ANALYSIS OF STRUCTURES CONTROLLED WITH A NEW VARIABLE DAMPING SEMI-ACTIVE (VDSA) DEVICE

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ABSTRACT :

A new semiactive device is proposed in this paper. It is composed of two fixed-orifice viscous fluid dampers installed in the form of *V* whose top ends are attached to the upper floor and their lower ends to a point that can move along a vertical rod. The mechanism is termed the Variable Damping Semi-Active (**VDSA**) device. By varying its position, an optimal instantaneous damping added to the structure to minimize the response is obtained. The position of the moving end of the device, an algorithm based on a variation of the Instantaneous Optimal control Theory and which includes a generalized *LQR* scheme. This modified algorithm, referred to as *Qv,* is based on the minimization of the performance index *J* quadratic in the state vector, the control force vector, and a vector of absolute velocity measured at selected points. Two variants of the algorithm are used to compute the seismic response of a single and a multi degree of freedom structure.

KEYWORDS: Structural Control, Earthquake, Semiactive Control, Control Algorithm, Damping.

1. INTRODUCTION

The traditional approached followed for the design of civil engineering structures, which is depends upon their strength and ductility to withstand the large forces imposed by strong earthquakes, is now slowly changing. One of the agents of change is the modern mechanical devices proposed in the last two decades aim at reducing the structural response. They are known collectively as protective devices and they include added viscoelastic dampers, viscous fluid dampers, frictional dampers, tuned-mass dampers, and base isolation systems. The devices themselves and their design methodology are referred to as passive control systems. At the highest level of sophistication for seismic protection are the active control systems.

There is an intermediate alternative between passive and fully active control systems: they are referred to as semi-active systems, and they are the topic of this paper. As its name indicates, a semi-active control scheme combines the features of active and passive systems to reduce the dynamic response of structures. The semiactive systems, in turn, can be divided into two types: active variable stiffness and active variable damping devices.

Semi-active control systems have only recently been considered for applications to large civil structures. We believe that the first application of these systems to civil engineering structures was reported by Hrovat et al. (1983). Several changeable damping devices, such as variable orifice dampers (Symans and Constantinou 1997, Kurata et al. 1999) and hydraulics dampers (Kawashima et al. 1992, Patten et al. 1996) have been developed. Variable stiffness devices have also proposed by (Kobori et al. 1993, Nagarajaiah et al. 1998, Gluck et al. 2000).

The present paper describes the implementation of a new variable damping semi-active control (VDSA) device. Contrary to semi-active dampers described in the technical literature, the damper coefficient c is not controlled by modifying the size of an orifice in the piston, but by changing the position of the damper. The required damping coefficient is calculated by means of two instantaneous optimal control algorithms (*closed-loop control* and *closed-open-loop control*). It is shown that both algorithms are effective in reducing the response.

The damping coefficient $c(t)$ can be adjusted between an upper limit c_{max} and a lower value c_{min}. The effectiveness of the proposed device is verified via numerical simulations.

2. THE MODIFIED ALGORITHM Q_V

The equations of motion of a structure modeled as a multi-degree of freedom (MDOF) system, outfitted with r semi-active dampers, and subjected to a base acceleration $\ddot{x}_g(t)$ at all its supports is:

$$[M]_{nxn}\{\ddot{x}(t)\}+[C]_{nxn}\{\dot{x}(t)\}+[K]_{nxn}\{x(t)\}=-[M]_{nxn}\{E\}\ddot{x}_g(t)+[D]_{nxr}\{u(t)\} \tag{1}$$

where $[M]$, $[C]$ y $[K]$ are the mass, damping and stiffness matrix, respectively, the vectors $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$ and $\{x(t)\}$ contain the acceleration, velocity and displacement of each dynamic degree of freedom of the structure, $\{E\}$ is the vector of influence coefficients, and n is the number of degrees of freedom. The matrix $[D]$ defines the locations of the controllers, r is the number of controllers and $\{u(t)\}$ is the r-dimensional control force vector.

