Table 1: Subjected to El Centre earthquake at t=4.26 sec.**

Building type	Level	Max. compressive value (KPa)
Uniform	15	-1599
$R_h=2/30$	21	-1589
$R_h=5/30$	20	-1864
$R_h=7/30$	19	-2225

**Table 2: Subjected to Hanshin earthquake at t=37.42 sec.**

Building type	Level	Max. compressive value (KPa)
Uniform	20	5261
$R_h=2/30$	22	4652
$R_h=5/30$	19	5638
$R_h=7/30$	7	9428

Note:  $R_h$  is the level of setback ratio.

## 5. CONCLUSIONS

This study has employed the Rayleigh-Ritz method combined with a substructure technique and simplified linear response analysis to evaluate the behavior of the thirty storey buildings subjected to large horizontal shaking during the El Centro and Hanshin earthquake. These methods are useful for highrise building systems that have many degrees of freedom and a large structure can be effectively and economically reduced. A complete finite element program that has substructure capabilities and degrees of freedom condensation of the global structure for static and dynamic analysis has been developed for the above studies.

## 6. REFERENCES

- [1] Biggs, John M. *Introduction to Structural dynamic*, McGraw-Hill, 1964.
- [2] Meek, John L. *Computer Methods in structure analysis*. E & FN SPON, 1991.
- [3] Meek, John L. and Zhang, Xiaojun 'Approximate analysis of unframed highrise buildings for seismic loading', *The fifth international conference on tall buildings*, Hong Kong, Vol. 2, 1998, pp. 675-680.