To solve the system of equations of motion (1) by transforming them into a set of uncoupled equations, it is convenient to change it into a system of $2n$ first order differential equations. Introducing the following response vector and matrices,

$$\{z(t)\}=\left\{\begin{array}{c}\{x(t)\}\\ \hline \{\dot{x}(t)\}\end{array}\right\} \quad ; \quad [A]=\left[\begin{array}{c|c}0 & I\\ \hline -M^{-1}K & -M^{-1}C\end{array}\right] \quad ; \quad [B]=\left[\begin{array}{c}0\\ \hline M^{-1}D\end{array}\right] \quad ; \quad [H]=\left[\begin{array}{c}0\\ \hline -E\end{array}\right] \tag{2}$$

Eq. 1 can be written in the form:

$$\{\dot{z}(t)\}_{2nx1}=[A]_{2nx2n}\{z(t)\}+[B]_{2nx2n}\{u(t)\}+[H]_{2nx1}\ddot{x}_g(t) \tag{3}$$

To define the variation of the control forces in $\{u(t)\}$ one needs to select a control algorithm. In this study, two algorithms (closed-loop control and closed-open-loop control) have been developed based on the Instantaneous Optimal Control Theory. They are referred here as the modified algorithms Qv. As usual, this type of algorithms is based on the minimization of a performance index J quadratic in the state vector $\{z(t)\}$ and in the control force $\{u(t)\}$. However, in the modified algorithm a quadratic form of the absolute velocity is added a J. A penalty is imposed through the matrix **Q** on the state vector, through a matrix **R** on the control vector and through a matrix **Qv** on the absolute velocity vector. **Q** and **Qv** are two symmetric positive semi-definite weighting matrices of size $2n$ x $2n$ and n x n, respectively, and **R** is an r x r positive definite weighting matrix. The performance index takes the form:

$$J=\int_0^{t_f}\left[\{z(t)\}^T[\mathbf{Q}]\{z(t)\}+\{\dot{x}_a(t)\}^T[\mathbf{Q_v}]\{\dot{x}_a(t)\}+\{u(t)\}^T[\mathbf{R}]\{u(t)\}\right]dt \tag{4}$$

where t_f is the duration of excitation.

The absolute velocity vector is computed as

$$\{\dot{x}_a(t)\}=[A_v]_{nx2n}\{z(t)\}+\{S_v\}_{nx1}\dot{x}_g(t) \tag{5}$$

where $[A_v]=[0 \mid I]$, $\{S_v\}=\{1\}$, [I] is an nxn identity matrix and $\{1\}$ is a vector of 1's of length n, and $\dot{x}_g(t)$ is the ground velocity.

The procedure to define the control and response vectors in the modified algorithm Qv can be found in Cundumi (2005). Here only the final results are reported. For the Closed-loop control case, the variables $\{u(t)\}$ and $\{z(t)\}$ can be obtained as follows:

$$\{u(t)\} = -\frac{\Delta t}{2}[R]^{-1}[B]^T\left[[A_2]\{z(t)\}+[A_3]\dot{x}_g(t)\right] \tag{6}$$

$$\{z(t)\}=\left[[I]+\frac{\Delta t^2}{4}[B][R]^{-1}[B]^T[A_2]\right]^{-1}\left[\begin{array}{r}[T]\{d(t-\Delta t)\}-\dfrac{\Delta t^2}{4}[B][R]^{-1}[B]^T[A_3]\dot{x}_g(t)\\ +\dfrac{\Delta t}{2}[H]\ddot{x}_g(t)\end{array}\right] \tag{7}$$

where Δt is the constant time step, $A_2 = Q + A_V^T Q_V A_V$, $A_3 = A_V^T Q_V S_V$ and $\{d(t-\Delta t)\}$ contains all the dynamic quantities at time t-Δt.

For the Closed-open-loop control case, $\{u(t)\}$ and $\{z(t)\}$ are calculated with the following equations:

$$\{u(t)\}=\frac{\Delta t}{4}[R]^{-1}[B]^T\left[[P]\{z(t)\}+\{p(t)\}\right] \tag{8}$$

$$\{z(t)\}=\left[[I]-\frac{\Delta t^2}{8}[B][R]^{-1}[B]^T[P]\right]^{-1}\left[\begin{array}{r}[T]\{d(t-\Delta t)\}+\dfrac{\Delta t^2}{8}[B][R]^{-1}[B]^T p(t)\\ +\dfrac{\Delta t}{2}[H]\ddot{x}_g(t)\end{array}\right] \tag{9}$$

In Eq. 8 and 9, P is the Riccati matrix and $p(t)$ represents the Open-loop control.

$$[P]=-\left[[Q]+2[A_V]^T[Q_V][A_V]\right]\left[[I]+\frac{\Delta t^2}{8}[Q][B][R]^{-1}[B]^T\right]^{-1} \tag{10}$$

$$\{p(t)\}=-\left[\frac{\Delta t^2}{8}[Q][B][R]^{-1}[B]^T+[I]\right]^{-1}\left[\begin{array}{r}[Q]\left[[T]\{d(t-\Delta t)\}+\dfrac{\Delta t}{2}[H]\ddot{x}_g(t)\right]\\ +2[A_V]^T[Q_V]\{S_V\}\dot{x}_g(t)\end{array}\right] \tag{11}$$

3. EQUATIONS OF MOTION OF STRUCTURES CONTROLLED WITH THE VDSA DEVICE

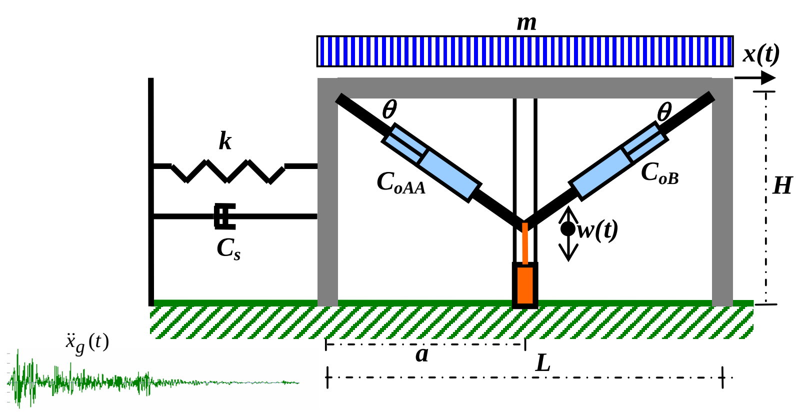

Figure 1 Single degree of freedom model of a structure with a VDSA device.

To present the concept in a simple way, the single degree of freedom system with a VDSA device shown in Fig. 1 is considered first. The model is an idealization of a one-story building with a mass m distributed at the roof level and a massless frame that provides a stiffness k to the system. The natural (or inherent) damping of the structure is represented by a damper with constant C_s. The dampers of the VDSA device have fixed damping coefficients Co_A and Co_B.

The velocities $\dot{x}(t)$ of the mass and $\dot{w}(t)$ of the lower end of the device are shown in Fig. 2. Using Fig. 2, it is straightforward to show that the equation of motion for the structure subjected to an horizontal ground acceleration $\ddot{x}_g(t)$ is:

$$m\ddot{x}(t)+\left(C_s+\left(C_{O_A}+C_{O_B}\right)\cos^2\theta(t)\right)\dot{x}(t)+kx(t)=-m\ddot{x}_g(t)+\frac{1}{2}\left(C_{O_A}-C_{O_B}\right)\sin 2\theta(t)\,\dot{w}(t) \qquad (12)$$

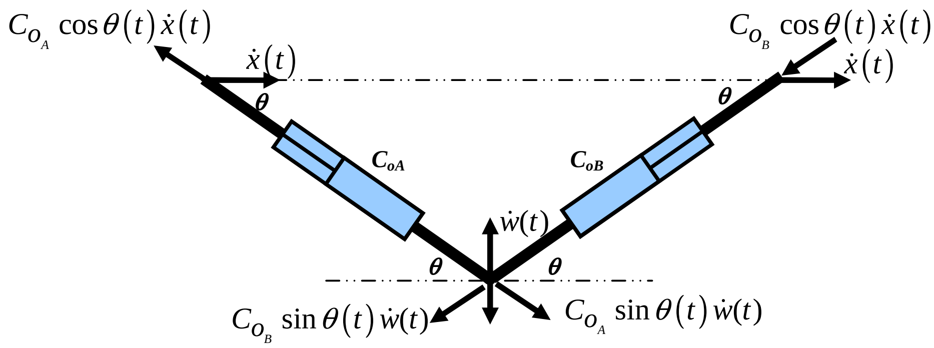

Figure 2 End velocities of the VDSA device installed in a SDOF structure.

where: $\cos^2\theta(t)=\dfrac{a^2}{a^2+\left[H-w(t)\right]^2}$; $\sin 2\theta(t)=\dfrac{a\left[H-w(t)\right]}{a^2+\left[H-w(t)\right]^2}$; $a=\dfrac{L}{2}$.

For a structure with two dampers in a fixed position, the second term in the right hand side of the equation of motion (12) vanishes. This term arises due to the component of the velocity of the lower end of the dampers in the direction of the axis of the device. Rewriting Eq. 12 in a space-state representation leads to

$$\begin{Bmatrix}\dot{z}_1(t)\\ \dot{z}_2(t)\end{Bmatrix}=\left[\begin{array}{c|c}0 & 1\\ \hline -m^{-1}k & -m^{-1}\left(C_s+\left(C_{O_A}+C_{O_B}\right)\cos^2\theta(t)\right)\end{array}\right]\begin{Bmatrix}z_1(t)\\ z_2(t)\end{Bmatrix}+\begin{bmatrix}0\\ \hline \frac{1}{2}m^{-1}\left(C_{O_A}-C_{O_B}\right)\sin 2\theta(t)\end{bmatrix}\dot{w}(t)+\begin{bmatrix}0\\ \hline -1\end{bmatrix}\ddot{x}_g(t) \qquad (13)$$

where $z_1(t)=x(t)$ and $z_2(t)=\dot{x}(t)$.

These equations can be solved by decoupling them with the complex eigenvectors of the matrix in the right hand side, provided that the displacement $w(t)$ of the bottom support of the dampers is known. The term $w(t)$ is determined by using one of the two modified algorithms Qv described in the previous section. For practical reasons, the position $w(t)$ of the common joint of the VDSA device (which governs the damping provided to the structure), must be limited between two w_{min} and w_{max} values.

The application of the VDSA device to a multi-degree of freedom systems is similar to the SDOF case. When the VDSA device is installed in a MDOF structure at a given floor (other then the first one), the damping force is related to the velocities of two consecutive floors $\dot{x}_i(t)$ and $\dot{x}_{i+1}(t)$, in addition to $\dot{w}(t)$. The equation of motion for a MDOF system with the device installed between the i^{th} and $(i+1)^{th}$ floor is

$$[M]\{\ddot{x}(t)\}+\left([C_s]+[C_1]+[C_2]\right)\{\dot{x}(t)\}+[K]\{x(t)\}=-[M]\{r\}\ddot{x}_g(t)-\{D\}\dot{w}(t) \qquad (14)$$

For a model with one degree of freedom per floor, the matrices $[C_1]$ and $[C_2]$, and the vector $\{D\}$ can be defined in terms of three vectors with only one or two non-zero elements. These vectors $\{e_1\}$, $\{e_2\}$ and $\{e_3\}$ with length n are:

$$\{e_1\}^T = [0,0,...,0,1,0,...,0] \text{ with 1 at column } "i+1"$$
$$\{e_2\}^T = [0,0,...,0,-1,0,...,0] \text{ with -1 at column } "i" \tag{15}$$
$$\{e_3\}^T = [0,0,...,0,-1,1,...,0] \text{ with -1 at column } "i", \text{ 1 at column } "i+1"$$

Using the three vectors in Eq. 15, the matrices $[C_1]$ and $[C_2]$ and the vector $\{D\}$ can be written as:

$$[C_1] = \left(C_{o_A} + C_{o_B}\right)\cos^2\theta(t)\{e_1\}\{e_3\}^T \quad ; \quad [C_2] = \left(C_{o_A} + C_{o_B}\right)\cos^2\theta(t)\{e_2\}\{e_1\}^T$$
$$\{D\} = \frac{1}{2}\left(C_{o_A} - C_{o_B}\right)\sin 2\theta(t)\{e_1\} \tag{16}$$

A detailed study of application of the VDSA device in multi-degree of freedom systems is presented by Cundumi (2005).

4. NUMERICAL EXAMPLE

To demonstrate the effectiveness of the VDSA device in reducing the seismic response, a MDOF structure is used as examples The response obtained by applying the *closed-loop* and the *closed-open-loop control* algorithms *Qv* are compared to the response of the uncontrolled structures. In addition, the response of the structures fitted with passive dampers was included in the comparisons. The structure was subjected to the horizontal component of three earthquakes: the 1940 El Centro, California (PGA = 0.348g), the 1971 San Fernando, California (PGA = 1.007g) and the 1976 Friuli, Italy, (PGA = 0.478g) records.

A model of a six-story building with one DOF per floor is used. The lateral stiffness coefficients of the columns are k_i = 5315 kip/in and the floor weights are W_i = 2205 kip for all floors. To obtain the response of the uncontrolled structure the damping ratio is assumed to be 5% for all modes. The natural periods of the structure are 0.5309, 0.1805, 0.1126, 0.0855, 0.0723 and 0.0659 sec. The weighting matrix **Q** and **Qv** are selected as $[I]\text{x}10^4$ and $[I]\text{x}10^2$, respectively, where $[I]$ is identity matrix, and **R** is selected as a scalar equal to 10^{-1}. The same values were used for *closed-loop* and *closed-open-loop control* algorithms. The coefficients of the dampers A and B in the VDSA device are 25 kip.sec/in and 10 kip.sec/in, respectively. The VDSA device was installed in the fourth floor. This location was selected by trial and error to maximize the response reduction.

Figures 3 and 4 show the relative displacement time histories of the first and top floors of the uncontrolled structure and controlled with the VDSA device, for the El Centro and San Fernando record, respectively. The results presented here were obtained by using *closed-loop control* modified algorithm *Qv*. Fig. 5 displays the variation of the bottom end of the semiactive device for the El Centro and San Fernando.

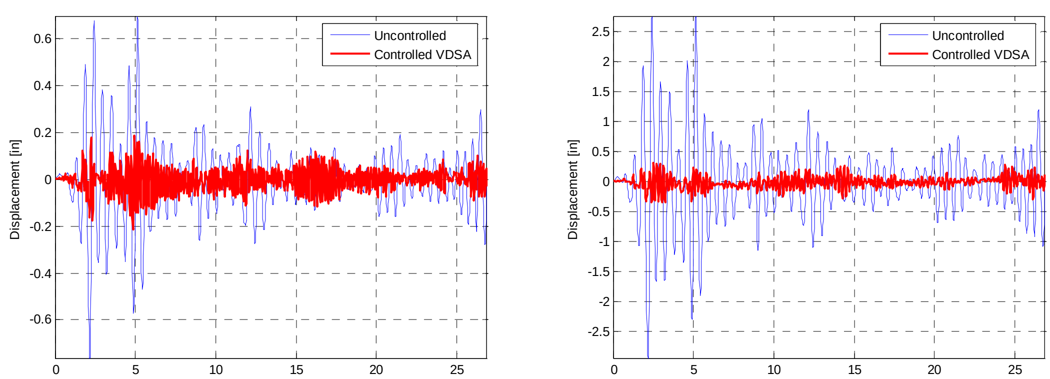

Figure 3:Relative displacements of the 6-story building for the El Centro record
(a) First floor, and (b) Top floor - Uncontrolled vs. VDSA (*closed-loop control*).

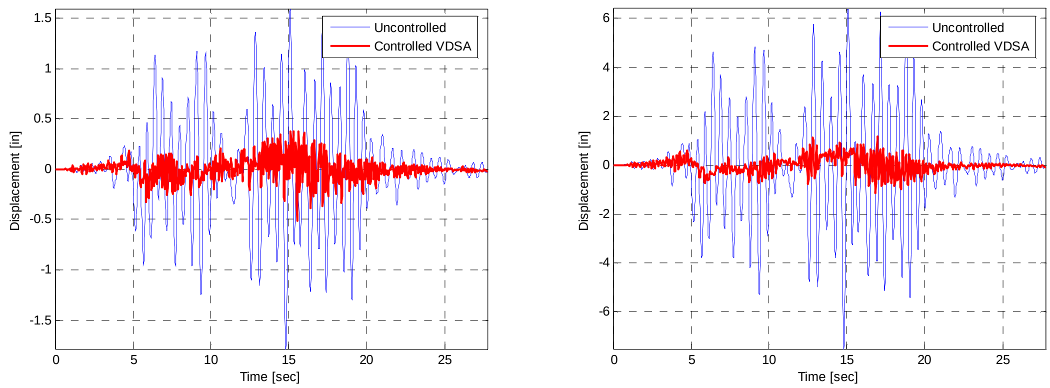

Figure 4 Relative displacements of the 6- story building for the San Fernando record
(a) First floor, (b) Top floor - Uncontrolled vs. VDSA (*closed-loop control*).

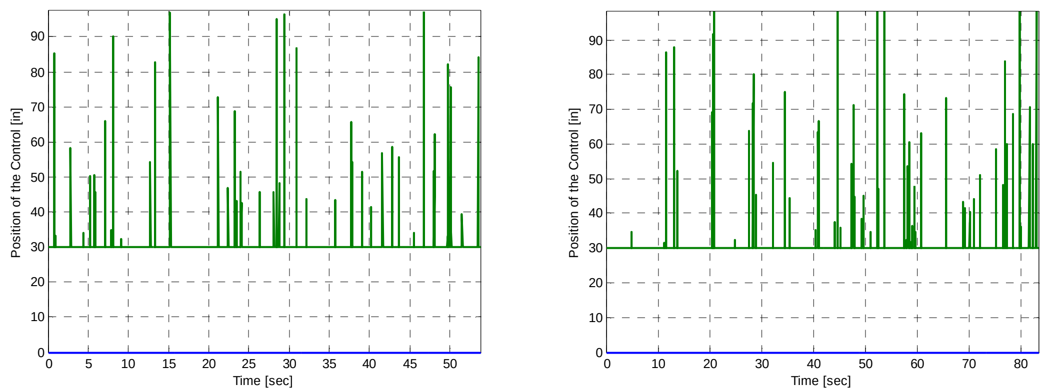

Figure 5 Variation of the position of the VDSA device in the 6-story building for the
(a) El Centro, and (b) San Fernando records.

Table 1 shows a summary of the maximum relative displacements of the 6-story building subjected to the El Centro, San Fernando and Friuli records, when the VDSA device is controlled with the *closed-loop* algorithm. A reduction in the top floor displacement of 88% for El Centro, 84.4% for San Fernando and 92.8% for Friuli was obtained with the VDSA device installed in the fourth floor. Similar reductions were obtained for the lower floors. For instance, for the first floor the reduction was of 71.7, 70.9 and 87.2% for the El Centro, San Fernando and Friuli records, respectively.

Table 1 Maximum displacements of the 6-story building without control and controlled with the VDSA
For the El Centro, San Fernando and Friulli records (*closed-loop control*).

	Displacement [inches]					
	El Centro record		**San Fernando record**		**Friuli record**	
Floor	**Uncontrolled**	**Controlled**	**Uncontrolled**	**Controlled**	**Uncontrolled**	**Controlled**
6th	2.9392	0.3519	7.5278	1.1778	5.9511	0.4279
5th	2.7857	0.2523	7.0656	0.8003	5.6169	0.2710
4th	2.4832	0.1415	6.1855	0.5192	4.9605	0.1322
3rd	2.0359	0.2733	4.9567	0.5854	4.0097	0.2100
2nd	1.4544	0.3198	3.4634	0.7011	2.8128	0.2690
1st	0.7622	0.2154	1.7855	0.5192	1.4521	0.1858

5. CONCLUSIONS

A semiactive dampers system referred to as the VDSA device consisting of two dampers in a ***V*** configuration was proposed as a seismic protective system. The bottom joint of the device moves up and down in such a way to achieve optimal damping in the structural system at each instant of time. The results of the numerical simulations indicate that the VDSA device is capable of significantly attenuating the seismic response of the single and multiple degree of freedom models. Two optimal control methodologies, the *closed-loop* and the *closed-open-loop control* modified algorithms *Qv*, were proposed applied to define the position of the movable end of the VDSA device.

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